TPS

17 c) $y' + 3x^2 y = x^2$

$$y_g(x) = e^{-\int 3x^2dx}\left[\int e^{\int 3x^2dx} \cdot x^2\,dx + C\right] =$$

$$= e^{-x^3}\left[\int e^{x^3} x^2\,dx + C\right] =$$

$u = x^3$

$du = 3x^2dx$

$$= e^{-x^3}\left[\int e^{u} \cdot \frac{1}{3}\,du + C\right] =$$

$$= e^{-x^3}\left[\frac{1}{3}e^{u} + C\right] =$$

$$= e^{-x^3}\left[\frac{1}{3}e^{x^3} + C\right] =$$

$$\boxed{y_p(x) = \frac{1}{3} + Ce^{-x^3}}$$

$D = I = \mathbb{R}$

$= (-\infty, \infty)$

(24) $(4xy + 3x^2)dx + (2y + 2x^2)dy = 0$

(a) $\left.\begin{array}{l} M_y = 4x \\ N_x = 4x \end{array}\right\} =$ cont es forma dif Exacta.

$f(x,y) = \int (4xy + 3x^2)\,dx = 2x^2y + x^3 + \alpha(y)$

$f(x,y) = \int (2y + 2x^2)\,dy = 2x^2y + y^2 + \beta(x)$

$f(x,y) = 2x^2y + x^3 + y^2 + K$

$\boxed{2x^2y + x^3 + y^2 = C}$

(b) $y(0) = -2$

$2 \cdot 0 \cdot (-2) + 0 + (-2)^2 = C$

$\boxed{C = 4}$

$y(1) = 1$

$2 \cdot 1 \cdot 1 + 1 + 1 = C$

$\boxed{C = 4}$

(c) $2x^2y + x^3 + y^2 = 4$

$$y = \frac{-2x^2 \pm \sqrt{(2x^2)^2 - 4 \cdot 1 (x^3 - 4)}}{2 \cdot 1}$$

$$= \frac{-2x^2 \pm \sqrt{4x^4 - 4x^3 + 16}}{2}$$

$$= -x^2 \pm \sqrt{x^4 - x^3 + 4}$$

$y_1(x) = -x^2 - \sqrt{x^4 - x^3 + 4}$

$y_1(0) = -2$

$y_2(x) = -x^2 + \sqrt{x^4 - x^3 + 4}$

$y_2(1) = 1$

(29)

(a) $y' + \frac{1}{x} y = 1$ L si Bernoulli ✓

V.S no.

$y' = \frac{x-y}{x}$

$x\,dy = (x-y)\,dx$ Exacto si

$(y-x)\,dx + x\,dy = 0$

$\left.\begin{array}{l} M_y = 1 \\ N_x = 1 \end{array}\right\}$

(b) $\frac{dy}{dx} = \frac{1}{y-x}$ L no B no Exacto no.

V.S no.

$\frac{dx}{dy} = y - x$ Lineal si B ✓

Exacto no V.S. no.

(c) $(x+1)\frac{dy}{dx} = -y+10$

$(x+1)y' + y = 10$ L si B ✓

$(x+1)\,dy = (-y+10)\,dx$

$(y+10)\,dx + (x+1)\,dy = 0$ Exacto si

$\frac{1}{10-y}\,dy = \frac{1}{x+1}\,dx$ V.S. si

(d) $(y^2+1)\,dx = y\sec^2 x\,dy$ L no B ✓

$\cos^2 x\,dx = \frac{y}{y^2+1}\,dy$ V.S ✓

Exacto ✓

$y + \frac{1}{y} = \sec^2 x\,y'$ $y' - \cos^2 x\,y = \cos^2 x\,y^{-1}$

(e) $y(\ln x - \ln y)\,dx = (x\ln x - x\ln y - y)\,dy$ L no

B no

V.S. no

E no.

$M_y = (\ln x - \ln y) + y \cdot -\frac{1}{y}$

$= \ln x - \ln y + 1.$

$N_x = -(1 \cdot \ln x + x \cdot \frac{1}{x} - \ln y)$

$= -\ln x - 1 + \ln y$

(15) $\underbrace{\frac{dx}{dt} = \underbrace{4}_{h(t)}\underbrace{(x^2+1)}_{f(x)}}$, $\underbrace{x\left(\frac{\pi}{4}\right) = 1}$

$y' = f(x)\,h(y)$

$x'(t) = f(x) \cdot h(t)$

$$\int \frac{1}{x^2+1}\,dx = \int 4\,dt$$

$\arctan(x) = 4t + C$

$x(t) = \tan(4t + C)$

$\tan\left(4 \cdot \frac{\pi}{4} + C\right) = 1$

$\tan(\pi + C) = 1$

$\pi + C = \pi/4 \to C = \pi/4 - \pi = -\frac{3}{4}\pi$

$$\boxed{x(t) = \tan\left(4t - \tfrac{3}{4}\pi\right)}$$

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$y = Ae^{x} + Be^{-x}$ $\qquad y(0) = 1 \qquad y'' - y = 0$

$y' = Ae^{x} - Be^{-x}$ $\qquad y'(0) = 2$

$$\left.\begin{aligned} y(0) &= Ae^{0} + Be^{-0} = \\ y'(0) &= Ae^{0} - Be^{-0} = \end{aligned}\right. \boxed{\begin{aligned} A + B &= 1 \\ A - B &= 2 \end{aligned}}$$

$$A = 3/2 \qquad B = -1/2$$

$$\boxed{y = \frac{3}{2}e^{x} - \frac{1}{2}e^{-x}}$$

(5) $y = e^{mx}$ $\quad y'' - 5y' + 6y = 0$

$y' = me^{mx}$

$y'' = m^2 e^{mx}$

$$m^2 e^{mx} - 5(me^{mx}) + 6e^{mx} = 0$$

$$e^{mx}(m^2 - 5m + 6) = 0$$

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0 \rightarrow m = 2, m = 3$$

$y_1 = e^{2x}$, $\quad y_2 = e^{3x}$

$y_1' = 2e^{2x}$

$y_1'' = 4e^{2x}$

$$\boxed{\begin{gathered} 4e^{2x} - 5(2e^{2x}) + 6e^{2x} = 0 \\ e^{2x}(\underbrace{4 - 10 + 6}_{0}) = 0 \end{gathered}}$$

$$\boxed{y_c = c_1 e^{2x} + c_2 e^{3x}}$$ ← Ver que tb es solución!

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i 3er prófico
ii 1er prófico
iii 2do prófico

(29) (f) $t\frac{dQ}{dt}+Q=t^4\cdot\ln t$

L si B no
E si V.S no.

$Q'+\frac{1}{t}Q=t^3\ln t$

(g) $(2x+y+1)y'=1$

L no B no
E no V.S no