Two-Degrees-of-Freedom PID controllers

Dr. Ing. Rodrigo Gonzalez

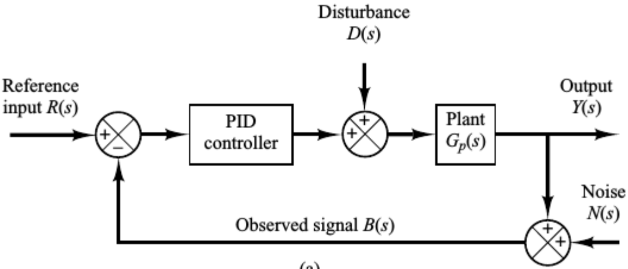
rodrigo.gonzalez@ingenieria.uncuyo.edu.ar

Control y Sistemas

Ingeniería Mecatrónica,
Facultad de Ingeniería,
Universidad Nacional de Cuyo

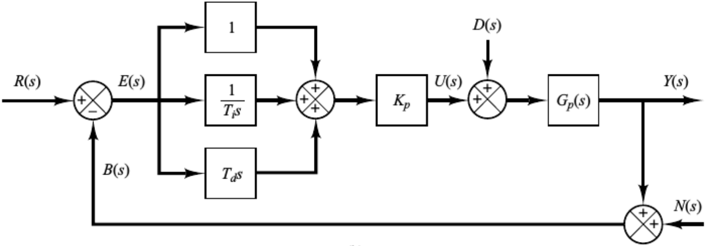
June 2020

- In practical cases, there may be one requirement on the *response to disturbance input* and another requirement on the *response to reference input*.
- Often these two requirements conflict with each other and cannot be satisfied in the single-degree-of-freedom case.
- By increasing the degrees of freedom, we are able to satisfy both, response to disturbance and response to reference input.



(a)

- If reference input is a step function, the derivative term in the control action will produce that $u(t)$ will involve an impulse function.
- Such a phenomenon is called *set-point kick*.

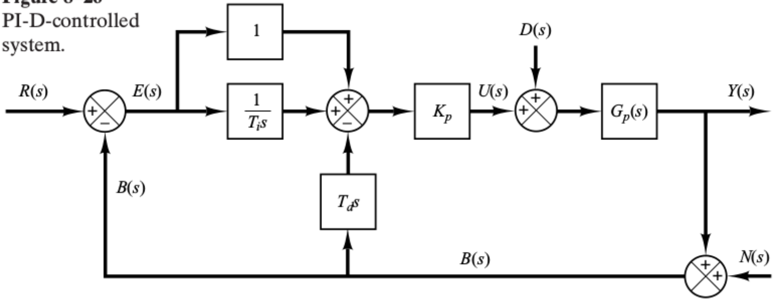


(b)

To avoid the set-point kick phenomenon (a pulse input), we may wish to operate the derivative action only in the feedback path so that differentiation occurs only on the feedback signal and not on the reference signal.

$$U(s) = K_p\left(1 + \frac{1}{T_i s}\right)R(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)B(s)$$

Figure 8–26
PI-D-controlled system.



Notice that in the absence of the disturbances and noises, the closed-loop transfer function of the basic PID control system [shown in Figure 8–25(b)] and the PI-D control system (shown in Figure 8–26) are given, respectively, by

$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s} + T_d s\right) \frac{K_p G_p(s)}{1 + \left(1 + \frac{1}{T_i s} + T_d s\right) K_p G_p(s)}$$

and

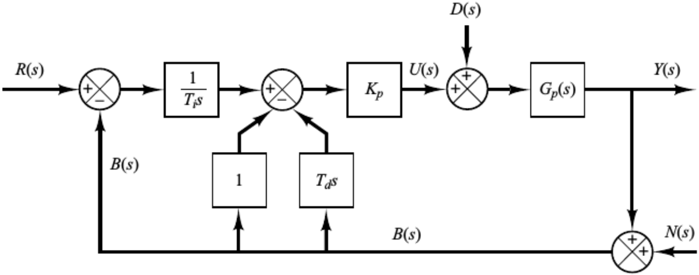
$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s}\right) \frac{K_p G_p(s)}{1 + \left(1 + \frac{1}{T_i s} + T_d s\right) K_p G_p(s)}$$

It is important to point out that in the absence of the reference input and noises, the closed-loop transfer function between the disturbance $D(s)$ and the output $Y(s)$ in either case is the same and is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

- If reference input is a step function, both PID control and PI-D control involve a step function in the manipulated signal.
- The proportional action and derivative action are moved to the feedback path so that these actions affect the feedback signal only.

$$U(s) = K_p \frac{1}{T_i s} R(s) - K_p \left(1 + \frac{1}{T_i s} + T_d s\right) B(s)$$



The closed-loop transfer function $Y(s)/R(s)$ in the absence of the disturbance input and noise input is given by

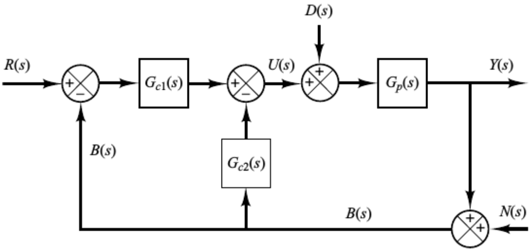
$$\frac{Y(s)}{R(s)} = \left(\frac{1}{T_i s}\right) \frac{K_p G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

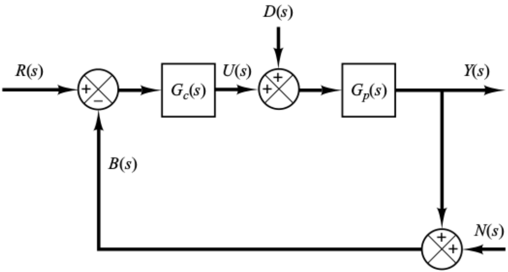
It is noted that in the absence of the reference input and noise signals, the closed-loop transfer function between the disturbance input and the output is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

This expression is the same as that for PID control or PI-D control.

- Instead of moving the entire derivative control action or proportional control action to the feedback path, it is possible to move only portions of these control actions to the feedback path.
- The characteristics of PI-PD control scheme lie between PID control and I-PD control.
- Similarly, PID-PD control can be considered.
- We will have one controller in the feedforward path and another controller in the feedback path. Such control schemes lead us to a two-degrees-of-freedom control scheme.





For this system, three closed-loop transfer functions $Y(s)/R(s) = G_{yr}$, $Y(s)/D(s) = G_{yd}$, and $Y(s)/N(s) = G_{yn}$ may be derived. They are

$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_c G_p}{1 + G_c G_p}$$

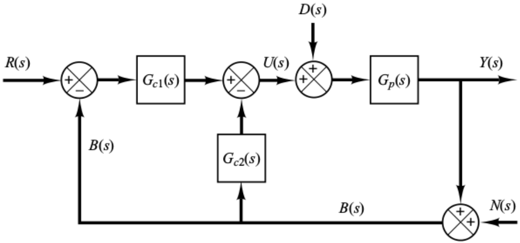
$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_c G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_c G_p}{1 + G_c G_p}$$

$$G_{yr} = \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

How many of these closed-loop transfer functions are independent?



$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{(G_{c1} + G_{c2})G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yr} = G_{c1}G_{yd}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

How many of these closed-loop transfer functions are independent?

Bibliography

- Ogata, Katsuhiko. Modern Control Engineering. Fifth Edition. Prentice Hall. 2009. Chapter 8.