$f(t)$ sinônimos

$f = f(x,y)$

$\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$



- $\dot{p} = f(D(p) - S(p))$

$\begin{cases} f(0)=0 \\ f'(0)>0 \end{cases}$ $\quad f(D(p^*) - S(p^*)) = 0$ $\quad p^*$

$F(x) \approx F(a) + F'(a)(x-a)$

$F(p) = f(D(p) - S(p))$, $a = p^*$

$\dot{p} = f(D(p) - S(p))$

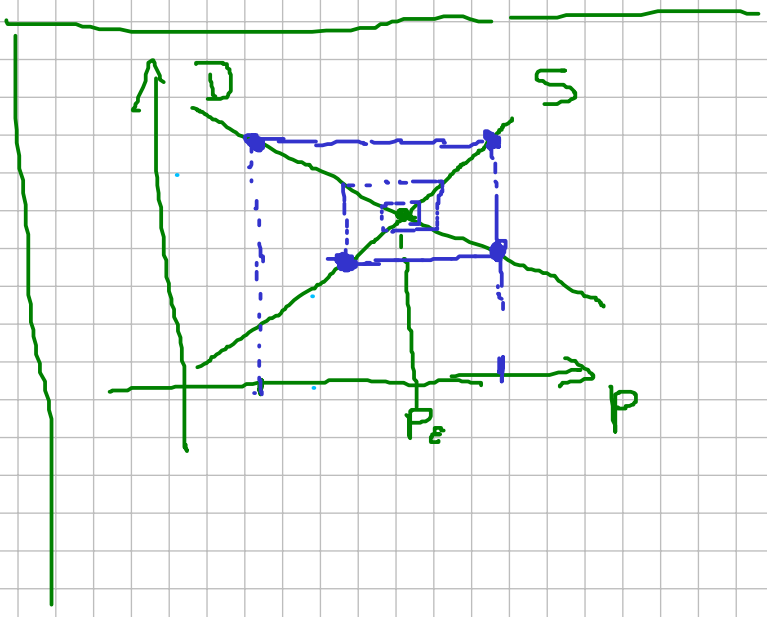
$\approx \underbrace{f(D(p^*) - S(p^*))}_{0} + \underbrace{f'(D(p^*) - S(p^*)) \cdot (D'(p^*) - S'(p^*))}_{a_1} (p - p^*)$

$\dot{p} \approx a_1 (p - p^*)$

$\dot{p} = a_1 p - a_1 p^*$

$\dot{p} = a_1 p - \underbrace{a_1 p^*}_{a_2}$

$\dot{p} = a_1 p - a_2$



$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\det A = a_{11}a_{22} - a_{21}a_{12}$$

$$\det A = (-1)^{1+1} a_{11}a_{22} + (-1)^{1+2} a_{12}a_{21}$$

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 10 & -2 & 4 \end{bmatrix}$$

$$Av = \lambda v \quad , \lambda \in \mathbb{C} \ , \ v \neq 0.$$

$$Av - \lambda I v = 0$$

$$(A - \lambda I)\, v = 0$$

No tenga inversa

$$\det(A - \lambda I) = 0$$

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = \det\left(\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right)$$

$$= \det \begin{bmatrix} -\lambda & -1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$= (-1)^{3+3} (1-\lambda) \det \begin{pmatrix} -\lambda & -1 \\ 1 & -\lambda \end{pmatrix}$$

$$= (1-\lambda)\cdot(\lambda^2 + 1) = 0$$

$\lambda_1 = 1 \; ; \; \lambda_2 = i \; ; \; \lambda_3 = -i$

$i^2 = -1$

$S^{-1} A S = D \qquad S = \left[v_1 \; v_2 \cdots v_n \right]$

$\boxed{AS = SD}$

$$A \cdot \underbrace{\left[v_1 \; v_2 \cdots v_n \right]}_{AS} = \left[Av_1 \; Av_2 \cdots Av_n \right]$$

$$= \left[\lambda_1 v_1 \; \lambda_2 v_2 \cdots \lambda_n v_n \right]$$

$$= \underbrace{\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}}_{D} \cdot \underbrace{\left[v_1 \cdots v_n \right]}_{S}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\det\left(A - \lambda I \right) = \det\left(\begin{bmatrix} -\lambda & 1 \\ 0 & -\lambda \end{bmatrix} \right) = \lambda^2 = 0$$

$$\boxed{\lambda_1 = \lambda_2 = 0}$$

$$\underbrace{\left(A - 0I \right)}_{} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \underbrace{0x_1 + x_2 = 0}_{0x_1 + 0x_2 = 0}$$

$$\boxed{v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}} \qquad \cancel{\begin{bmatrix} -1 \\ 0 \end{bmatrix}} \qquad v = \begin{bmatrix} x_1 \\ 0 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\det(A - \lambda I) = \det \begin{pmatrix} 4-\lambda & 1 \\ -1 & 2-\lambda \end{pmatrix} = (4-\lambda)\cdot(2-\lambda) + 1$$

$$= \lambda^2 - 6\lambda + 9 = (\lambda - 3)^2 = 0$$

$$\lambda_1 = \lambda_2 = 3$$

$$(A - 3I)\cdot v = 0 \qquad \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

$$v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} \leftarrow$$

Vectores propios generalizados

$$\dim \ker (A-3I) = 1$$

$$\dim \ker (A-3I)^2$$

Si $v \in \ker (A-3I)$

$$(A-3I)v = 0$$

$$(A-3I)\cdot(A-3I)v = 0$$

$$(A-3I)^2 v = 0$$

$$\ker (A-3I) \subset \ker (A-3I)^2 = \mathbb{R}^2$$

$$(A-3I)^2 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Buscamos un vector w tal que

$$(A-3I)^2 w = 0$$

$$(A-3I)\underbrace{(A-3I)w}_{\bar{w}} = 0 \qquad (A-3I)\bar{w} = 0$$

$$\bar{w} \in \ker (A-3I)$$

$\overline{w} = v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$(A - 3I) \cdot w = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\boxed{x_1 + x_2 = -1}$ $\quad x_1 = -1 - x_2 \,, \; x_2 \in \mathbb{R}$

$-x_1 - x_2 = 1$

Por ejemplo: $\quad w = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad [x_2 = 0]$

$v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $w = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$S = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$, $S^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$

$S^{-1} A S = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$

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(1) $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$ $\qquad \lambda = ?$

$\det(A - \lambda I) = \det \begin{pmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & -1 & 2-\lambda \end{pmatrix}$

$= (2-\lambda) \cdot (2-\lambda)^2 = 0$

$\lambda = 2$

$(A - 2I) \cdot V = 0$

$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\boxed{x_2 = 0}$

$$V = \begin{pmatrix} x_1 \\ 0 \\ x_3 \end{pmatrix} = x_1 \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{V_1^{(1)}} + x_3 \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{V_2^{(1)}}$$

$$\boxed{\delta_1 = 2}$$

$$B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \text{ es base d } \ker(A - 2I).$$

(2) $\delta_2 = \dim \ker (A-2I)^2$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{\delta_2 = 3}$$

Elegimos $\left\{ V_j^{(1)} \right\}_{j=1}^{2}$ base de $\ker (A-2I)$

tal que

$$\underbrace{(A-2I)}\, V_j^{(2)} = V_{j=1}^{(1)} \quad , j = 1 = \delta_2 - \delta_1$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 \\ 0 \\ c_2 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ 0 \\ -x_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ 0 \\ c_2 \end{pmatrix}$$

$$\boxed{x_2 = c_1 \ , \ -x_2 = c_2} \Rightarrow \boxed{c_1 = -c_2}$$

Elegimos: $c_2 = 1$, $c_1 = -1$

$$V_1^{(1)} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \ , \ V_1^{(2)} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \ , \ V_2^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$M = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \qquad M^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M^{-1} A M = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -2 & -1 & -1 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

λ:

$$\det(B - \lambda I) = \det \begin{bmatrix} -\lambda & -2 & -1 & -1 \\ 1 & 2-\lambda & 1 & 1 \\ 0 & 1 & 1-\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{bmatrix} = 0$$

$\lambda = 1$ Única raiz del polinomio característico

$$B - 1I = \begin{bmatrix} -1 & -2 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\dim \ker (B - 1I) = 2$

S_1

Vectores propios:

Resolver:

$$(B - 1I)\, v = 0$$

$$v_1^{(1)} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad v_2^{(1)} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\delta_2 = \dim \ker \underbrace{(B-1I)^2}$$

$$\begin{bmatrix} -1 & -2 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & -2 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{(B-1I)^2}$$

$$\boxed{\delta_2 = 3} > \boxed{\delta_1 = 2}$$

Elegimos una base $\{V_1^{(1)}; V_2^{(1)}\}$ de $\ker(B-1I)$

$$(B-1I)V_1^{(2)} = V_1^{(1)}$$

$$\begin{bmatrix} -1 & -2 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = c_1 \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{V_1^{(1)}} + c_2 \underbrace{\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{V_2^{(1)}}$$

$$\begin{cases} -x_1 - 2x_2 - x_3 - x_4 = -c_1 - c_2 \\ x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 = c_1 \\ 0 = c_2 \end{cases}$$

Elegimos:

$$\boxed{c_1 = 1} \Rightarrow V_1^{(1)} = V_1^{(1)}$$

$$\boxed{x_2 = 1} \quad \boxed{x_1 = -1} \quad \boxed{x_3 = x_4 = 0}$$

$$\left\{ \underbrace{V_1^{(1)}}_{1}, V_1^{(2)}, \underbrace{V_2^{(1)}}_{2} \right\} = \left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\delta_3 = \dim \ker \underbrace{(B - 1I)^3}_{0} = 4 > \delta_2 = 3$$

Elegir una base $\left\{ V_j^{(2)} \right\}_{j=1}^{3}$ de $\ker (B - 1I)^2$ tal que:

$$V_1^{(2)} \in \text{span} \left\{ V_1^{(2)} \right\}$$

$$\Rightarrow (B - 1I) V_1^{(3)} = V_1^{(2)}$$

$$\begin{bmatrix} -1 & -2 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$-X_1 - 2X_2 - X_3 - X_4 = -1$$

$$\rightarrow X_1 + \cancel{X_2} + X_3 + X_4 = 1$$

$$X_2 = 0$$

$$X_1 = 1 \qquad X_3 = X_4 = 0$$

$$V_1^{(3)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Luego,

$$B = \left\{ V_1^{(1)} ; V_1^{(2)} ; V_1^{(3)} ; V_2^{(1)} \right\}$$

$$= \left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} ; \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$M = \begin{bmatrix} -1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad M^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M^{-1}BM = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$BM = \begin{bmatrix} BM_1 & BM_2 & BM_3 & BM_4 \end{bmatrix}$$

BM_4 → $1\,M_4$

$$MJ = \begin{bmatrix} MJ_1 & MJ_2 & MJ_3 & MJ_4 \end{bmatrix}$$

Afirmación: tres V.P.

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & \Phi 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

$$M^{-1} A M = J$$

$$J = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} ; \quad \begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix} ; \quad \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$$

$$X' = A X \longrightarrow Y' = J Y$$

$$X = M Y$$