

Sistemas de Automatización
AÑO 2021

UNIDAD 5
Sustitución directa

Profesores:

Ing. María Susana Bernasconi-

sbernasc@uncu.edu.ar

susybernasconi@gmail.com

Ing Fernando Geli

fernandogeli@gmail.com

Bibliografía:

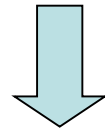
Ingeniería de Control- W. BOLTON

Ingeniería de Control Moderna-K.OGATA

Control Automático de Procesos- C.SMITH, A. CORRIPIO

Criterio de estabilidad

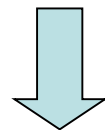
$$c(t) = b_1 e^{r_1 t} + b_2 e^{r_2 t} + \dots + b_n e^{r_n t} + (\text{términos de entrada})$$



Para raíces reales: Si $r < 0$, entonces $e^{rt} \rightarrow 0$ conforme $t \rightarrow \infty$

Para raíces complejas: $r = \sigma + i\omega$ $e^{rt} = e^{\sigma t} (\cos \omega t + i \operatorname{sen} \omega t)$

Si $\sigma < 0$, entonces $e^{\sigma t} (\cos \omega t + i \operatorname{sen} \omega t) \rightarrow 0$ conforme $t \rightarrow \infty$



Para que el circuito de control con retroalimentación sea estable, todas las raíces de su ecuación característica deben ser números reales negativos o números complejos con partes reales negativas.

Prueba de Routh

$$900.s^3 + 420.s^2 + 43.s + (1 + 0.80.Kc) = 0$$

fila 1	900		43		0
fila 2	420	+	0.80K_c	-	0
fila 3	b₁		0		0
fila 4	+	0.80K_c			0

$$b_1 = \frac{(420).(43) - 900.(1 + 0.80.Kc)}{420} = \frac{17160 - 720.Kc}{420}$$

$$b_1 \geq 0 \rightarrow 17160 - 720.Kc \geq 0 \rightarrow Kc \leq 23.8$$

$$1 + 0.80.Kc \geq 0 \rightarrow 0.80.Kc \geq -1 \rightarrow Kc \geq -1.25$$



$$Kc \leq 23.8$$

Método de sustitución directa

$$900.s^3 + 420.s^2 + 43.s + (1 + 0.80.Kc) = 0$$

$$s = i\omega$$

$$900.(i\omega)^3 + 420.(i\omega)^2 + 43.(i\omega) + (1 + 0.80.Kc) = 0$$

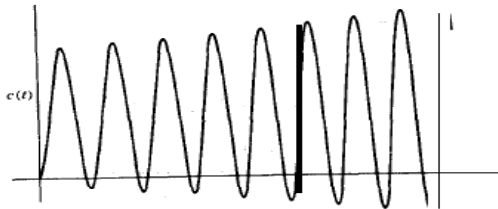
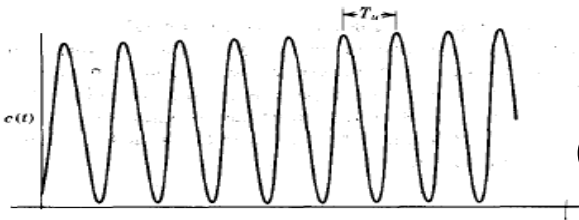
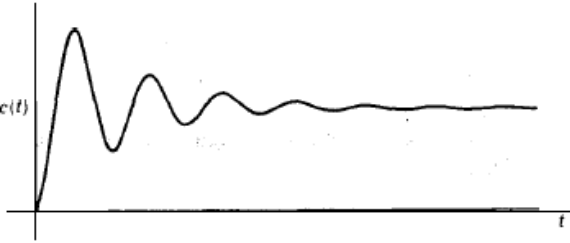
$$(-420.\omega_u^2 + 1 + 0.80.Kc_u) + i(-900.\omega_u^3 + 43\omega_u) = 0 + i0$$

$$(-420.\omega_u^2 + 1 + 0.80.Kc_u) = 0$$

$$(-900.\omega_u^3 + 43\omega_u) = 0$$

$$\omega_u = 0 \rightarrow Kc_u = -1.25 \frac{\%}{\%}$$

$$\omega_u = 0.22 \text{ rad / seg} \rightarrow Kc_u = 23.8 \frac{\%}{\%}$$



Efecto del tiempo muerto:

$$e^{-t_0 s} \doteq \frac{1 - \frac{1}{2}t_0 s}{1 + \frac{1}{2}t_0 s}$$

Aproximacion de Pade:

$$G(s) = \frac{K e^{-t_0 s}}{\tau s + 1} \left\{ \mathbf{1} + \mathbf{G.Kc} = \mathbf{0} \longrightarrow \mathbf{1} + \left(\frac{K \cdot e^{-t_0 s}}{\tau \cdot s + 1} \right) \cdot (\mathbf{Kc}) = \mathbf{0} \right.$$

$$G_c(s) = K_c \left. \right\} \mathbf{1} + \left(\frac{K}{\tau \cdot s + 1} \right) \left(\frac{1 - \frac{t_0 s}{2}}{1 + \frac{t_0 s}{2}} \right) \cdot (\mathbf{Kc}) = \mathbf{0}$$

$$\mathbf{1} + \frac{K \cdot Kc \left(1 - \frac{t_0 s}{2} \right)}{(\tau \cdot s + 1) \left(1 + \frac{t_0 s}{2} \right)} = \mathbf{0}$$

$$\mathbf{1/2 \cdot t_0 \tau \cdot s^2 + (\tau + 1/2 \cdot t_0 - 1/2 \cdot K \cdot Kc \cdot t_0) s + 1 + K \cdot Kc = 0}$$

$$(K \cdot Kc)_u = 1 + 2 \frac{\tau}{t_0}$$

$$\omega_u = \frac{2}{t_0} \sqrt{\frac{t_0}{\tau} + 1}$$

1. se desconectan las acciones integral y derivativo del controlador, de manera de tener un controlador proporcional. En algunos modelos no es posible desconectar la acción integral, se iguala R al valor máximo.
2. con el controlador cerrando el circuito, se incrementa la acción proporcional constante. Luego se registra el valor de K_{cu} . Los incrementos deben ser pequeños, en especial al acercarse al valor de oscilación permanente.
3. del registro del tiempo de la variable controlada, se registra y mide el período de oscilación como T_u , período último, según se muestra en la figura 3.13

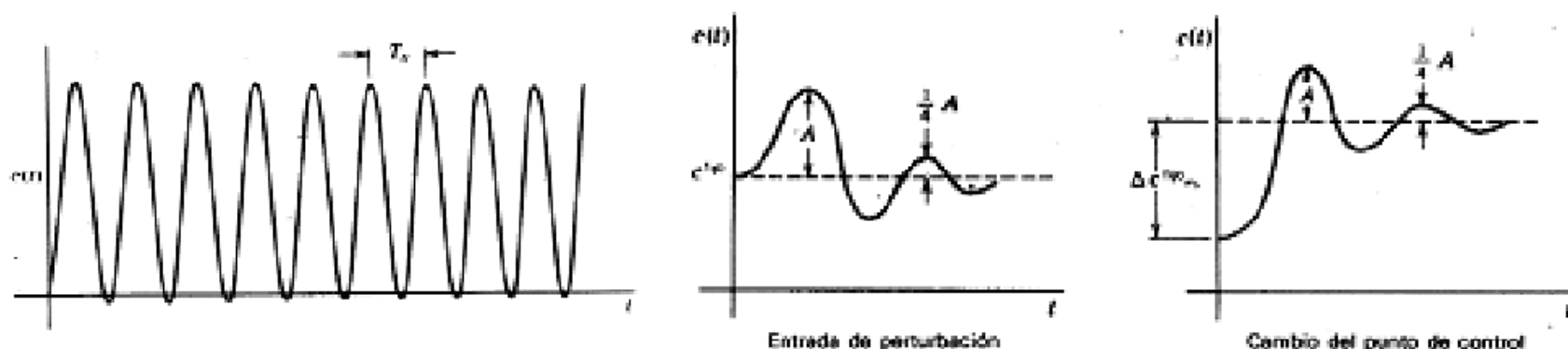


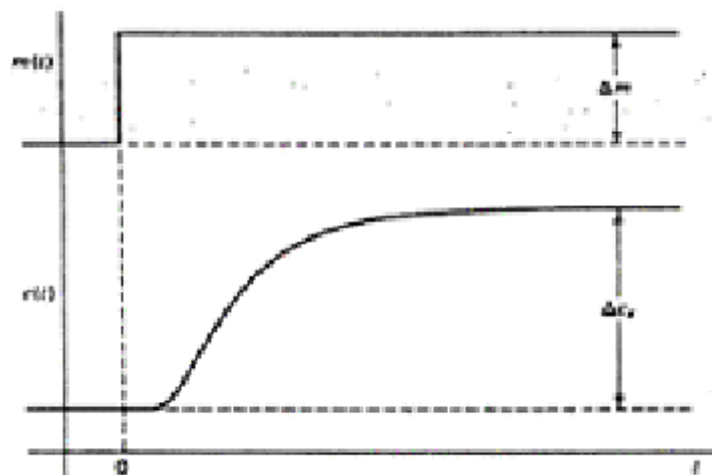
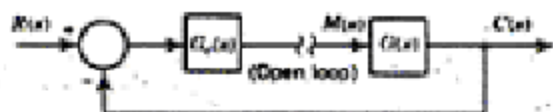
Tabla 6-1 Fórmulas para ajuste de razón de asentamiento de un cuarto.

Tipo de controlador		Ganancia proporcional K_C	Tiempo de integración τ_I	Tiempo de derivación τ_D
Proporcional	P	$K_{cu} / 2$	—	—
Proporcional-integral	PI	$K_{cu} / 2.2$	$T_u / 1.2$	—
Proporcional-integral-derivativo	PID	$K_{cu} / 1.7$	$T_u / 2$	$T_u / 8$

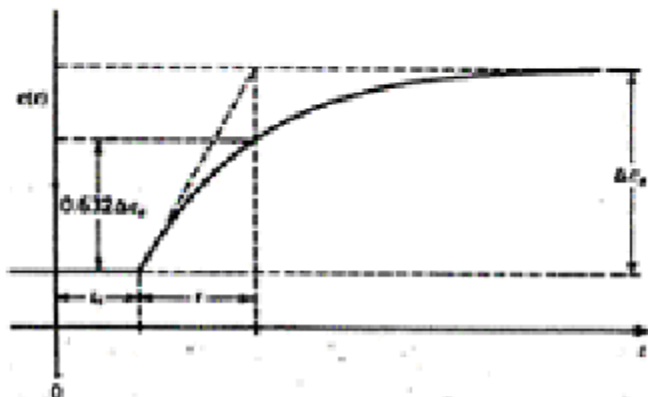
Caracterización del proceso

$$C(s) = G(s) \frac{\Delta m}{s}$$

$$C(s) = \frac{K \cdot e^{-t_0 s}}{\tau \cdot s + 1} \frac{\Delta m}{s}$$

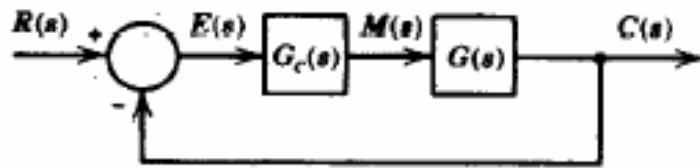


$$K = \frac{\Delta C_s}{\Delta m}$$



Controller Type		Proportional Gain K_c	Integral Time τ_I	Derivative Time τ_D
Proportional only	P	$\frac{1}{K} \left(\frac{\tau}{1 + \tau} \right)$	—	—
Proportional-integral	PI	$\frac{0.9}{K} \left(\frac{\tau}{1 + \tau} \right)$	$3.33 t_0$	—
Proportional-integral-derivative	PID	$\frac{1.2}{K} \left(\frac{\tau}{1 + \tau} \right)$	$2.0 t_0$	$\frac{1}{3} t_0$

Método de síntesis directa o ajuste de Dahlin



$$\frac{C(s)}{R(s)} = \frac{G_c(s) G(s)}{1 + G_c(s) G(s)}$$

$$G_c(s) = \frac{1}{G(s)} \frac{C(s) / R(s)}{1 - [C(s) / R(s)]} \longrightarrow G_c(s) = \frac{1}{G(s)} \frac{1}{1 - 1} = \frac{1}{G(s)} \frac{1}{0}$$

$$\frac{C(s)}{R(s)} = \frac{1}{\tau_c s + 1} \longrightarrow G_c(s) = \frac{1}{G(s)} \frac{1}{\tau_c s}$$

★ Si $G_p = \frac{1}{\tau s + 1}$ (proceso de primer orden)

$$G_c(s) = \frac{\tau}{K \tau_c} \left(1 + \frac{1}{\tau s}\right) \longrightarrow K_c = \frac{\tau}{K \tau_c} \quad \tau_i = \tau$$

★ Si $G_p = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$ (proceso de segundo orden) $\longrightarrow G_c(s) = \frac{\tau_1}{K \tau_c} \left(1 + \frac{1}{\tau_1 s}\right) (\tau_2 s + 1)$

$$\text{Modelo del proceso: } G(s) = \frac{K e^{-\tau s}}{\tau s + 1}$$

Controlador proporcional (P): $G_c(s) = K_c$

Integral del error	ICE	IAE	IAET
$K_c = \frac{a}{K} \left(\frac{t_0}{\tau} \right)^a$	$a = 1.411$	0.902	0.490
	$b = -0.917$	-0.985	-1.084

Controlador proporcional-Integral (PI)

$$G_c(s) = K_c \left(1 + \frac{1}{\tau_I s} \right)$$

Integral de error	ICE	IAE	IAET
$K_c = \frac{a_1}{K} \left(\frac{t_0}{\tau} \right)^{a_1}$	$a_1 = 1.305$	0.984	0.859
	$b_1 = -0.959$	-0.986	-0.977
$\tau_I = \frac{\tau}{a_2} \left(\frac{t_0}{\tau} \right)^{a_2}$	$a_2 = 0.492$	0.608	0.674
	$b_2 = 0.739$	0.707	0.680

Controlador proporcional-Integral-derivativo (PID):

$$G_c(s) = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s \right)$$

Integral de error	ICE	IAE	IAET
$K_c = \frac{a_1}{K} \left(\frac{t_0}{\tau} \right)^{a_1}$	$a_1 = 1.485$	1.435	1.357
	$b_1 = -0.945$	-0.921	-0.947
$\tau_I = \frac{\tau}{a_2} \left(\frac{t_0}{\tau} \right)^{a_2}$	$a_2 = 1.101$	0.878	0.842
	$b_2 = 0.771$	0.749	0.738
$\tau_D = a_3 \tau \left(\frac{t_0}{\tau} \right)^{a_3}$	$a_3 = 0.560$	0.482	0.381
	$b_3 = 1.006$	1.137	0.995

Ziegler - Nichols	Ziegler-Nichols	IAE	Dahlin
$K_c = \frac{1,2}{K} \left(\frac{\tau}{\tau_d} \right)$	$K_c = \frac{K_{cm}}{1,7}$	$K_c = \frac{a_1}{K} \left(\frac{\tau_d}{\tau} \right)^{b_1}$ $a_1 = 1,435$ $b_1 = -0,921$	$K_c = \frac{\tau}{K(\tau_d + \tau_c)}$ $\tau_c = \frac{1}{5} \tau_d$
$R = 2\tau_d$	$R = \frac{\tau_u}{2}$	$R = \frac{\tau}{a_2} \left(\frac{\tau_d}{\tau} \right)^{b_2}$ $a_2 = 0,878$ $b_2 = 0,749$	$R = \tau$
$D = \frac{\tau_d}{2}$	$D = \frac{\tau_u}{8}$	$D = a_3 \tau \left(\frac{\tau_d}{\tau} \right)^{b_3}$ $a_3 = 0,482$ $b_3 = 1,137$	$D = \frac{\tau_d}{2}$

Lugar de raíces

$$G(s) = \frac{Gc.Gp}{1 + Gc.Gp}$$



Función de transferencia de lazo cerrado

Si: $Gc(s) = Kc$ $Gp(s) = \frac{1}{s(s+1)}$

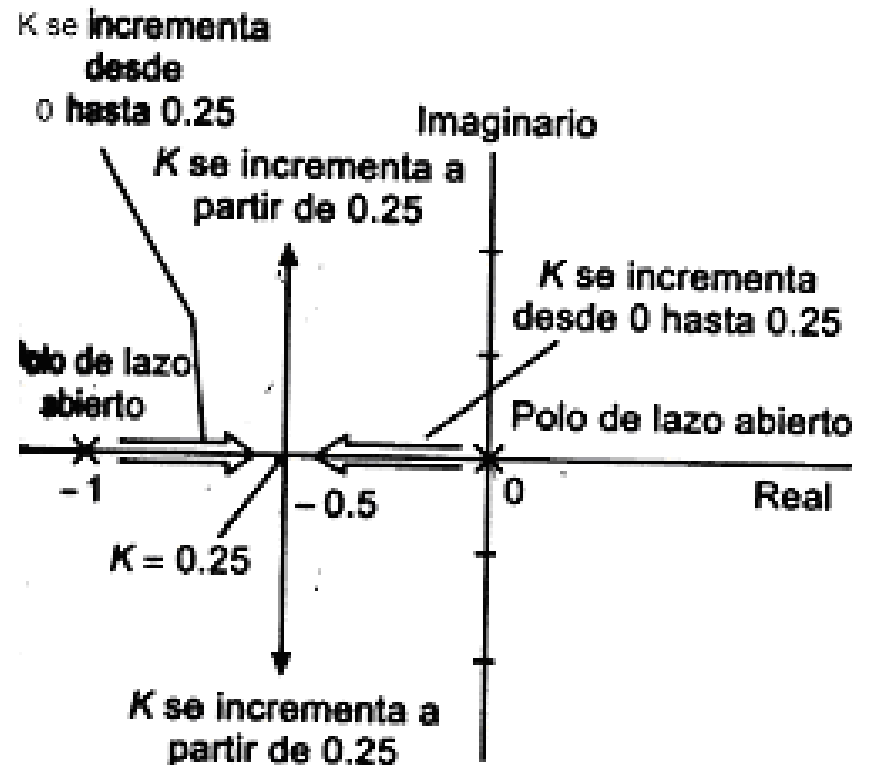
$$G(s) = \frac{Kc / [s(s+1)]}{1 + Kc / [s(s+1)]}$$

$$G(s) = \frac{Kc}{s^2 + s + Kc}$$

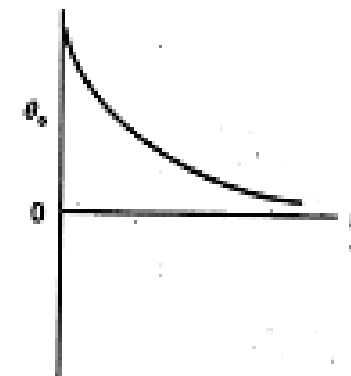
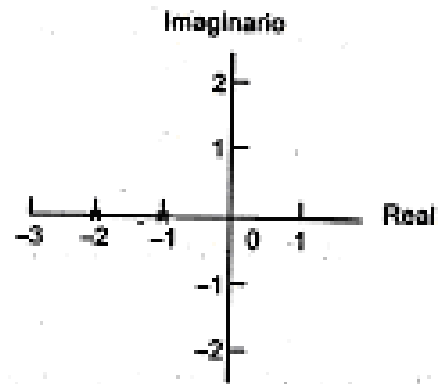
Las raíces del polinomio del denominador de la función de transferencia son:

$$p = \frac{-1 \pm \sqrt{1 - 4Kc}}{2}$$

$$p = -\frac{1}{2} \pm \frac{1}{2} \sqrt{1 - 4Kc}$$

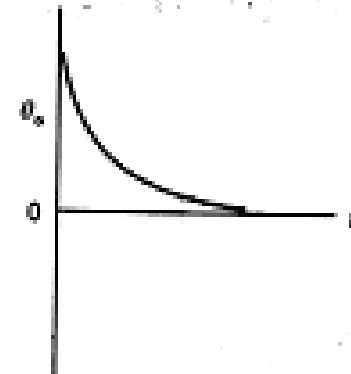
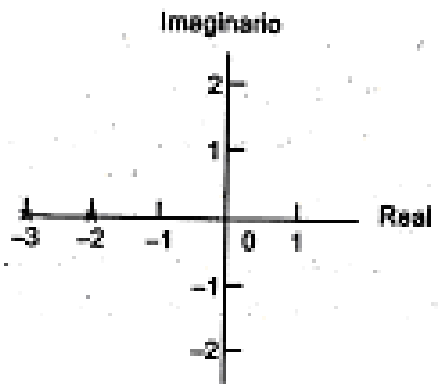


Raíz= -1



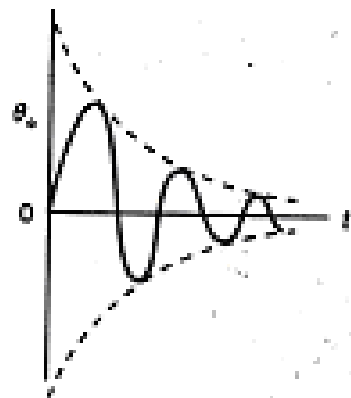
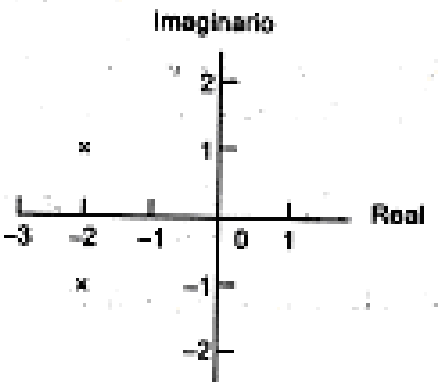
a)

Raíz= -2

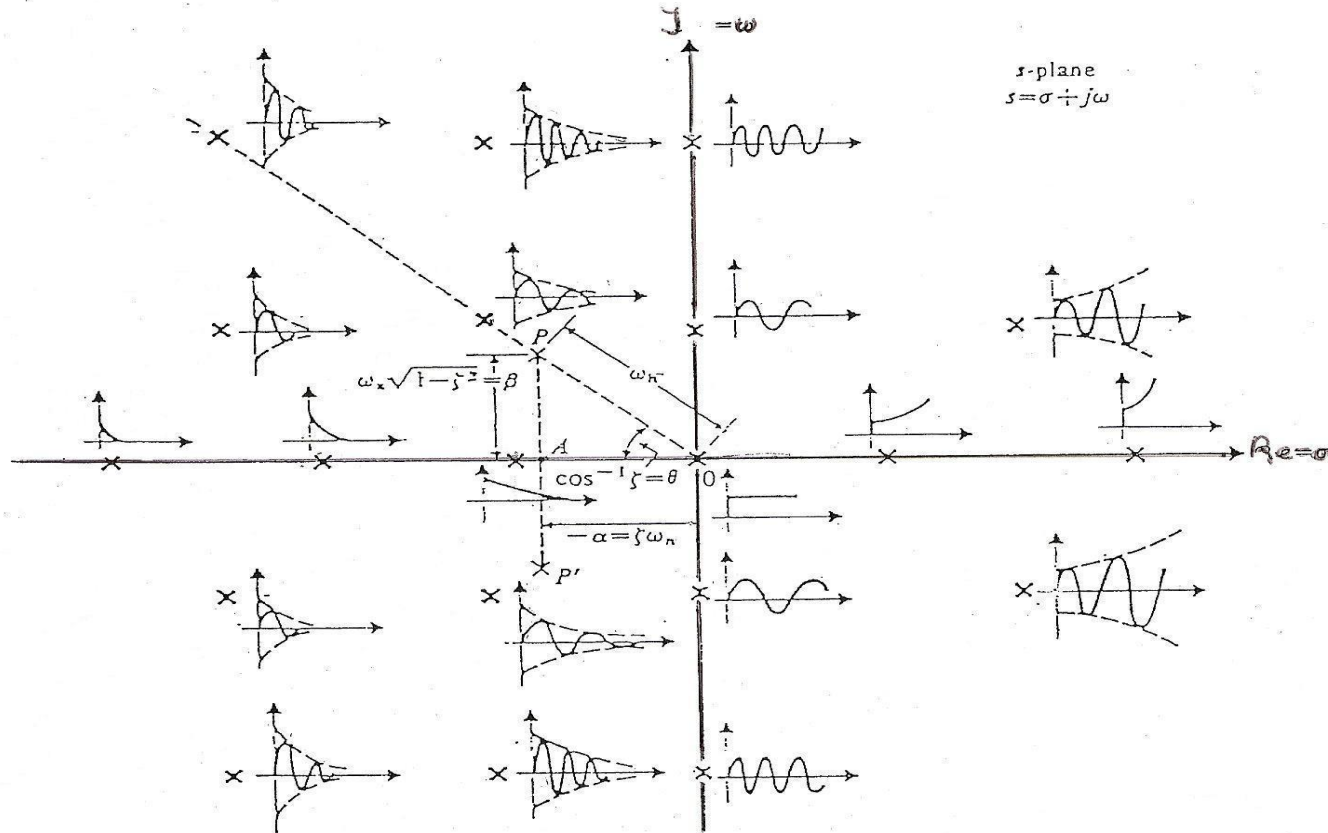


b)

Raíces= $-2+1j$
 $-2-1j$



Scalar input-output linear systems and feedback control



s-plane
 $s = \sigma + j\omega$

Significance of root location in the s-plane.

$$G(s) = K \frac{\prod (s - z_j)}{\prod (s - p_i)} \frac{\prod [(s + \alpha_j)^2 + \beta_j^2]}{\prod [(s + \alpha_i)^2 + \beta_i^2]} = \frac{K \prod (1 + z_j s) \prod [1 + 2\xi_j z_j s + (\zeta_j s)^2]}{\prod (1 + p_i s) \prod [1 + 2\xi_i \zeta_i s + (\zeta_i s)^2]}$$