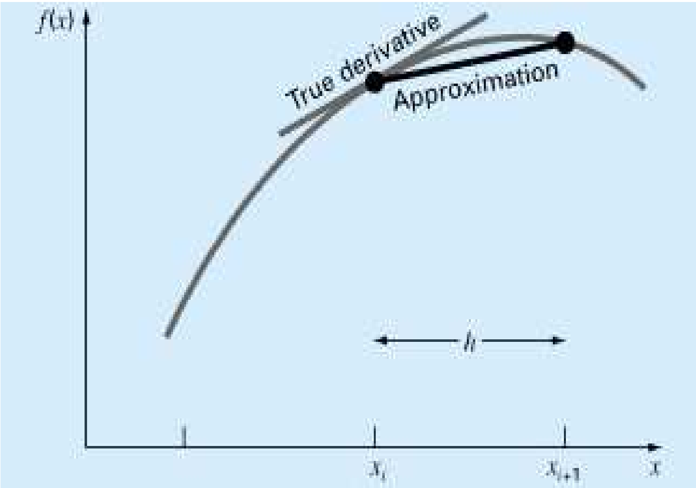
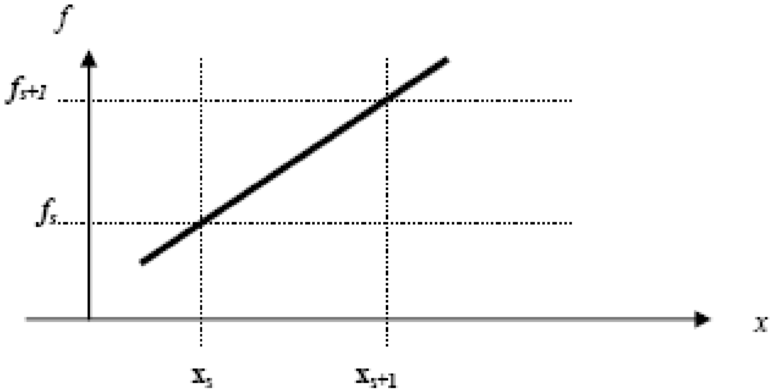
RESUMEN

DERIVADAS PRIMERAS

Hacia adelante

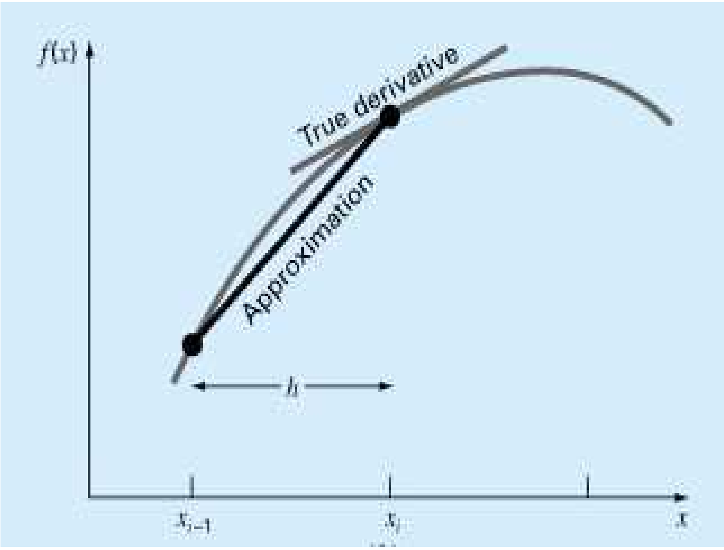
$$f_s' = \frac{1}{h}\left[f_{s+1} - f_s\right] - O(h)$$

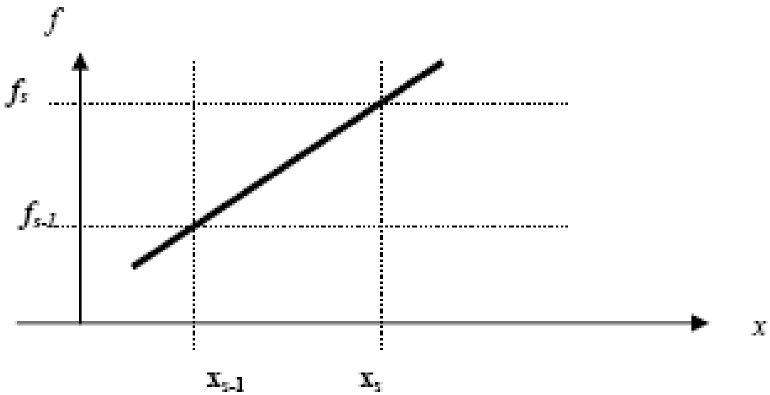




Hacia atrás

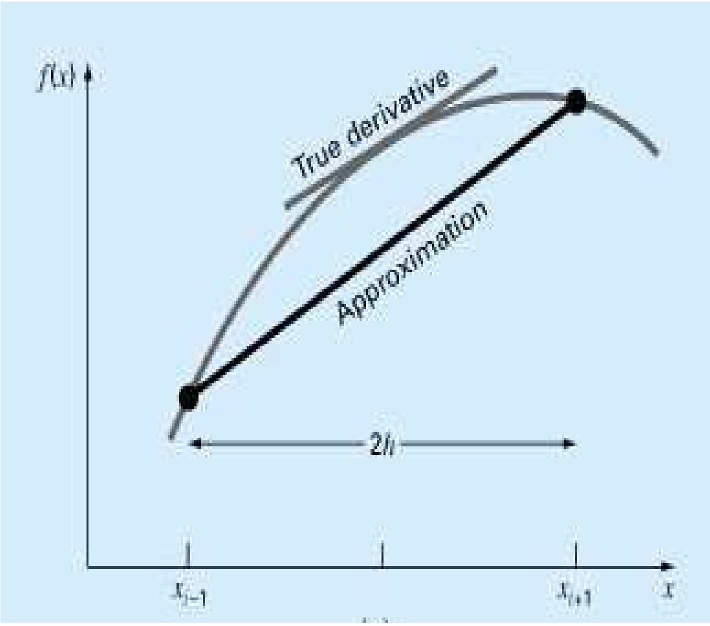
$$f'_{s-1} = \frac{1}{h}\left[f_s - f_{s-1}\right] + O(h)$$

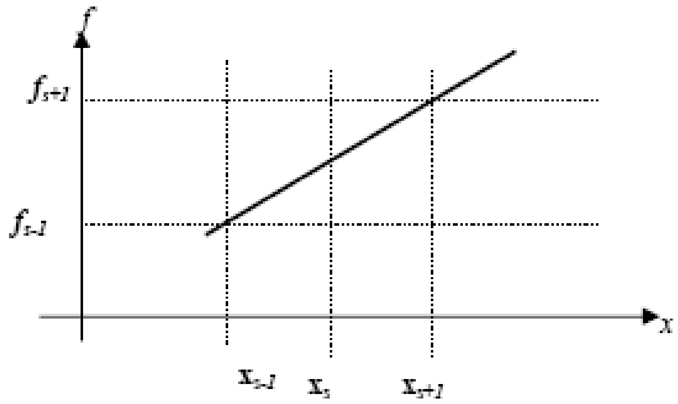




Central

$$f'_s = \frac{[f_{s+1} - f_{s-1}]}{2h} + O(h^2)$$





3 DERIVADAS SEGUNDAS

$$f''_s = \frac{1}{h^2}[f_{s+1} - 2f_s + f_{s-1}] + O(h^2)$$

4 DERIVADA TERCERA

$$f'''_s = \frac{1}{2h^3}[-f_{s-2} + 2f_{s-1} - 2f_{s+1} + f_{s+2}] - O(h^2).$$

DERIVADA CUARTA

Considerando los desarrollos en serie de Taylor de la función para

$n = -2$ $$f_{s-2} = f_s - 2h\,f_s' + 2h^2\,f_s'' - \frac{4}{3}h^3\,f_s''' + \frac{2}{3}h^4\,f_s^{iv} - \frac{4}{15}h^5\,f_s^{v} + \cdots$$

$n = -1$ $$f_{s-1} = f_s - h\,f_s' + \frac{h^2}{2}f_s'' - \frac{h^3}{6}f_s''' + \frac{h^4}{24}f_s^{iv} - \frac{h^5}{120}f_s^{v} + \cdots$$

$n = 0$ $$f_s = f_s$$

$n = +1$ $$f_{s+1} = f_s + h\,f_s' + \frac{h^2}{2}f_s'' + \frac{h^3}{6}f_s''' + \frac{h^4}{24}f_s^{iv} + \frac{h^5}{120}f_s^{v} + \cdots$$

$n = +2$ $$f_{s+2} = f_s + 2h\,f_s' + 2h^2\,f_s'' + \frac{4}{3}h^3\,f_s''' + \frac{2}{3}h^4\,f_s^{iv} + \frac{4}{15}h^5\,f_s^{v} + \cdots$$

Si se truncan las series en el término de cuarto orden se obtiene el siguiente sistema:

$$\begin{Bmatrix} f_{s-2} \\ f_{s-1} \\ f_s \\ f_{s+1} \\ f_{s+2} \end{Bmatrix} = \begin{bmatrix} 1 & -2h & 2h^2 & -\frac{4}{3}h^3 & \frac{2}{3}h^4 \\ 1 & -h & \frac{h^2}{2} & -\frac{h^3}{6} & \frac{h^4}{24} \\ 1 & 0 & 0 & 0 & 0 \\ 1 & h & \frac{h^2}{2} & \frac{h^3}{6} & \frac{h^4}{24} \\ 1 & 2h & 2h^2 & \frac{4}{3}h^3 & \frac{2}{3}h^4 \end{bmatrix} \begin{Bmatrix} f_s \\ f_s' \\ f_s'' \\ f_s''' \\ f_s^{\prime v} \end{Bmatrix}$$

La solución del sistema es:

$$f_s^{\prime v} = \frac{1}{h^4}\left[f_{s-2} - 4f_{s-1} + 6f_s - 4f_{s+1} + f_{s+2}\right]$$

$$f_s''' = \frac{1}{2h^3}\left[-f_{s-2} + 2f_{s-1} + 0f_s - 2f_{s+1} + f_{s+2}\right]$$

$$f_s'' = \frac{1}{h^2}\left[-\frac{1}{12}f_{s-2} + \frac{4}{3}f_{s-1} - \frac{5}{2}f_s + \frac{4}{3}f_{s+1} - \frac{1}{12}f_{s+2}\right]$$

$$f_s' = \frac{1}{h}\left[\frac{1}{12}f_{s-2} - \frac{2}{3}f_{s-1} + 0f_s + \frac{2}{3}f_{s+1} - \frac{1}{12}f_{s+2}\right]$$

DERIVADA PRIMERA ASIMÉTRICA

OBTENER:
Fórmula de derivada primera hacia delante con orden de error superior a uno.

⇩

Se consideran tres puntos equidistantes:

$X_s \quad X_{s+1} \quad X_{s+2}$

⇩

Plantear derivada primera como combinación lineal de los valores de la función, cuya derivada se pretende calcular, en esas abscisas. Se propone:

$$f_s' = \left[\alpha \cdot f_s + \beta \cdot f_{s+1} + \gamma \cdot f_{s+2}\right] \quad (1)$$

Considerando S. de Taylor de f en dichas abscisas:

$$n = +1 \qquad f_{s+1} = f_s + h\,f_s' + \frac{h^2}{2} f_s'' + \frac{h^3}{6} f_s''' + \frac{h^4}{24} f_s^{iv} + \frac{h^5}{120} f_s^{v} + \cdots$$

$$n = +2 \qquad f_{s+2} = f_s + 2\,h\,f_s' + 2\,h^2\,f_s'' + \frac{4}{3} h^3\,f_s''' + \frac{2}{3} h^4 f_s^{iv} + \frac{4}{15} h^5 f_s^{v} + \cdots$$

reemplazando en (1) y agrupando:

$$f_s' = [\alpha + \beta + \gamma]\cdot f + f_s' \cdot [\beta \cdot h + \gamma \cdot 2h] + \frac{h^2}{2} f_s'' \cdot [\beta + 4 \cdot \gamma] + \frac{h^3}{6} f_s''' \cdot [\beta + 8 \cdot \gamma] + \frac{h^4}{24} f_s^{iv} [\beta + 16 \cdot \gamma] + \cdots \qquad (1')$$

Para que la nueva serie obtenida sea igual a la derivada primera en X_s, se debe cumplir que

$$0 = [\alpha + \beta + \gamma]$$
$$1 = [\beta \cdot h + \gamma \cdot 2h] \qquad (2)$$

siendo el error de truncamiento,

$$Er = \frac{h^2}{2} f_\xi'' \cdot [\beta + 4 \cdot \gamma] \qquad (2')$$

De (2):

$$\alpha = [-1/h + \gamma]$$
$$\beta = [1/h - 2 \cdot \gamma] \qquad (3)$$

De (3) en (1):

$$f_s' = [[-1/h + \gamma]\cdot f_s + [1/h - 2 \cdot \gamma]\cdot f_{s+1} + \gamma \cdot f_{s+2}]$$

(4)

con error ((3) en (2´)):

$$Er = \frac{h^2}{2} f_\xi'' \cdot [1/h + 2\cdot\gamma] \qquad (5)$$

Válido para todo γ .

Para $\gamma = 0$ => se recupera la derivada primera adelante y su error de truncamiento

Para $\gamma \neq 0$ => error de truncamiento local depende γ y de h linealmente.

<u>Caso $\gamma = -1/(2h)$</u>

Reemplanando en (4):

$$f_s' = \{[-3/(2h)]\cdot f_s + [2/h]\cdot f_{s+1} + [-1/(2h)] f_{s+2}\}$$

y el error de trocamiento resulta nulo (ver al reemplazar $\gamma = -1/(2h)$ en (5)).

Si ahora (recordando (1´)):

$$f_s' = [\alpha+\beta+\gamma]\cdot f + f_s'\cdot[\beta\cdot h + \gamma\cdot 2h] + \frac{h^2}{2} f_s''\cdot[\beta + 4\cdot\gamma] + \frac{h^3}{6} f_s'''\cdot[\beta + 8\cdot\gamma] + \frac{h^4}{24} f_s^{iv}[\beta + 16\cdot\gamma] + \cdots$$

el error de truncamiento resulta:

$$Er = \frac{h^3}{6} f_s'''\cdot[\beta + 8\cdot\gamma] = \frac{h^3}{6} f_s'''\cdot[2/h] = \frac{h^2}{3} f_s'''\cdot$$

=> derivada primera hacia adelante considerando tres puntos es exacta hasta polinomios de grado 2 y el orden del error de truncamiento local es de h^2.

RESUMEN DE DERIVADAS ASIMÉTRICAS Y CENTRALES

$$f_s' = (1/(2h))\cdot\left[-3\cdot f_s + 4\cdot f_{s+1} - 1\cdot f_{s+2}\right]$$

$$f_s'' = (1/h^2)\cdot\left[2\cdot f_s - 5\cdot f_{s+1} + 4\cdot f_{s+2} - 1\cdot f_{s+3}\right]$$

$$f_s' = (1/(2h))\cdot\left[-f_{s-1} + 0\cdot f_s + f_{s+1}\right]$$

$$f_s'' = (1/h^2)\cdot\left[f_{s-1} - 2\cdot f_s + f_{s+1}\right]$$

$$f_s' = (1/(2h))\cdot\left[3\cdot f_s - 4\cdot f_{s-1} + 1\cdot f_{s-2}\right]$$

$$f_s'' = (1/h^2)\cdot\left[2\cdot f_s - 5\cdot f_{s-1} + 4\cdot f_{s-2} - 1\cdot f_{s-3}\right]$$