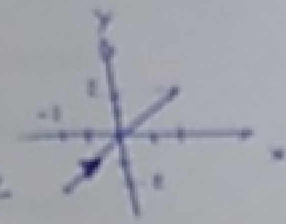
T.P.1

1. ⓐ $r(t)=(t,t)\quad -2\le t\le 2$

$\begin{cases} x=t \\ y=t \\ -2\le t\le 2 \end{cases}$ $\quad r_0(-2)=(-2,-2)$ $\quad r_f(2)=(2,2)$ $\quad y=x,\ -2\le x\le 2$



3 ⓐ continuidad:

$\left.\begin{array}{l} x=t \\ y=t \end{array}\right\}$ fcs continuas luego $r(t)$ continua.

ⓑ $r'(t)=(x'(t),y'(t))=(1,1)$

r' continuo y $r'\neq(0,0)$ $\forall t$ luego $r(t)$ suave.

ⓒ rapidez $\|r'\|=\sqrt{x'^2+y'^2}=\sqrt{1^2+1^2}=\sqrt{2}$ ctte.

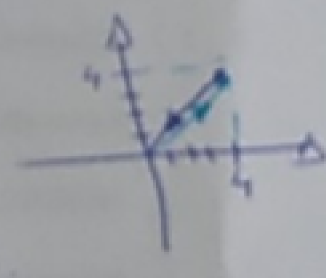
$a(t)=r''(t)=(0,0)$.

$\vec{T}(t)=\frac{1}{\|r'\|}r'=\frac{1}{\sqrt{2}}(1,1)=(1/\sqrt{2},1/\sqrt{2})$ ctte

nana

1 ⓑ $r(t)=(t^2,t^2)\quad -2\le t\le 2$

$\begin{cases} x=t^2 \\ y=t^2 \\ -2\le t\le 2 \end{cases}$ continuas $r(t)$ continua. $\quad r_0(-2)=(4,4)$ $\quad r_f(2)=(4,4)$ $\quad y=x,\ 0\le x\le 4$ (dos veces).



3 ⓐ $r(t)$ continua.

ⓑ $r'(t)=(2t,2t)$ continuos

pero $r'(0)=(0,0)$ luego no es suave

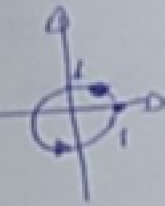
ⓒ rapidez $\|r'\|=\sqrt{(2t)^2+(2t)^2}=\sqrt{8t^2}=\sqrt{2^2\,2t^2}=2\sqrt{2}|t|$

$a(t)=(2,2)$ ctte.

$\vec{T}(t)$ no existe para $t=0$. (no es suave)

1 (d) $r(t) = (\cos t, \sin t) \quad 0 \le t \le 2\pi$

$$\begin{cases} x = \cos t \\ y = \sin t \\ 0 \le t \le 2\pi \end{cases} \qquad \begin{aligned} r_0(0) &= (1,0) \\ r_f(2\pi) &= (1,0) \end{aligned} \qquad x^2+y^2=1$$



3 (a) $\begin{matrix} x = \cos t \\ y = \sin t \end{matrix}$ continuas luego $r(t)$ continua.

(b) $r'(t) = (-\sin t, \cos t)$

$r'(t)$ continua y $r'(t) \neq (0,0) \ \forall t$.
r es suave.

(c) rapidez $\|r'\| = \sqrt{(-\sin t)^2 + (\cos t)^2} = \sqrt{1} = \underset{\text{cte}}{1}$.

aceleración $a(t) = (-\cos t, -\sin t)$. $a(\pi) = (+1, 0)$

$\vec{T}(t) = \frac{1}{\|r'\|} r' = \frac{1}{1}(-\sin t, \cos t) = (-\sin t, \cos t)$

$\vec{T}(\pi) = (0, 1)$
punto medio intervalo

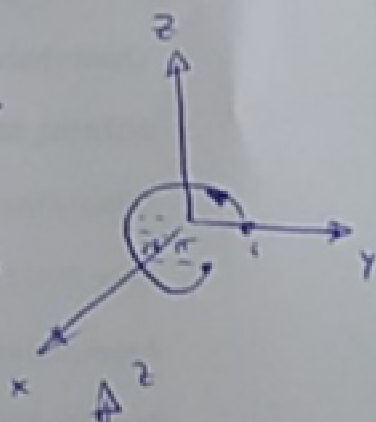
$\vec{N}(t) = \frac{\vec{T}'}{\|\vec{T}'\|} = (-\cos t, -\sin t)$.

$\vec{T}'(t) = (-\cos t, -\sin t)$ $\qquad \vec{N}(\pi) = (1, 0)$.

$\|\vec{T}'(t)\| = 1$

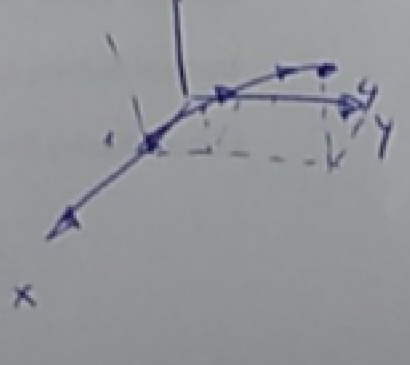
1 (g) $r(t) = (t, \cos t, \sin t) \quad 0 \le t \le 2\pi$

$$\begin{cases} x = t \\ y = \cos t \\ z = \sin t \end{cases} \qquad \begin{aligned} r_0(0) &= (0,1,0) \\ r_f(2\pi) &= (2\pi,1,0). \end{aligned} \qquad \begin{cases} x = t \\ y^2+z^2=1 \end{cases}$$



1 (h) $r(t) = (1, t^2, t) \quad 0 \le t \le 2$

$$\begin{cases} x = 1 \\ y = t^2 \\ z = t \end{cases} \qquad \begin{aligned} r_0(0) &= (1,0,0) \\ r_f(2) &= (1,4,2) \end{aligned} \qquad \begin{cases} x = 1 \\ y = z^2 \end{cases}$$



(6) $r(t)$ diferenciable sobre $x(t), y(t), z(t)$ continuas y derivables, $r' \neq \vec{0}$ si $\|r\| = c$ entonces $r' \perp r$ para todo t:

$$\|r(t)\| = c$$

$$\|r(t)\| = (r \cdot r)^{1/2} = c$$

derivando.

$$\frac{1}{2}(r \cdot r)^{-1/2} \cdot (r' \cdot r + r \cdot r') = 0$$

$$\frac{1}{\cancel{2}(r \cdot r)^{1/2}}(\cancel{2}\, r' \cdot r) = 0$$

$$\frac{1}{c}\, r' \cdot r = 0$$

$$r' \cdot r = 0 \ (\forall t)$$

luego $r' \perp r$ $(\forall t)$.