PID controllers

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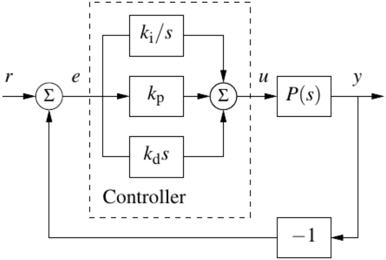
A survey of controllers for more than 100 boiler-turbine units in the Guangdong Province in China [1]:

- 94.4% of all controllers were PI,
- 3.7% PID,
- 1.9% used advanced control.



$$u(t) = k_p \cdot e(t) + k_i \int_0^t e(\tau)d\tau + k_d \frac{de(t)}{dt} = k_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{de(t)}{dt} \right) , \quad (1)$$

$$k_i = \frac{k_p}{T_i} , \quad (2)$$

$$k_d = k_p \cdot T_d . \quad (3)$$

[1] Li Sun, Donghai Li, and Kwang Y. Lee. Optimal disturbance rejection for pi controllerwith constraints on relative delay margin.ISA Transactions,2016.

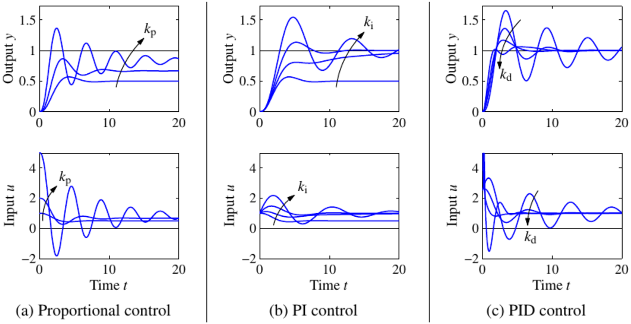


Figure 11.2: Responses to step changes in the reference value for a system with a proportional controller (a), PI controller (b) and PID controller (c). The process has the transfer function $P(s) = 1/(s+1)^3$, the proportional controller has parameters $k_p = 1$, 2 and 5, the PI controller has parameters $k_p = 1$, $k_i = 0$, 0.2, 0.5, and 1, and the PID controller has parameters $k_p = 2.5$, $k_i = 1.5$ and $k_d = 0$, 1, 2, and 4.

$$u = k_p \cdot e + k_i \int_0^t e(\tau)d\tau + k_d \frac{de}{dt} = k_p \left(e + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{de}{dt} \right) \qquad (4)$$

- If the plant mathematical model cannot be obtained at all, then an analytical or computational approach to the design of a PID controller is not possible.
- Then we must resort to experimental approaches to the tuning of PID controllers.
- Ziegler and Nichols suggested rules for tuning PID controllers (values K_p , T_i, and T_d) based on:
  - Experimental step responses (Method 1).
  - Based on the value of K_p that results in marginal stability when only proportional control action is used (Method 2).

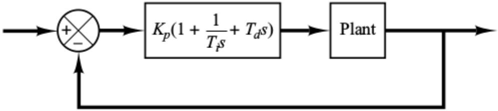
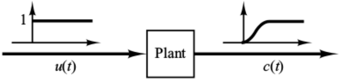


Figure 8–1
PID control of a plant.

- We obtain experimentally the response of the plant to a unit-step input.
- This method applies if the response to a step input exhibits an S-shaped curve.
- If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped.

Figure 8–2
Unit-step response of a plant.



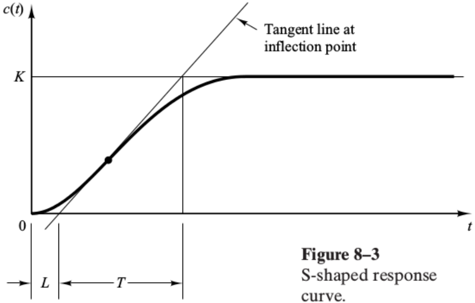


Figure 8–3
S-shaped response curve.

- The S-shaped curve may be characterized by two constants, delay time L and time constant T.

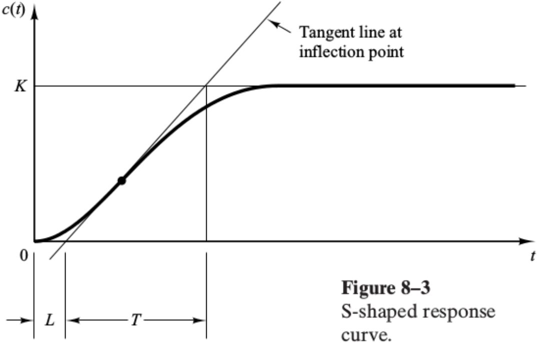


Figure 8–3
S-shaped response curve.

Table 8–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d according to the formula shown in Table 8–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\,\frac{\left(s + \frac{1}{L}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

PID Control Tuning
Method 2

- We first set $T_i = \infty$, and $T_d = 0$.
- Increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations.
- Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally determined.
- If the output does not exhibit sustained oscillations, then this method does not apply.

Figure 8–4 Closed-loop system with a proportional controller.

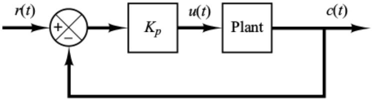


Figure 8–5 Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

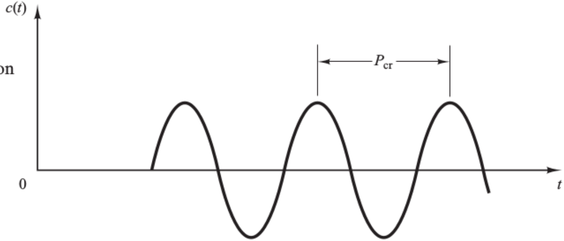


Table 8–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$
\begin{aligned}
G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\
&= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\
&= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}
\end{aligned}
$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

EXAMPLE 8–1 Consider the control system shown in Figure 8–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

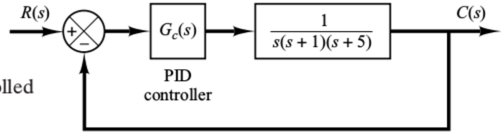
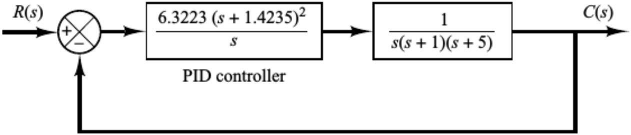


Figure 8–6 PID-controlled system.

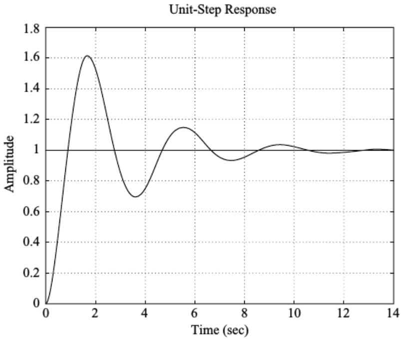
$$
\begin{aligned}
K_{cr} &= 30 \\
P_{cr} &= 2.8099 \\
K_p &= 0.6 \cdot K_{cr} = 18 \\
T_i &= 0.5 \cdot P_{cr} = 1.405 \\
T_d &= 0.125 \cdot P_{cr} = 0.35124
\end{aligned}
$$



Maximum overshoot is close to 62%.

MATLAB Program 8–1

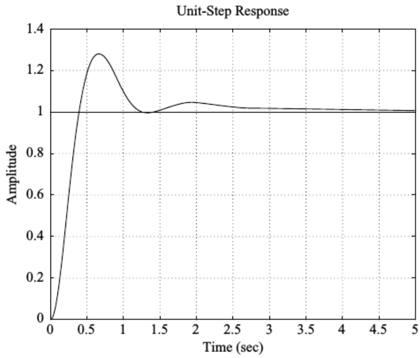
```
% ---------- Unit-step response ----------
num = [6.3223  18  12.811];
den = [1  6  11.3223  18  12.811];
step(num,den)
grid
title('Unit-Step Response')
```



The Ziegler–Nichols tuning rule has provided a starting point for fine tuning.

$$K_p = 39.42$$
$$T_i = 3.077$$
$$T_d = 0.7692$$

Maximum overshoot is fairly close to 25%.



- The value of Kp increases the speed of response.
- However, varying the location of the double zero has a significant effect on the maximum overshoot.

Bibliography

- Karl J. Astrom and Richard M. Murray *Feedback Systems*. Version v3.0i. Princeton University Press. September 2018. Chapter 11.
- Ogata, Katsuhiko. Modern Control Engineering. Fifth Edition. Prentice Hall. 2009. Chapter 8.