$f(t)$ sinônimos

$f = f(x,y)$

$\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$



- $\dot{p} = f(D(p) - S(p))$

$\begin{cases} f(0)=0 \\ f'(0)>0 \end{cases}$ $\quad f(D(p^*) - S(p^*)) = 0$ $\quad p^*$

$F(x) \approx F(a) + F'(a)(x-a)$

$F(p) = f(D(p) - S(p))$, $a = p^*$

$\dot{p} = f(D(p) - S(p))$

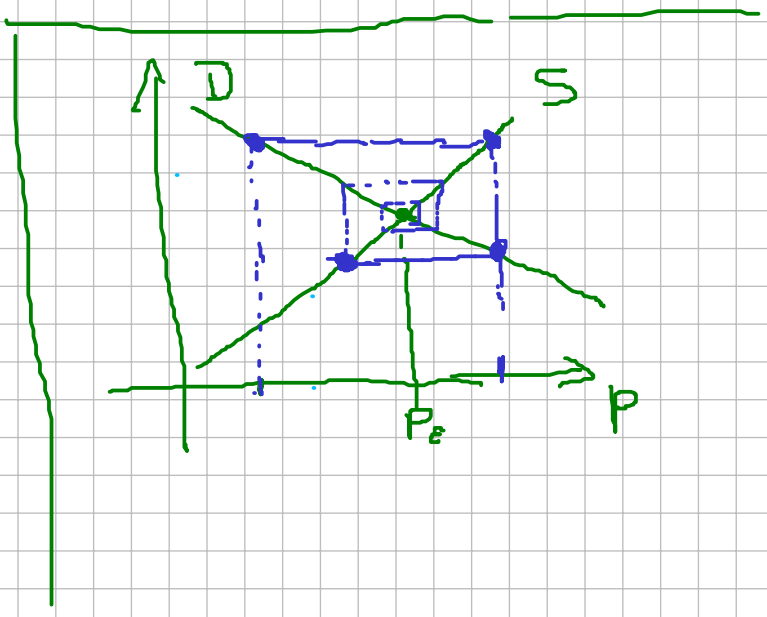
$\approx \underbrace{f(D(p^*) - S(p^*))}_{0} + \underbrace{f'(D(p^*) - S(p^*)) \cdot (D'(p^*) - S'(p^*))}_{a_1} (p - p^*)$

$\dot{p} \approx a_1 (p - p^*)$

$\dot{p} = a_1 p - a_1 p^*$

$\dot{p} = a_1 p - \underbrace{a_1 p^*}_{a_2}$

$\dot{p} = a_1 p - a_2$



$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\det A = a_{11}a_{22} - a_{21}a_{12}$$

$$\det A = (-1)^{1+1} a_{11}a_{22} + (-1)^{1+2} a_{12}a_{21}$$

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 10 & -2 & 4 \end{bmatrix}$$

$$Av = \lambda v \quad , \lambda \in \mathbb{C} \, , \, v \neq 0.$$

$$Av - \lambda I v = 0$$

$$(A - \lambda I)\, v = 0$$

No tenga inversa

$$\det(A - \lambda I) = 0$$

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = \det\left(\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right)$$

$$= \det \begin{pmatrix} -\lambda & -1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{pmatrix}$$

$$= (-1)^{3+3} (1-\lambda) \det \begin{pmatrix} -\lambda & -1 \\ 1 & -\lambda \end{pmatrix}$$

$$= (1-\lambda) \cdot (\lambda^2 + 1) = 0$$

$\lambda_1 = 1 \; ; \; \lambda_2 = i \; ; \; \lambda_3 = -i$

$i^2 = -1$

$S^{-1} A S = D$ $\qquad S = \left[v_1 \; v_2 \cdots v_n \right]$

$$AS = SD$$

$$A \cdot \underbrace{\left[v_1 \; v_2 \cdots v_n \right]}_{S} = \left[Av_1 \; Av_2 \cdots Av_n \right]$$

$$AS = \left[\lambda_1 v_1 \; \lambda_2 v_2 \cdots \lambda_n v_n \right]$$

$$= \underbrace{\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}}_{D} \cdot \underbrace{\left[v_1 \cdots v_n \right]}_{S}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\det\left(A - \lambda I\right) = \det\left(\begin{bmatrix} -\lambda & 1 \\ 0 & -\lambda \end{bmatrix}\right) = \lambda^2 = 0$$

$$\lambda_1 = \lambda_2 = 0$$

$$\underbrace{\left(A - 0I\right)}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \underbrace{0x_1 + x_2 = 0}_{0x_1 + 0x_2 = 0}$$

$$v = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \cancel{\begin{bmatrix} -1 \\ 0 \end{bmatrix}} \qquad v = \begin{bmatrix} x_1 \\ 0 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\det(A - \lambda I) = \det \begin{pmatrix} 4-\lambda & 1 \\ -1 & 2-\lambda \end{pmatrix} = (4-\lambda)\cdot(2-\lambda) + 1$$

$$= \lambda^2 - 6\lambda + 9 = (\lambda - 3)^2 = 0$$

$$\lambda_1 = \lambda_2 = 3$$

$$(A - 3I)\cdot v = 0 \qquad \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

$$v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} \leftarrow$$

Vectores propios generalizados

$$\dim \ker (A-3I) = 1$$

$$\dim \ker (A-3I)^2$$

Si $v \in \ker (A-3I)$

$$(A-3I)v = 0$$

$$(A-3I)\cdot(A-3I)v = 0$$

$$(A-3I)^2 v = 0$$

$$\ker (A-3I) \subset \ker (A-3I)^2 = \mathbb{R}^2$$

$$(A-3I)^2 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Buscamos un vector w tal que

$$(A-3I)^2 w = 0$$

$$(A-3I)\underbrace{(A-3I)w}_{\bar{w}} = 0 \qquad (A-3I)\bar{w} = 0$$

$$\bar{w} \in \ker (A-3I)$$

$\overline{w} = v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$(A - 3I) \cdot w = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\boxed{x_1 + x_2 = -1}$ $\quad x_1 = -1 - x_2 \,, \; x_2 \in \mathbb{R}$

$-x_1 - x_2 = 1$

Por ejemplo: $\quad w = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad [x_2 = 0]$

$v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $w = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$S = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$, $S^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$

$S^{-1} A S = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$

---

(1) $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$ $\qquad \lambda = ?$

$\det(A - \lambda I) = \det \begin{pmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & -1 & 2-\lambda \end{pmatrix}$

$= (2-\lambda) \cdot (2-\lambda)^2 = 0$

$\lambda = 2$

$(A - 2I) \cdot V = 0$

$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\boxed{x_2 = 0}$

$$V = \begin{pmatrix} x_1 \\ 0 \\ x_3 \end{pmatrix} = x_1 \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{v_1^{(1)}} + x_3 \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{v_2^{(1)}}$$

$$\boxed{\delta_1 = 2}$$

$$B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \text{ es base de } \ker(A - 2I).$$

(2) $\delta_2 = \dim \ker (A-2I)^2$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{\delta_2 = 3}$$

Elegimos $\left\{ v_j^{(1)} \right\}_{j=1}^{2}$ base de $\ker(A-2I)$

tal que

$$\underbrace{(A-2I)}\, v_j^{(2)} = v_{j=1}^{(1)} \quad , j = 1 = \delta_2 - \delta_1$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 \\ 0 \\ c_2 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ 0 \\ -x_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ 0 \\ c_2 \end{pmatrix}$$

$$\boxed{x_2 = c_1 \ , \ -x_2 = c_2} \Rightarrow \boxed{c_1 = -c_2}$$

Elegimos: $c_2 = 1$, $c_1 = -1$

$$v_1^{(1)} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \ , \ v_1^{(2)} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \ , \ v_2^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$M = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \qquad M^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M^{-1}AM = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$