



UNIVERSIDAD
NACIONAL DE CUYO



FACULTAD DE INGENIERIA
en acción continua...

ANALISIS ESTRUCTURAL I

UNIDAD 3

MÉTODO DE LAS FUERZAS

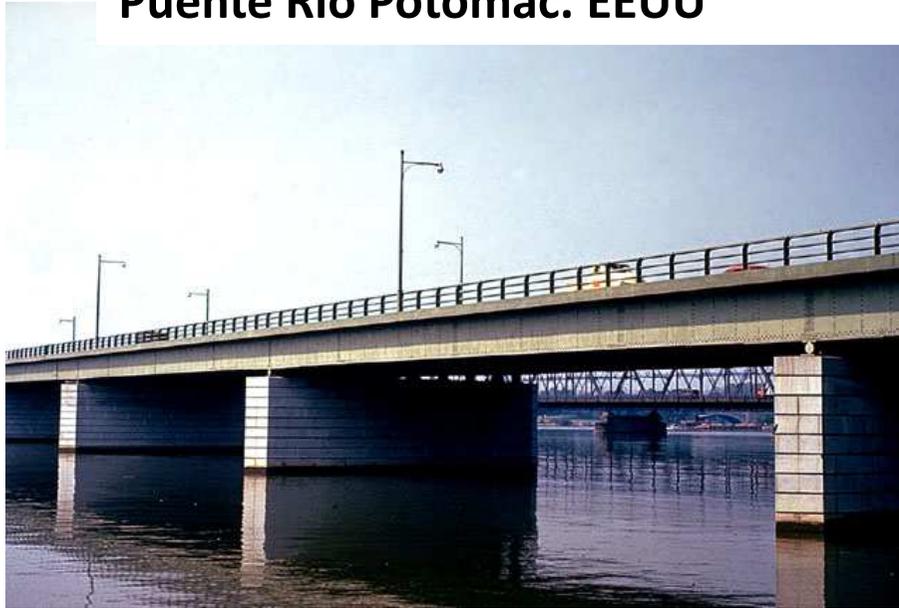
Curso 2.023

Mg. Ing. DANIEL E. LÓPEZ

ESTRUCTURAS HIPERESTÁTICAS

Introducción

Puente Rio Potomac. EEUU



Autopista Sacatekas. DF. México



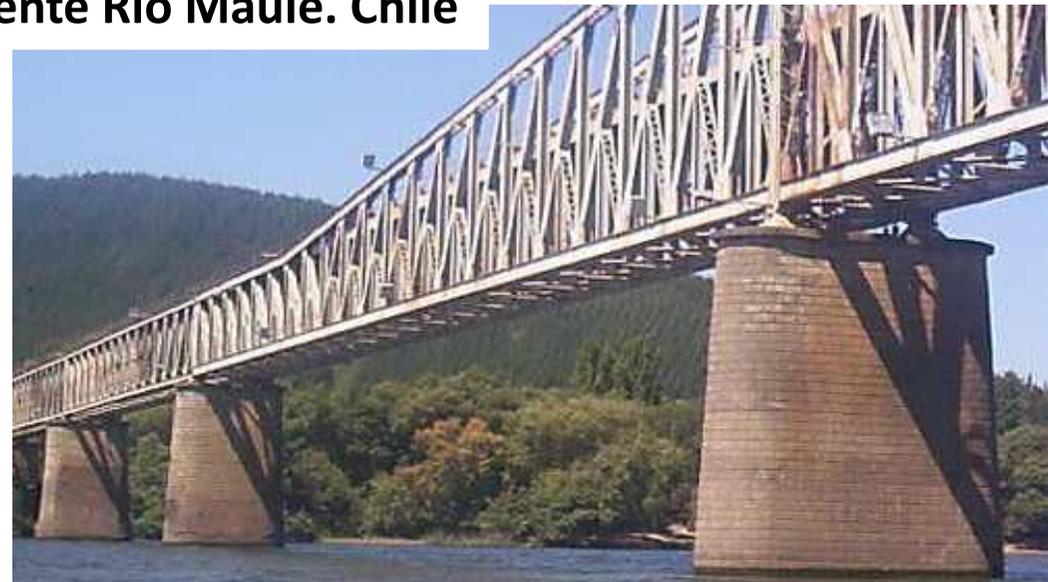
ESTRUCTURAS HIPERESTÁTICAS

Introducción

Nave Bodega en Mendoza

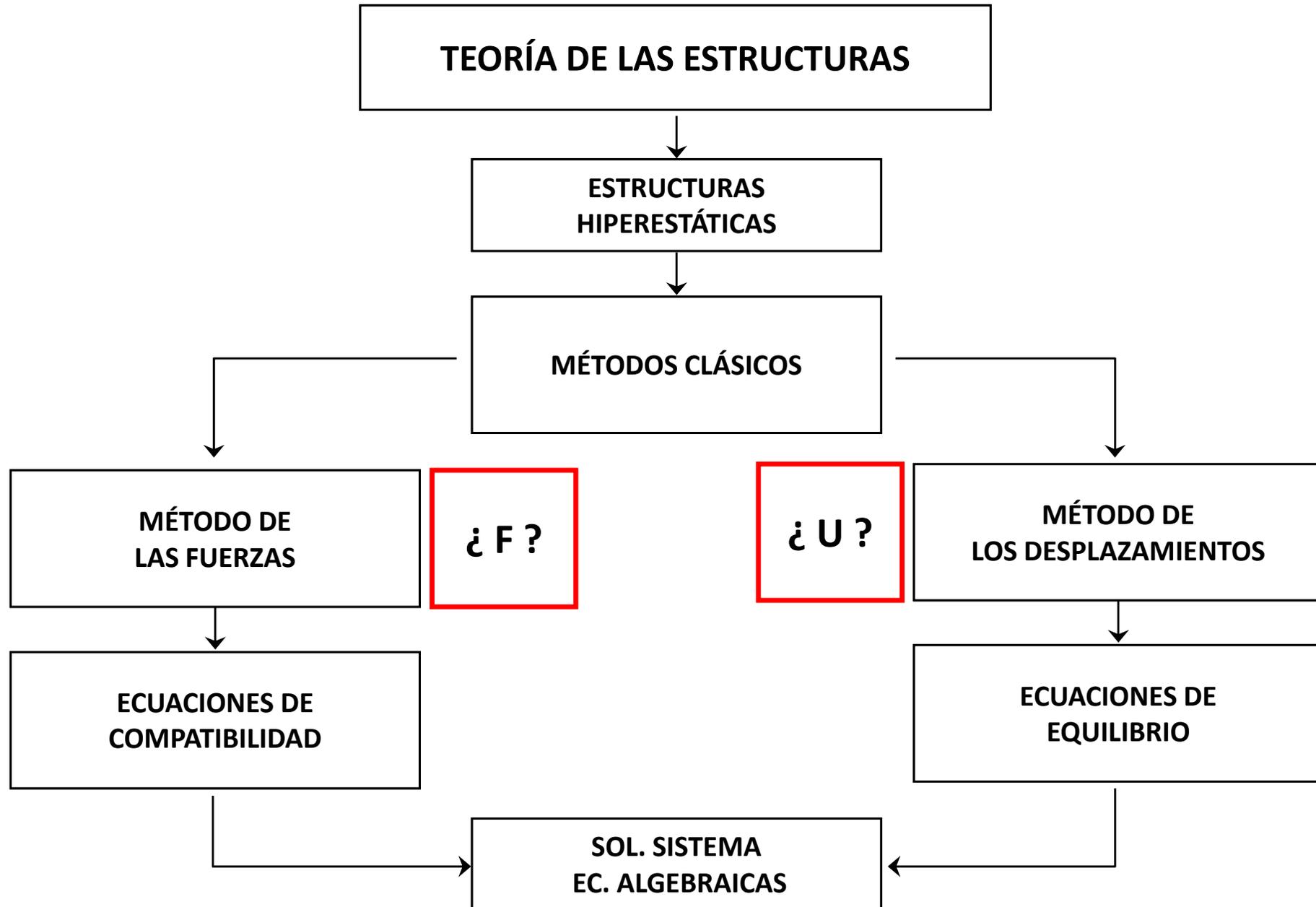


Puente Río Maule. Chile



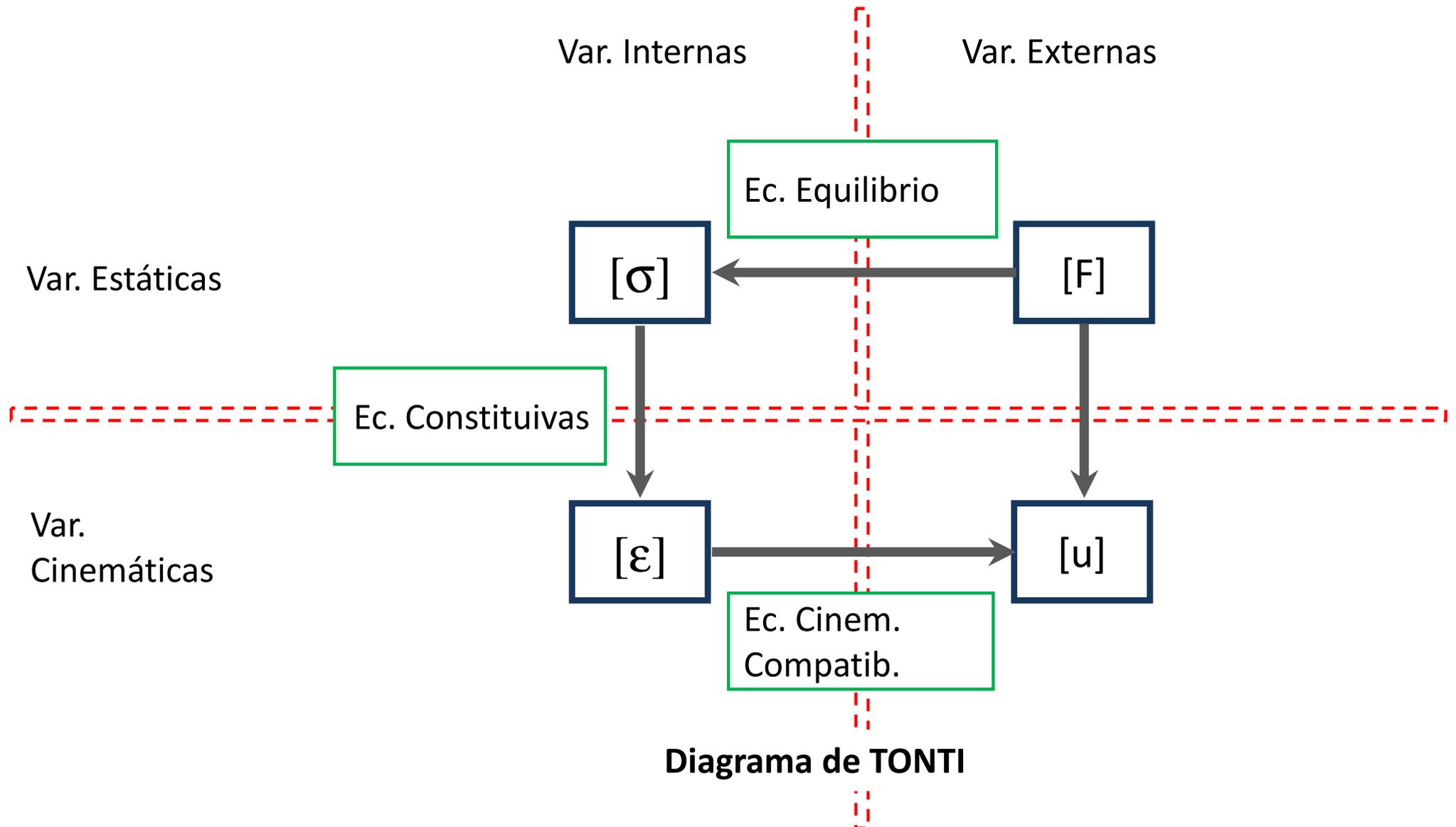
MÉTODOS CLÁSICOS

Introducción



MÉTODOS CLÁSICOS

Introducción



GRADO DE HIPERESTATICIDAD

Introducción

$$GH = I - E$$

GH : Grado de hiperestaticidad.

I : Cantidad de Incógnitas hiperestáticas.

E : Ecuaciones de la estática.

$GH < 0$: Estructura Hipostática

$GH = 0$: Estructura Isostática

$GH > 0$: Estructura Hiperestática

$GH < 0 \rightarrow$ Estructura Inestable

$GH \geq 0 \rightarrow$ Condición necesaria
Estructura Estable

GRADO DE HIPERESTATICIDAD

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E : Ecuaciones de la estática.

$$I = V_s + B + 2 A^1 + 3 R + 4 A^2$$

$$E = 3 N + 2 V$$

V_s : Vínculos simples

B : Barras tracción/compresión

A^1 : Articulación de 1° especie

R : Recintos cerrados / Unión rígida

A^2 : Articulación de 2° especie

N : Número Chapas

V : Número Vértices

GRADO DE HIPERESTATICIDAD

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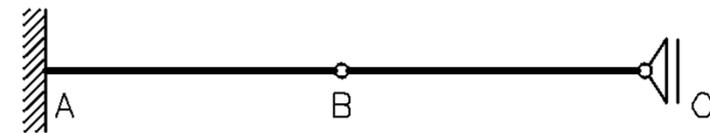
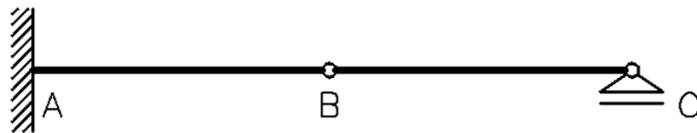
R : Recintos cerrados / Unión rígida

A^2 : Articulación de 2° especie

$$E = 3N + 2V$$

N : Número Chapas

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Isostática Inestable

GRADO DE HIPERESTATICIDAD

Introducción

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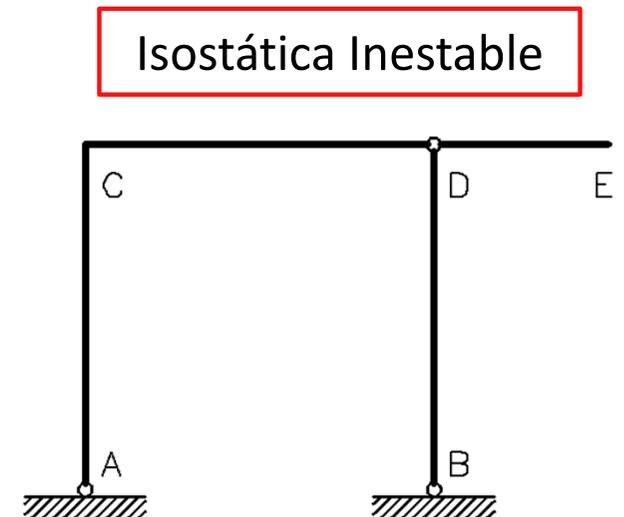
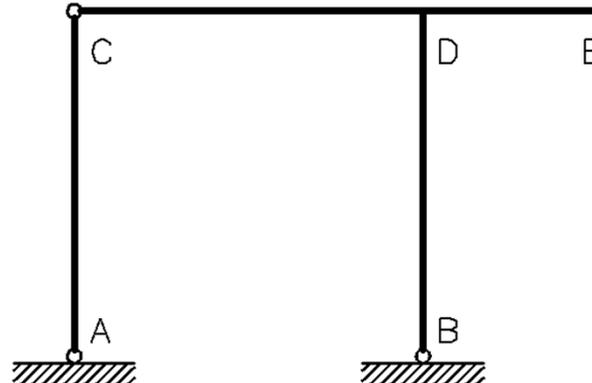
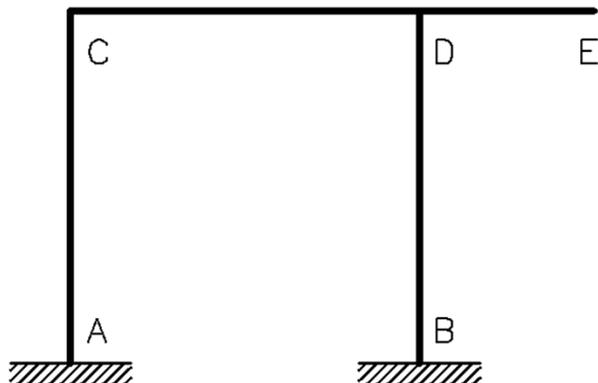
R : Recintos cerrados / Unión rígida

A^2 : Articulación de 2° especie

$$E = 3N + 2V$$

N : Número Chapas

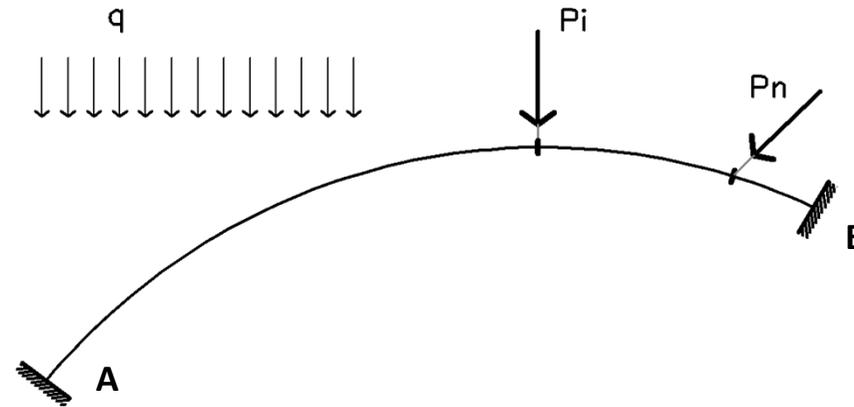
V : Número Vértices



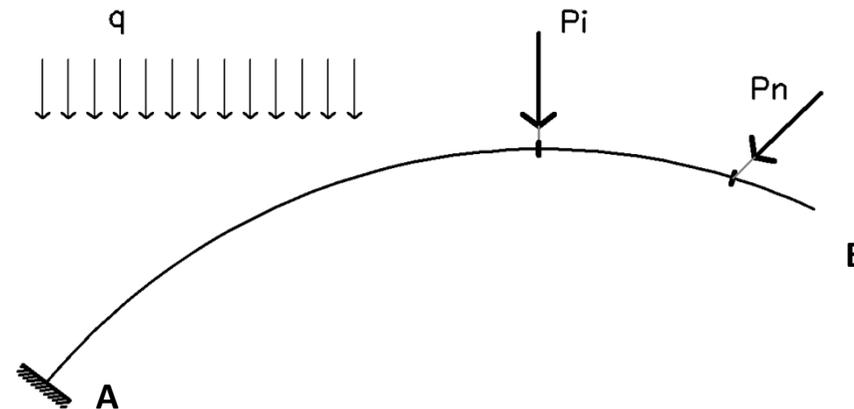
Isostática Inestable

MÉTODO DE LAS FUERZAS

Estructura Original



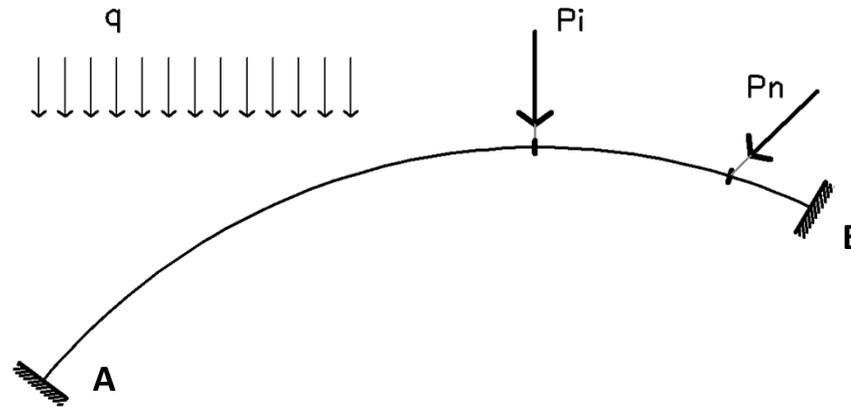
Sistema Fundamental



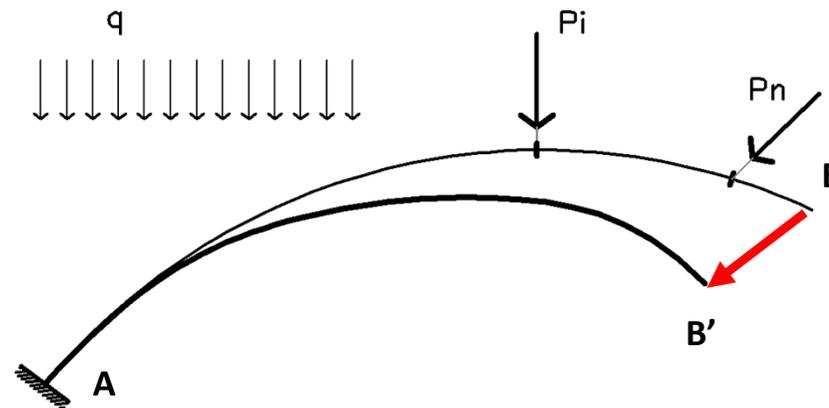
Sistema Fundamental: Se obtiene de la estructura original, eliminando vínculos (int/ext) hasta obtener una estructura isostática estable.

MÉTODO DE LAS FUERZAS

Estructura Original

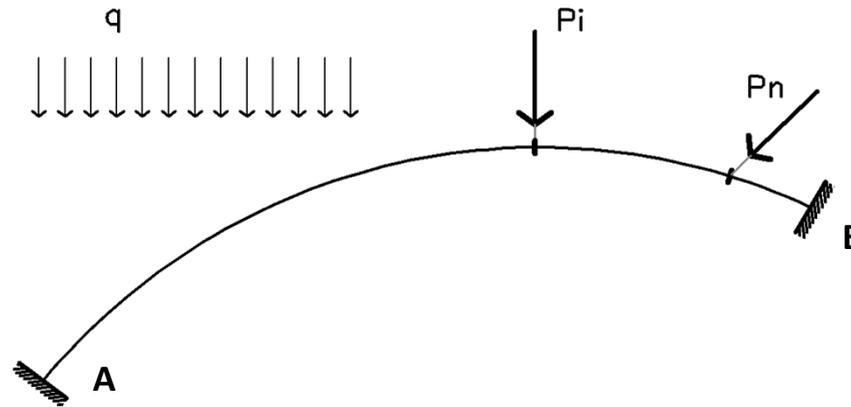


Sistema Fundamental

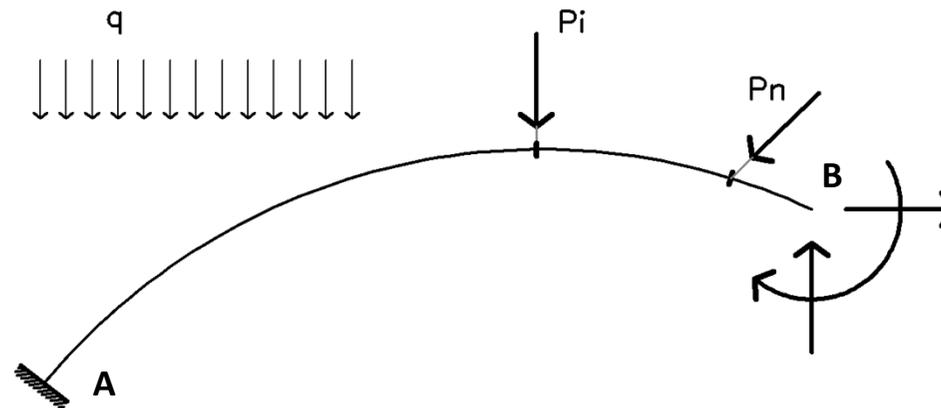


MÉTODO DE LAS FUERZAS

Estructura Original

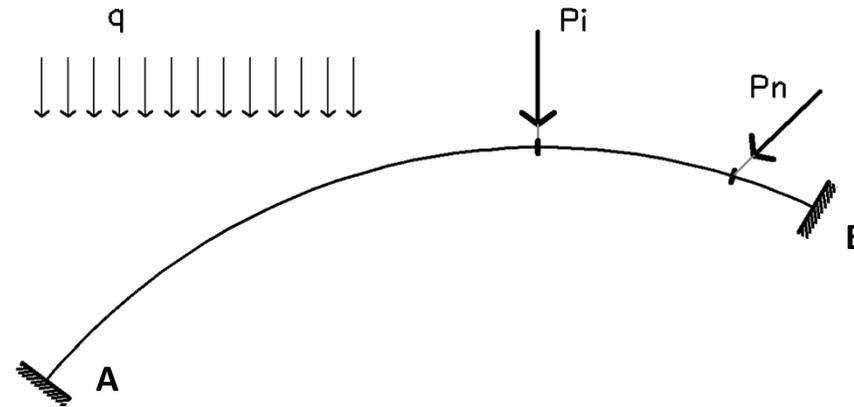


Sistema Fundamental

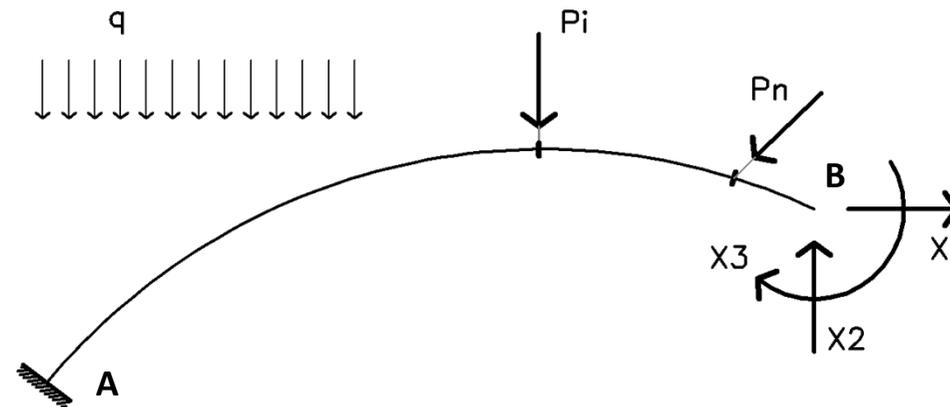


MÉTODO DE LAS FUERZAS

Estructura Original



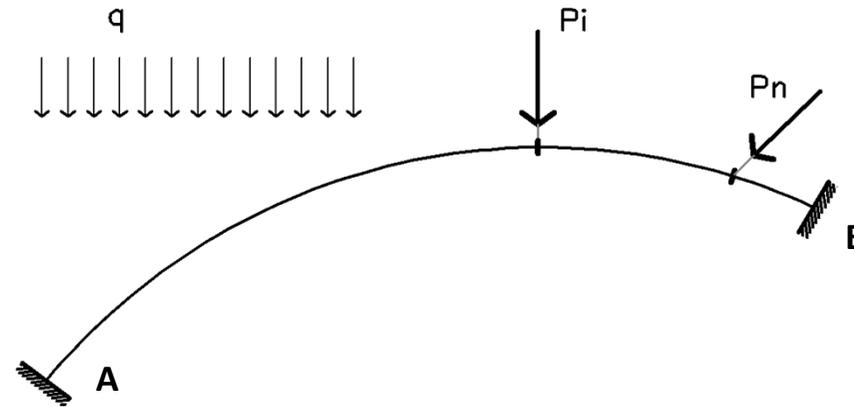
S. Isostático Equivalente



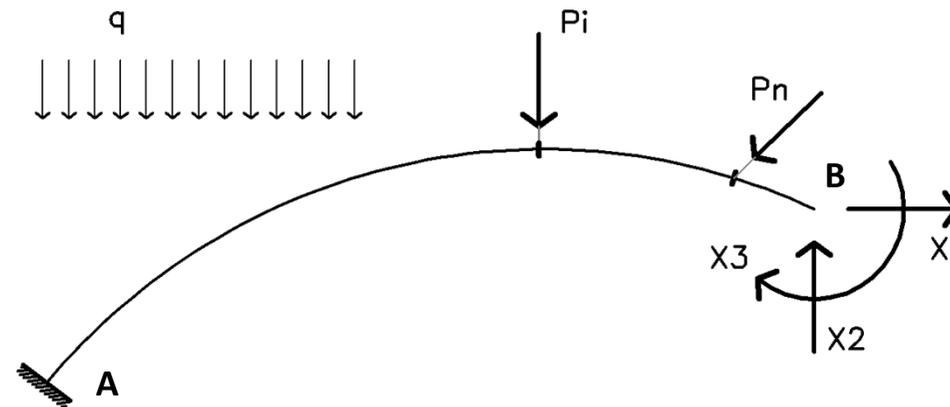
Los valores de las incógnitas X_i deben ser tales que restauren condiciones cinemáticas de la estructura original. Condiciones de Compatibilidad

MÉTODO DE LAS FUERZAS

Estructura Original



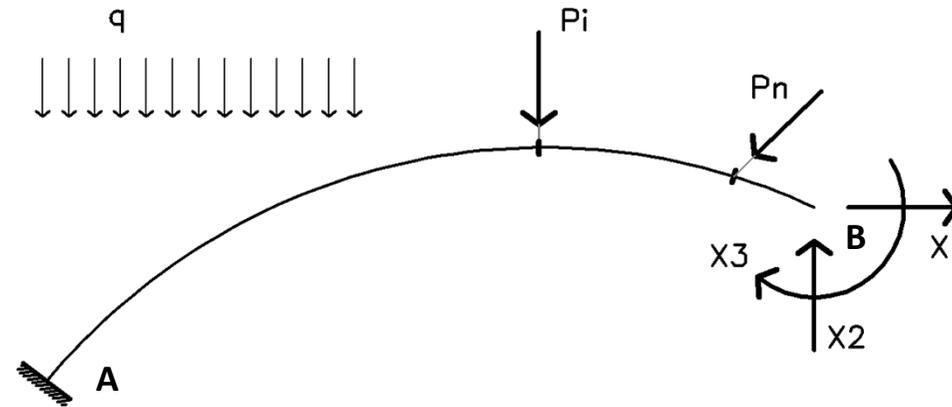
S. Isostático Equivalente



$$\Delta_1 = 0; \quad \Delta_2 = 0; \quad \Delta_3 = 0$$

MÉTODO DE LAS FUERZAS

S. Isostático
Equivalente

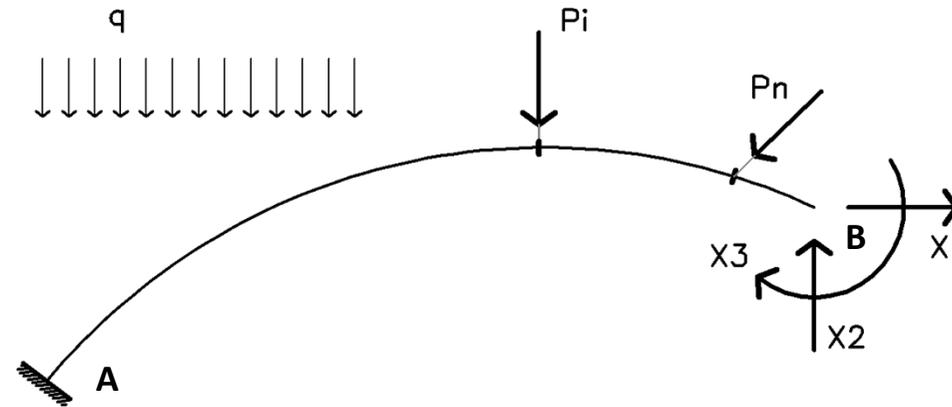


$$\Delta_1 = 0; \quad \Delta_2 = 0; \quad \Delta_3 = 0$$

¿ Quién ? P_0, X_1, X_2, X_3

MÉTODO DE LAS FUERZAS

S. Isostático
Equivalente



$$\Delta_1 = 0; \quad \Delta_2 = 0; \quad \Delta_3 = 0$$

PIASE

$$\Delta_1 = 0 \rightarrow \Delta_{10} + \Delta_{11} + \Delta_{12} + \Delta_{13} = 0$$

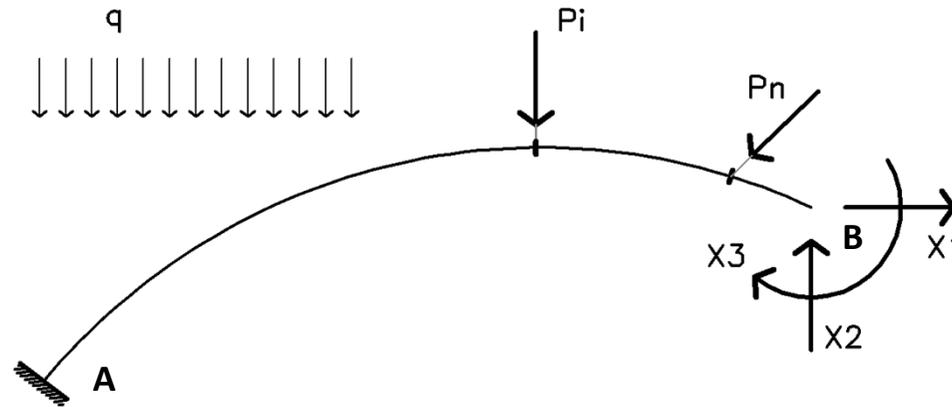
$$\Delta_2 = 0 \rightarrow \Delta_{20} + \Delta_{21} + \Delta_{22} + \Delta_{23} = 0$$

$$\Delta_3 = 0 \rightarrow \Delta_{30} + \Delta_{31} + \Delta_{32} + \Delta_{33} = 0$$

Δ_{ij} : Desplazamiento en la dirección de X_i provocado por una acción que actúa en la dirección X_j

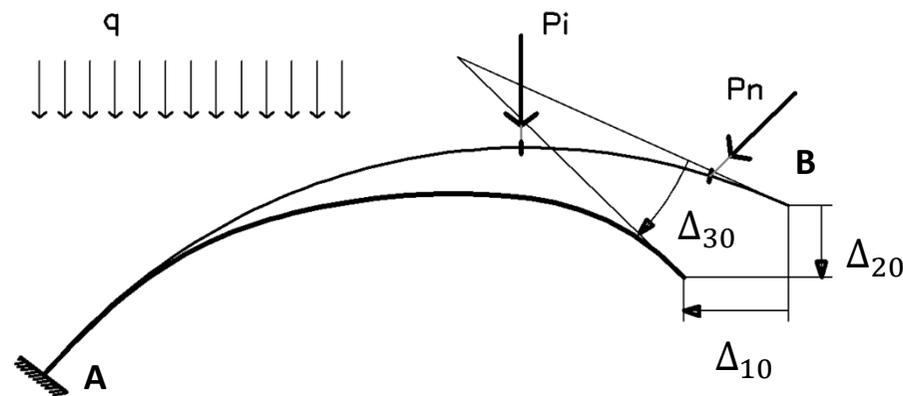
MÉTODO DE LAS FUERZAS

**S. Isostático
Equivalente**



Desplazamientos provocados por P_0 : Δ_{10} ; Δ_{20} ; Δ_{30}

P_0



¿ Donde ?

**Sistema
Fundamental**

MÉTODO DE LAS FUERZAS

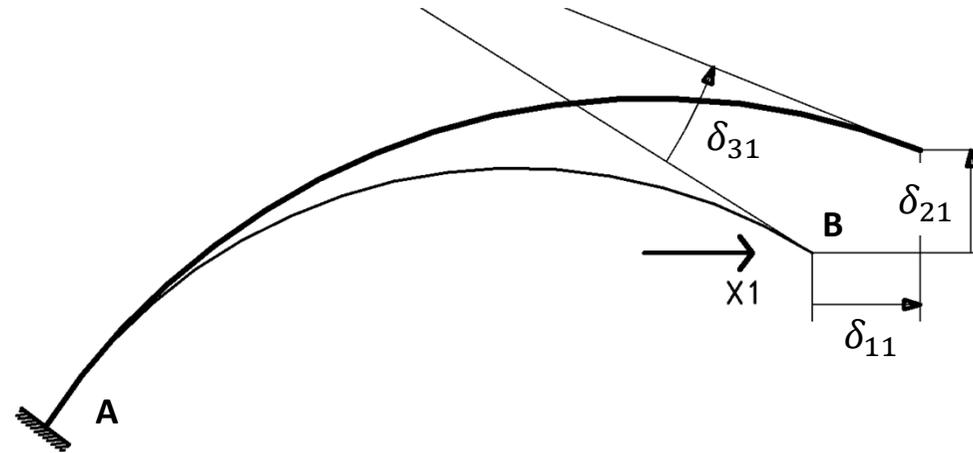
Desplazamientos provocados por $X_1, X_2, X_3 : \Delta_{ij}$

$$X_1 = ?; \quad X_2 = ?; \quad X_3 = ?$$

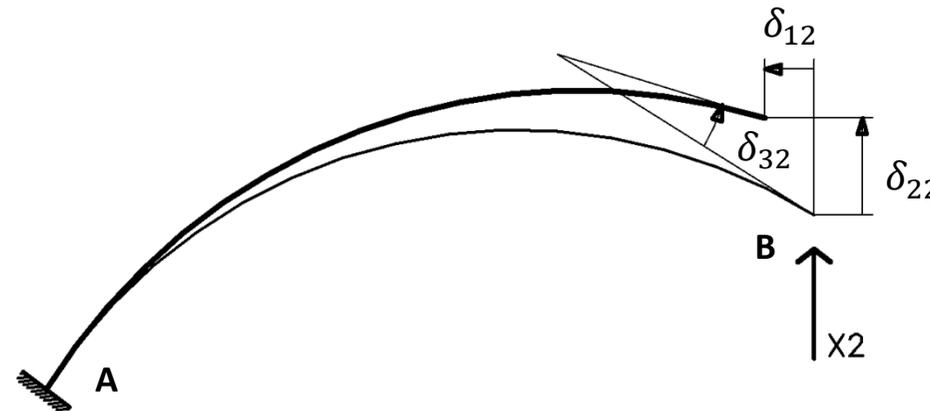
$$\delta_{ij} = \frac{\Delta_{ij}}{P_j}$$

$$\Delta_{ij} = \delta_{ij} X_j$$

$$X_1 = 1$$



$$X_2 = 1$$



**Sistema
Fundamental**

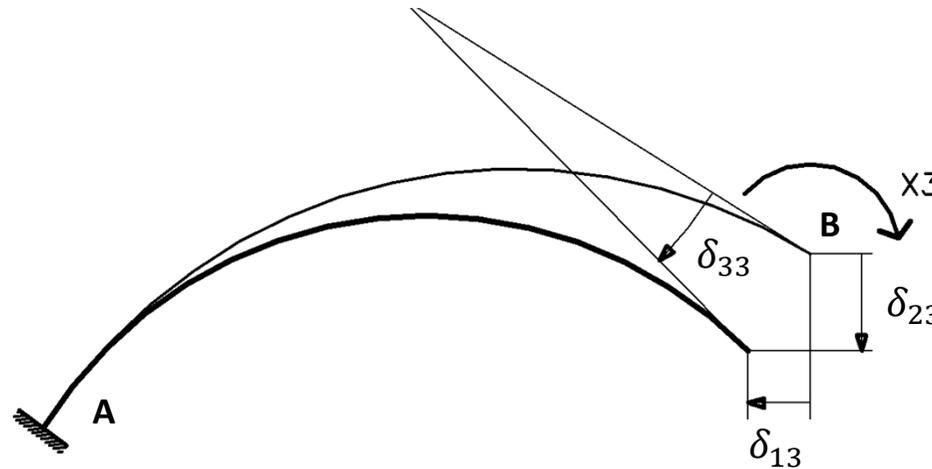
MÉTODO DE LAS FUERZAS

Desplazamientos provocados por $X_1, X_2, X_3 : \Delta_{ij}$

$$X_1 = ?; \quad X_2 = ?; \quad X_3 = ?$$

$$\Delta_{ij} = \delta_{ij} X_j$$

$$X_3 = 1$$



**Sistema
Fundamental**

δ_{ij} : Desplazamiento en la dirección de la incógnita X_i provocado por el valor unitario de la incógnita X_j

MÉTODO DE LAS FUERZAS

Ecuaciones de Compatibilidad

$$\Delta_1 = 0 \rightarrow \Delta_{10} + \Delta_{11} + \Delta_{12} + \Delta_{13} = 0$$

$$\Delta_2 = 0 \rightarrow \Delta_{20} + \Delta_{21} + \Delta_{22} + \Delta_{23} = 0$$

$$\Delta_3 = 0 \rightarrow \Delta_{30} + \Delta_{31} + \Delta_{32} + \Delta_{33} = 0$$

Ecuaciones de Compatibilidad \rightarrow Sistema de Ecuaciones

$$\Delta_1 = 0 \rightarrow \Delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 = 0$$

$$\Delta_2 = 0 \rightarrow \Delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 = 0$$

$$\Delta_3 = 0 \rightarrow \Delta_{30} + \delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 = 0$$

MÉTODO DE LAS FUERZAS

Generalización n Incógnitas

$$\Delta_1 = 0 \rightarrow \Delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 + \cdots + \delta_{1n}X_n = 0$$

$$\Delta_2 = 0 \rightarrow \Delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 + \cdots + \delta_{2n}X_n = 0$$

$$\begin{array}{ccccccc} \cdot & & \cdot & & \cdot & & \cdot \\ \cdot & + & \cdot & + & \cdot & + \cdots + & \cdot \\ \cdot & & \cdot & & \cdot & & \cdot \end{array} = 0$$

$$\Delta_n = 0 \rightarrow \Delta_{n0} + \delta_{n1}X_1 + \delta_{n2}X_2 + \cdots + \delta_{nn}X_n = 0$$

Notación Matricial

$$[\Delta_{i0}] + [\delta_{ij}] [X_j] = 0$$

MÉTODO DE LAS FUERZAS

Generalización n Incógnitas

$$[\Delta_{i0}] + [\delta_{ij}] [X_j] = 0$$

$[\Delta_{i0}]$: Vector de términos independientes.

Desplazamientos en la dirección de las incógnitas, provocados por el sistema de cargas, actuando sobre el sistema fundamental.

$[\delta_{ij}]$: Matriz de Flexibilidad.

Desplazamientos en la dirección de las incógnitas, provocados por los valores unitarios de estas, actuando sucesivamente sobre el sistema fundamental.

- Independiente de las cargas sobre la estructura.
- Depende del material, la geometría y de las condiciones de vínculo del sistema fundamental.
- Es una matriz cuadrada.
- Es una matriz simétrica $\delta_{ij} = \delta_{ji}$ (Teorema de Maxwell)
- En general es diagonal dominante, pero no siempre. $\delta_{ii} > |\delta_{ij}|$
- Es positiva definida. Es inversible.

$[X_j]$: Vector de Incógnitas.

Fuerzas en vínculos externos y/o internos. Reacciones y/o Esfuerzos Internos

APLICACIÓN MÉTODO DE LAS FUERZAS

Pasos

1. Determinar el GH
2. Definir el sistema fundamental. SF
3. Definir la incógnitas. SIE
4. Plantear las ecuaciones de compatibilidad. $\Delta_i = 0$
5. Calcular Δ_{i0} aplicando TTV
6. Calcular δ_{ij} aplicando TTV
7. Resolver el SEL. $[\Delta_{i0}] + [\delta_{ij}] [X_j] = 0$
8. Calcular los diagramas de esfuerzos internos.

APLICACIÓN MÉTODO DE LAS FUERZAS

Cálculo de Esfuerzos Internos

P I A S E

$$M = M_{(0)} + M_{(1)} + M_{(2)} + \dots + M_{(n)}$$

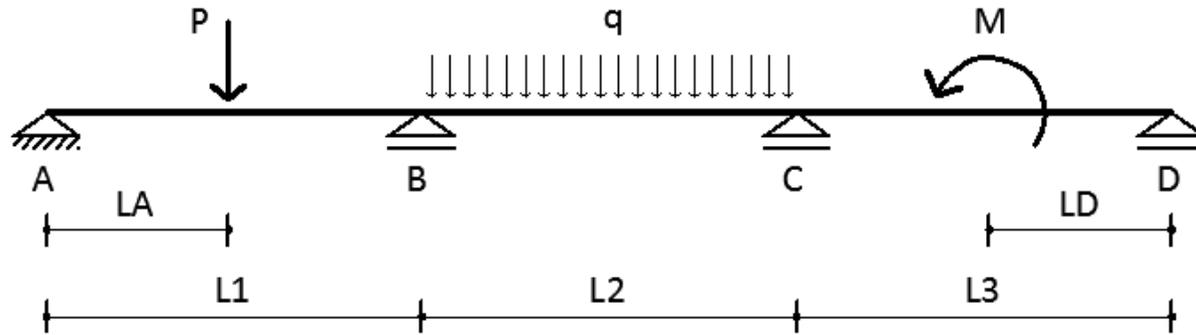
$$M = M_{(0)} + m_{(1)}X_1 + m_{(2)}X_2 + \dots + m_{(n)}X_n$$

$M_{(0)}$: Momentos debidos al sistema P0 actuando sobre el SF.

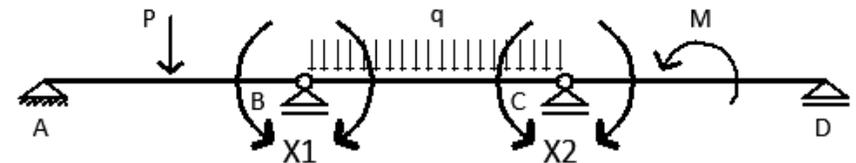
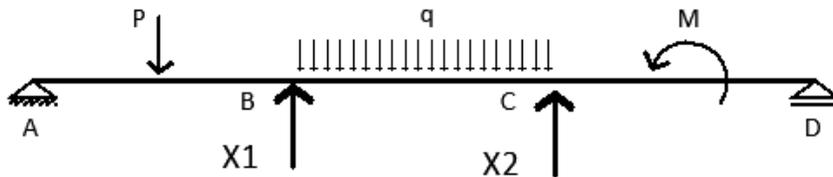
$m_{(i)}$: Momentos debidos a los valores unitarios de la incógnitas, actuando sobre el sistema fundamental.

X_i : Valor de la incógnita.

VIGA CONTINUA



SIE



$$\Delta_1 = 0; \Delta_2 = 0$$

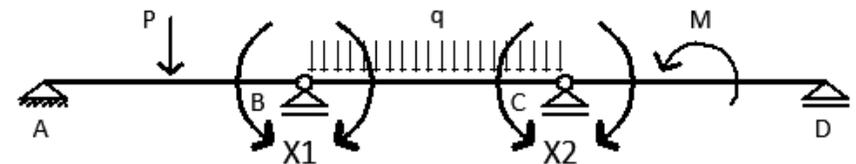
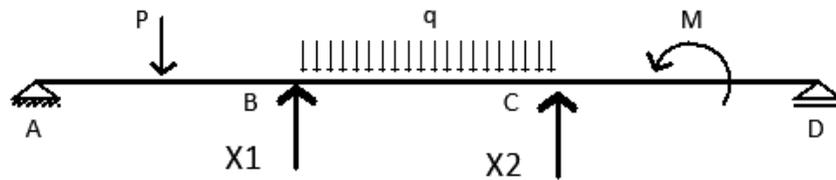
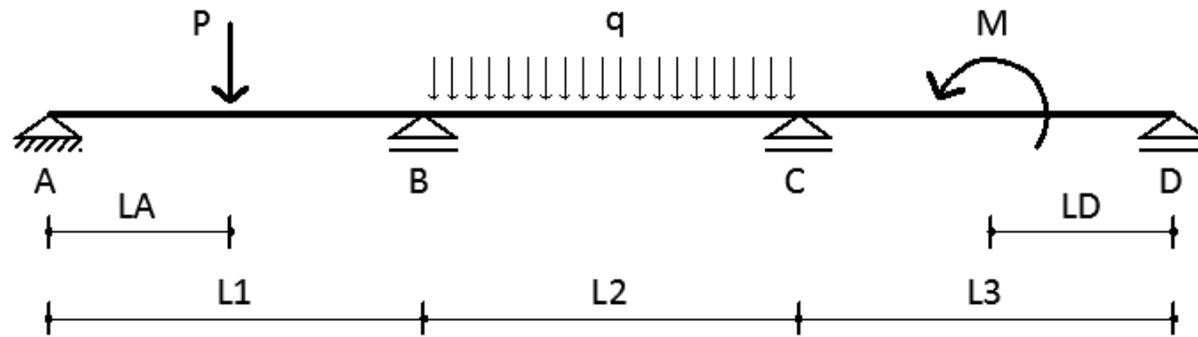
$$\Delta_{10} + \Delta_{11} + \Delta_{12} = 0$$

$$\Delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0$$

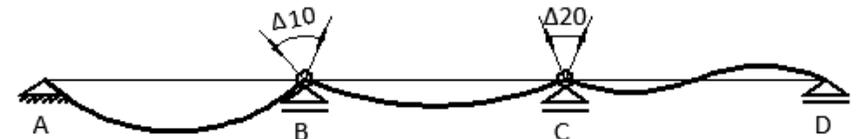
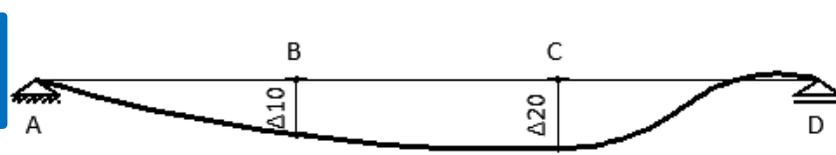
$$\Delta_{20} + \Delta_{21} + \Delta_{22} = 0$$

$$\Delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0$$

VIGA CONTINUA



P0



X1=1

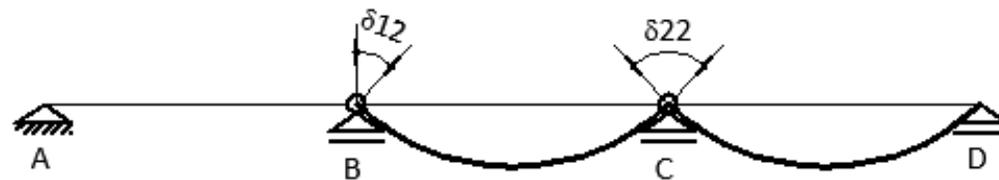
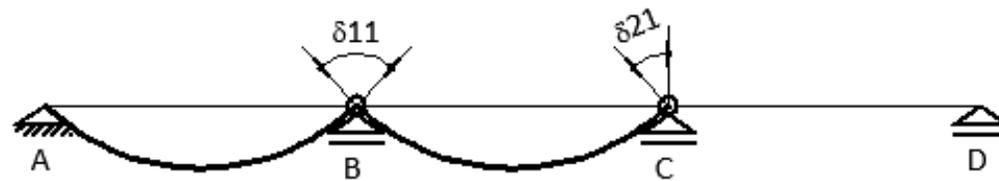
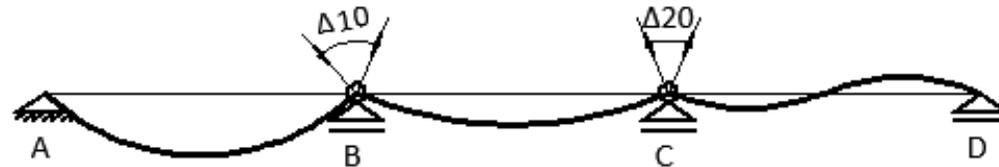
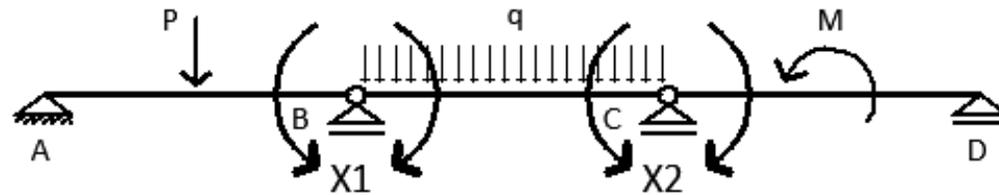


X2=1



VIGA CONTINUA

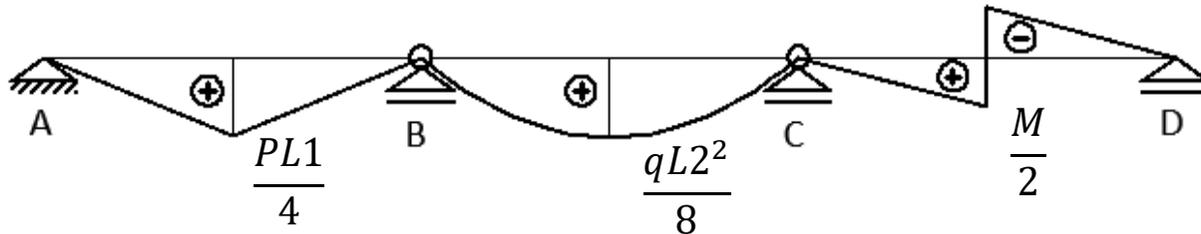
$$\bar{P} \eta = \int_L \bar{N} \left(\frac{N}{EF} + \frac{ti + ts}{2} \alpha \right) dx + \int_L \psi \frac{\bar{Q}Q}{GF} dx + \int_L \bar{M} \left(\frac{M}{EJ} + \frac{ti - ts}{h} \alpha \right) dx + \int_L \frac{\bar{M}tMt}{GJt} dx$$



VIGA CONTINUA

$$\bar{P} \eta = \int_L \bar{N} \left(\frac{N}{EF} + \frac{ti + ts}{2} \alpha \right) dx + \int_L \psi \frac{\bar{Q}Q}{GF} dx + \int_L \bar{M} \left(\frac{M}{EJ} + \frac{ti - ts}{h} \alpha \right) dx + \int_L \frac{\bar{M}tMt}{GJt} dx$$

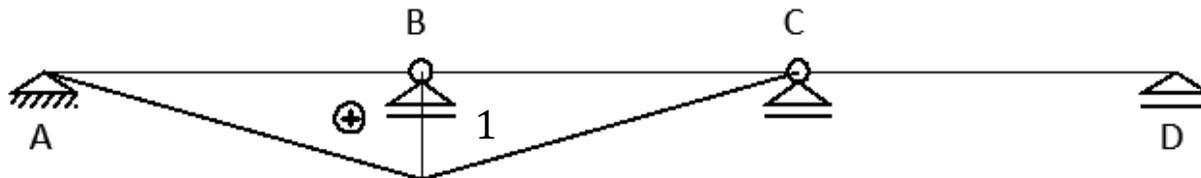
M0



$$\Delta_{10} = \int \frac{\bar{M}_1 M_0}{EJ} dx$$

$$\Delta_{20} = \int \frac{\bar{M}_2 M_0}{EJ} dx$$

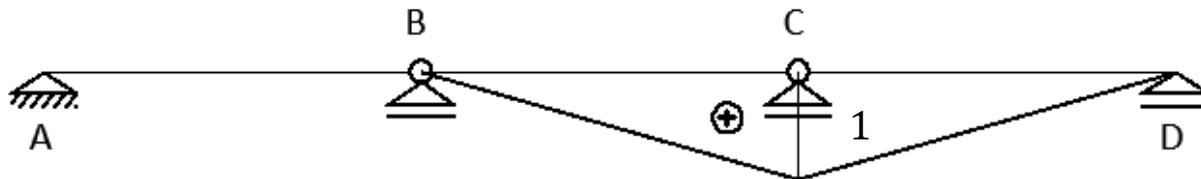
M1
X1=1



$$\delta_{11} = \int \frac{\bar{M}_1 M_1}{EJ} dx$$

$$\delta_{12} = \int \frac{\bar{M}_1 M_2}{EJ} dx$$

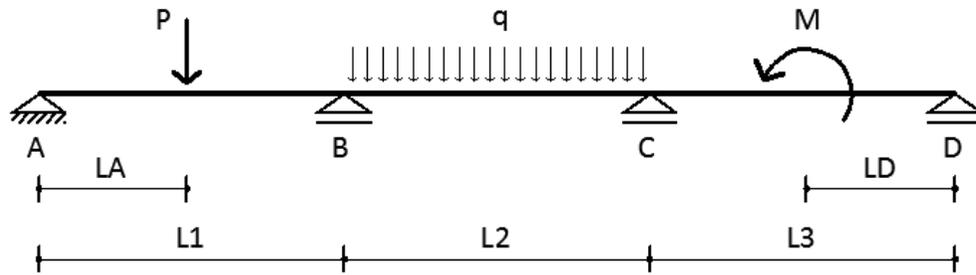
M2
X2=1



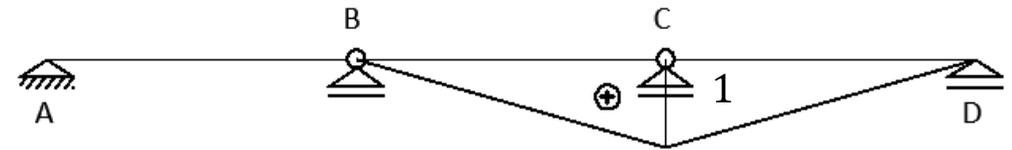
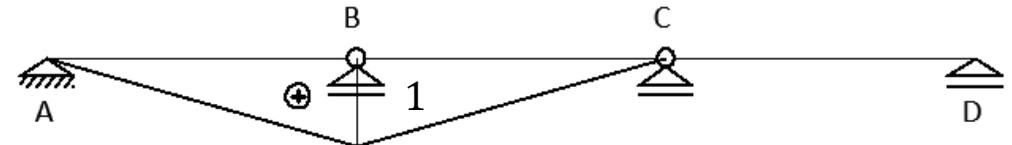
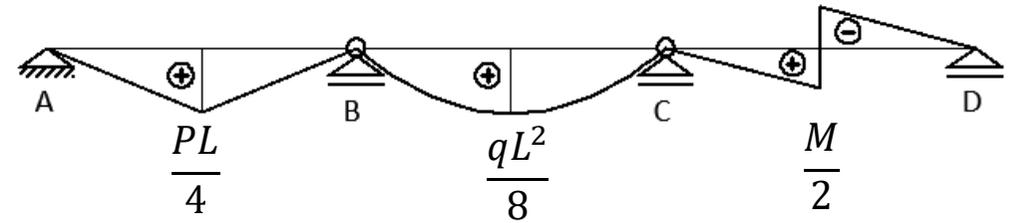
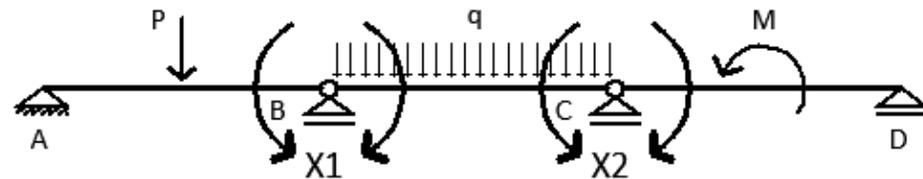
$$\delta_{21} = \int \frac{\bar{M}_2 M_1}{EJ} dx$$

$$\delta_{22} = \int \frac{\bar{M}_2 M_2}{EJ} dx$$

VIGA CONTINUA



$L1=L2=L3= 4.00\text{m}$, $LA=LD= 2.00\text{m}$
 $P= 20 \text{ kN}$, $q= 30 \text{ kN/m}$, $M= 120 \text{ kNm}$



$$\Delta_{10} = \frac{1}{4EJ} \frac{PL}{4} 1 L + \frac{1}{3EJ} \left(\frac{qL^2}{8} \right) 1 L = \frac{1}{EJ} (20 + 80) = \frac{100}{EJ}$$

$$\Delta_{20} = \frac{1}{3EJ} \left(\frac{qL^2}{8} \right) 1 L + \frac{1}{12EJ} \frac{M}{2} 1 L = \frac{1}{EJ} (80 + 20) = \frac{100}{EJ}$$

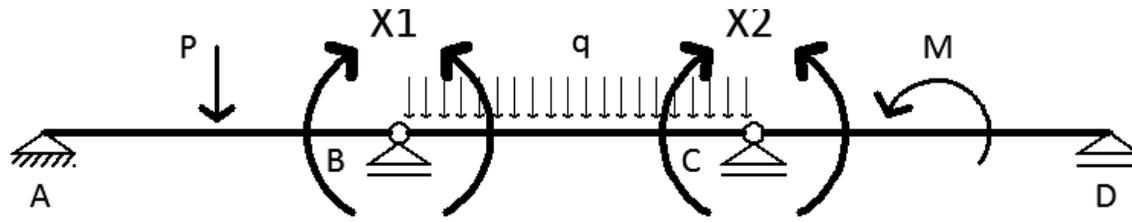
$$\delta_{11} = \frac{1}{3EJ} 1 1 L + \frac{1}{3EJ} 1 1 L = \frac{8}{3EJ}$$

$$\delta_{12} = \frac{1}{6EJ} 1 1 L = \frac{2}{3EJ}$$

$$\delta_{21} = \frac{1}{6EJ} 1 1 L = \frac{2}{3EJ}$$

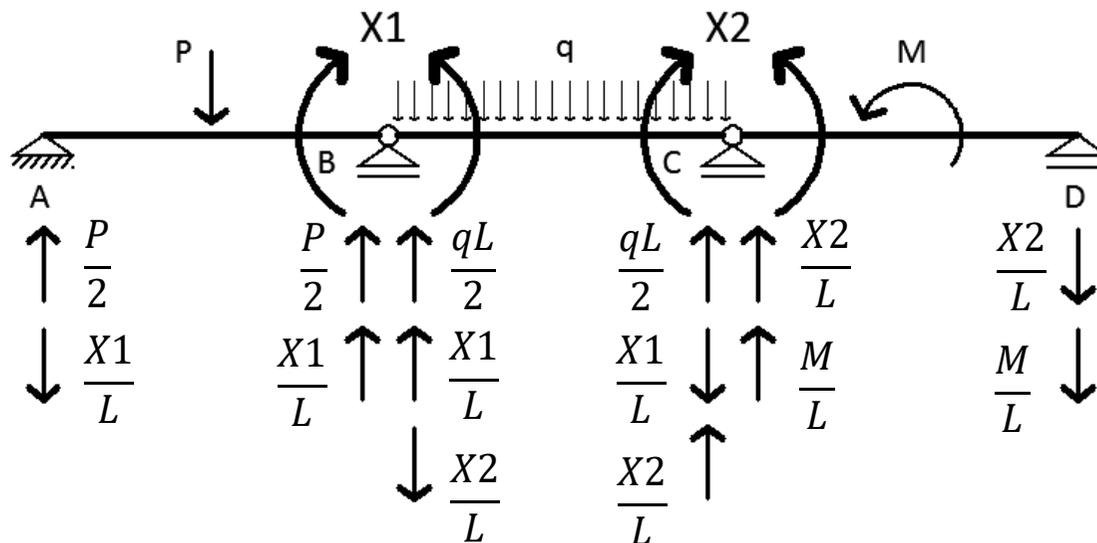
$$\delta_{22} = \frac{1}{3EJ} 1 1 L + \frac{1}{3EJ} 1 1 L = \frac{8}{3EJ}$$

VIGA CONTINUA



$$\frac{8}{3}X_1 + \frac{2}{3}X_2 = -100 \quad X_1 = -30kNm$$

$$\frac{2}{3}X_1 + \frac{8}{3}X_2 = -100 \quad X_2 = -30kNm$$



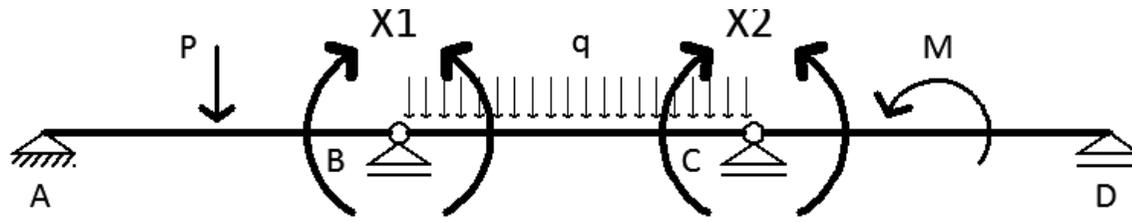
$$R_A = 2.50kN (\uparrow)$$

$$R_B = 77.50kN (\uparrow)$$

$$R_C = 97.50kN (\uparrow)$$

$$R_D = 37.50kN (\downarrow)$$

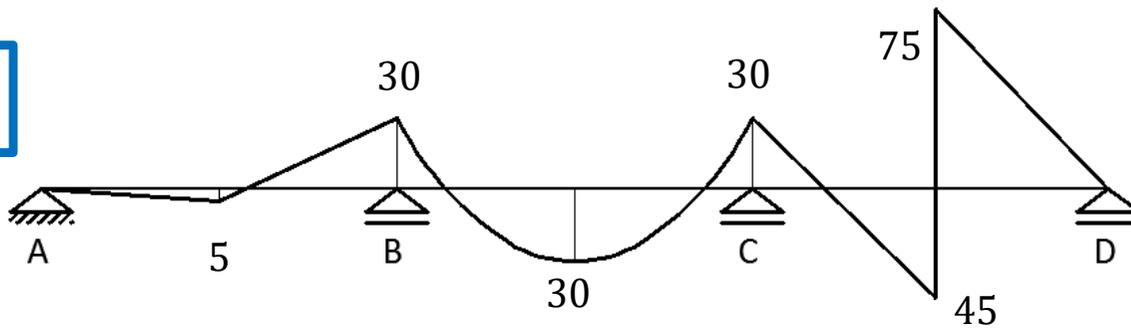
VIGA CONTINUA



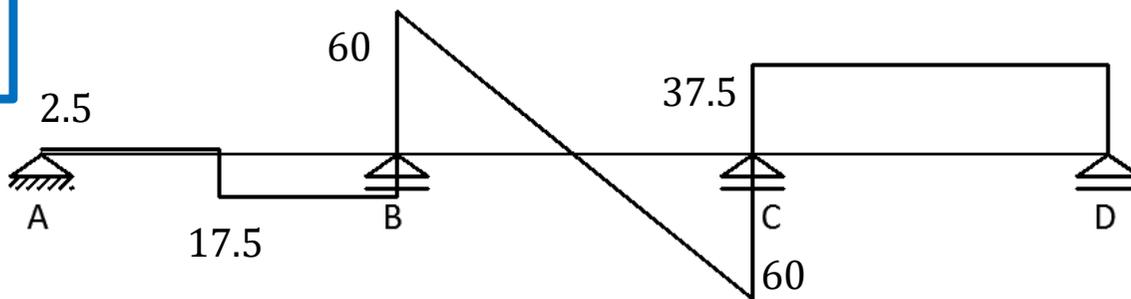
$$X_1 = -30kNm$$

$$X_2 = -30kNm$$

M



Q



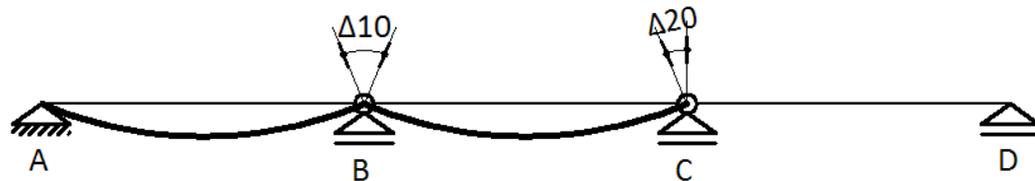
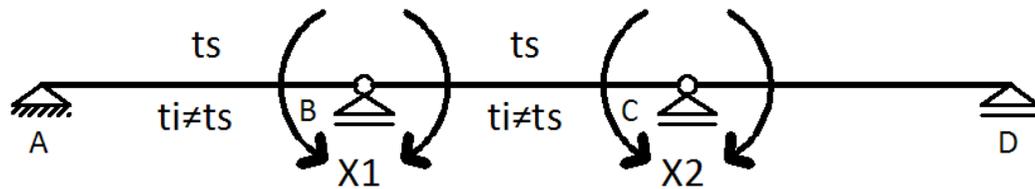
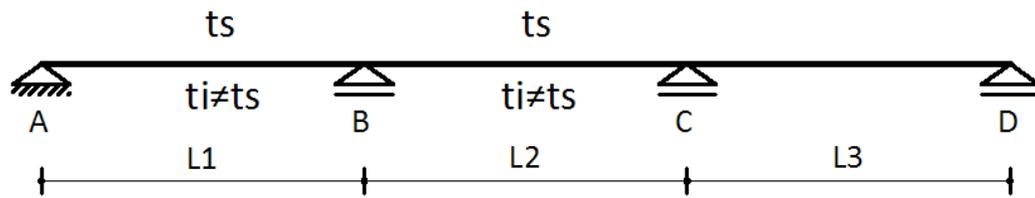
$$R_A = 2.50kN (\uparrow)$$

$$R_B = 77.50kN (\uparrow)$$

$$R_C = 97.50kN (\uparrow)$$

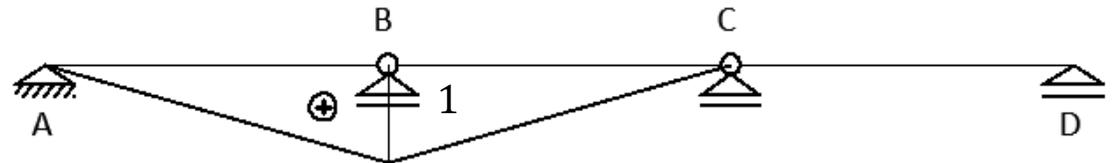
$$R_D = 37.50kN (\downarrow)$$

VARIACIÓN DE TEMPERATURA



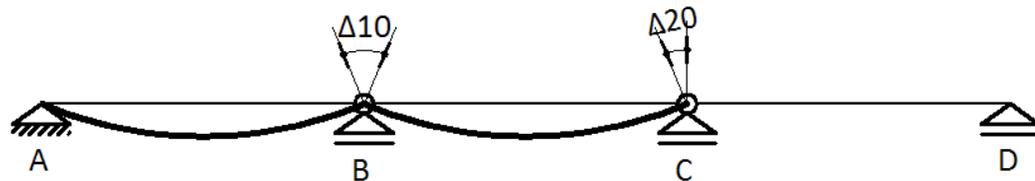
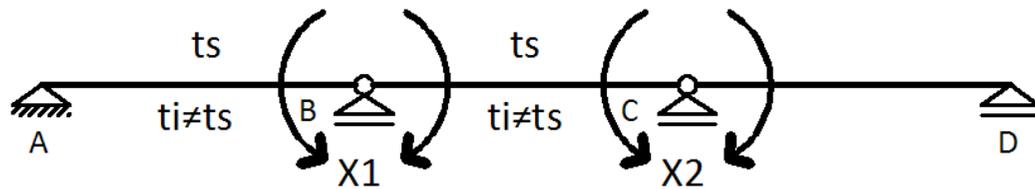
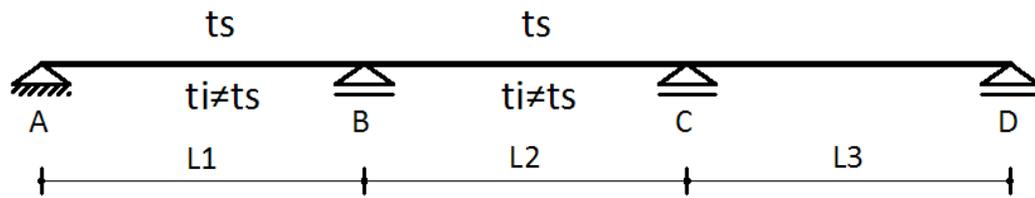
$$\bar{M} \theta = \int_L \bar{N} \left(\frac{N}{EF} + \frac{t_i + t_s}{2} \alpha \right) dx + \int_L \psi \frac{\bar{Q}Q}{GF} dx + \int_L \bar{M} \left(\frac{M}{EJ} + \frac{t_i - t_s}{h} \alpha \right) dx$$

$$\bar{M} \theta = \int_L \bar{M} \left(\frac{t_i - t_s}{h} \alpha \right) dx$$



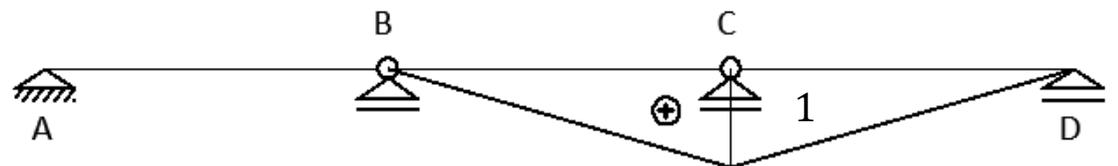
$$\Delta_{10} = \int_{L1} \frac{x}{L1} \nabla t \alpha dx + \int_{L2} \left(1 - \frac{x}{L2} \right) \nabla t \alpha dx$$

VARIACIÓN DE TEMPERATURA



$$\bar{M} \theta = \int_L \bar{N} \left(\frac{N}{EF} + \frac{t_i + t_s}{2} \alpha \right) dx + \int_L \psi \frac{\bar{Q}Q}{GF} dx + \int_L \bar{M} \left(\frac{M}{EJ} + \frac{t_i - t_s}{h} \alpha \right) dx$$

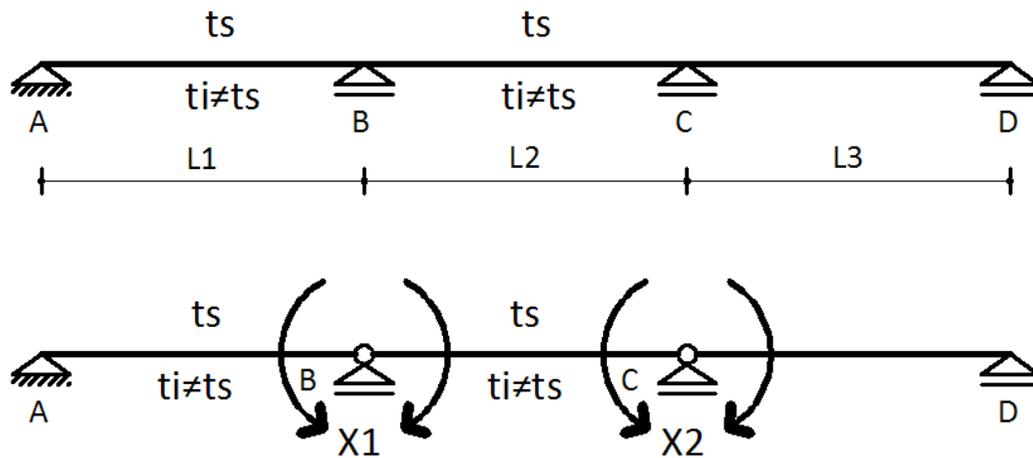
$$\bar{M} \theta = \int_L \bar{M} \left(\frac{t_i - t_s}{h} \alpha \right) dx$$



$$\Delta_{10} = \int_{L_1} \frac{x}{L_1} \nabla t \alpha dx + \int_{L_2} \left(1 - \frac{x}{L_2} \right) \nabla t \alpha dx$$

$$\Delta_{20} = \int_{L_2} \frac{x}{L_2} \nabla t \alpha dx$$

VARIACIÓN DE TEMPERATURA



$L1=L2=L3= 4.00\text{m}$
 $h= 0.50\text{m}, b=0.20\text{m}$
 $J= 2.083 \times 10^{-3} \text{ m}^4$
 $t_s= 50 \text{ }^\circ\text{C}$
 $t_i= 20 \text{ }^\circ\text{C}$
 $\alpha= 1.5 \times 10^{-5} \text{ 1/}^\circ\text{C}$
 $E= 20000 \text{ MPa}$

$$\nabla t = \frac{20 - 50 \text{ }^\circ\text{C}}{0.5 \text{ m}} = -60 \frac{\text{ }^\circ\text{C}}{\text{m}}$$

$$\Delta_{10} = \int_L \frac{x}{L} \nabla t \alpha dx + \int_L \left(1 - \frac{x}{L}\right) \nabla t \alpha dx = \alpha \nabla t \left(\frac{L}{2} + \frac{L}{2}\right) = \alpha \nabla t L = -0.0036$$

$$\Delta_{20} = \int_L \frac{x}{L} \nabla t \alpha dx = \alpha \nabla t \frac{L}{2} = -0.0018$$

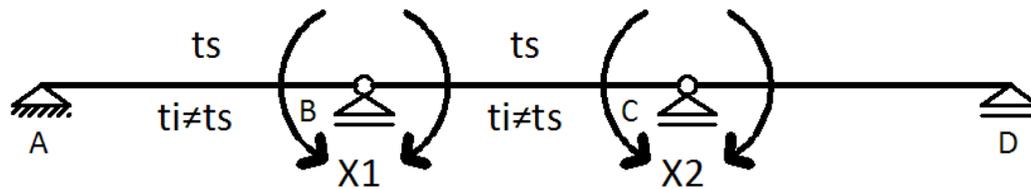
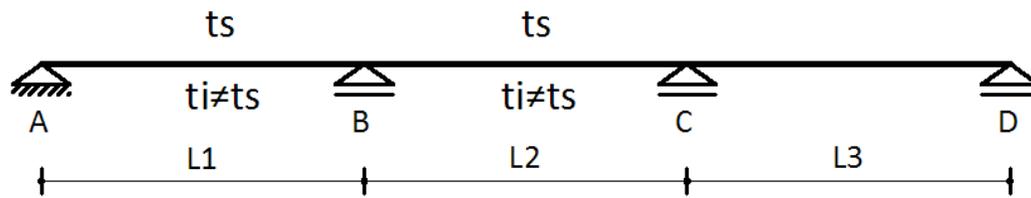
$$\delta_{11} = \frac{1}{3EJ} \int_0^L \int_0^L dx = \frac{8}{3EJ}$$

$$\delta_{12} = \frac{1}{6EJ} \int_0^L \int_0^L dx = \frac{2}{3EJ}$$

$$\delta_{21} = \frac{1}{6EJ} \int_0^L \int_0^L dx = \frac{2}{3EJ}$$

$$\delta_{22} = \frac{1}{3EJ} \int_0^L \int_0^L dx + \frac{1}{3EJ} \int_0^L \int_0^L dx = \frac{8}{3EJ}$$

VARIACIÓN DE TEMPERATURA



$$\frac{8}{3EJ}X1 + \frac{2}{3EJ}X2 = 0.0036$$

$$\frac{2}{3EJ}X1 + \frac{8}{3EJ}X2 = 0.0018$$

$$L1=L2=L3= 4.00m$$

$$h= 0.50m, b=0.20m$$

$$J= 2.083 \times 10^{-3} m^4$$

$$ts= 50 \text{ } ^\circ\text{C}$$

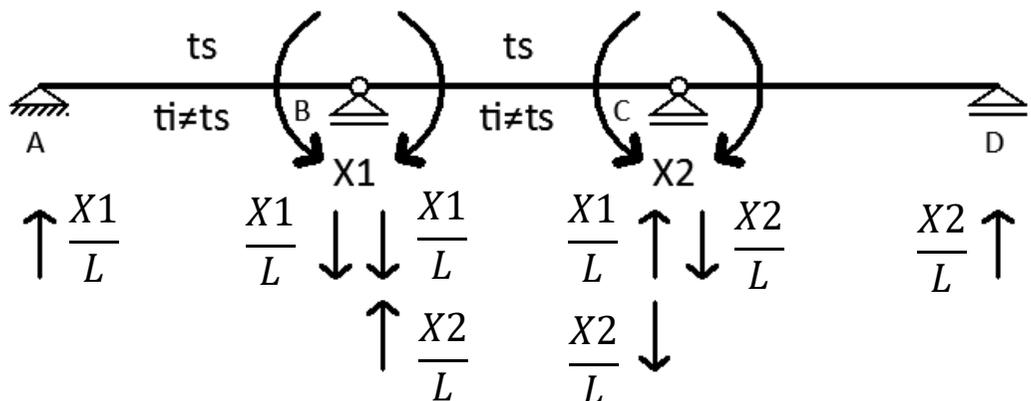
$$ti= 20 \text{ } ^\circ\text{C}$$

$$\alpha= 1.5 \times 10^{-5} 1/^\circ\text{C}$$

$$E= 20000 \text{ MPa}$$

$$X1 = 55.12 \text{ kNm}$$

$$X2 = 15.75 \text{ kNm}$$



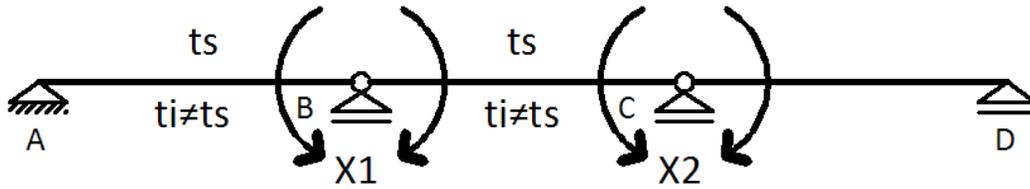
$$RA = 13.78 \text{ kN } (\uparrow)$$

$$RB = 26.63 \text{ kN } (\downarrow)$$

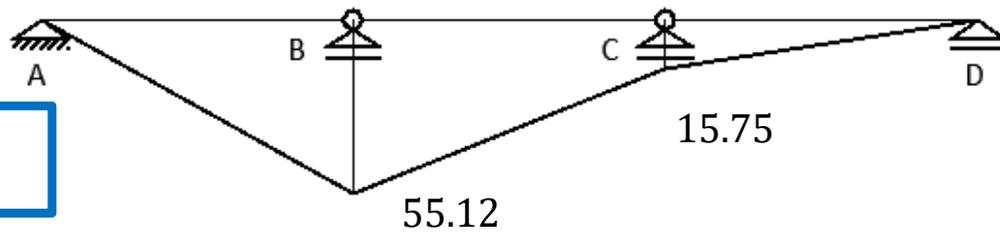
$$RC = 5.91 \text{ kN } (\uparrow)$$

$$RD = 3.94 \text{ kN } (\downarrow)$$

VARIACIÓN DE TEMPERATURA



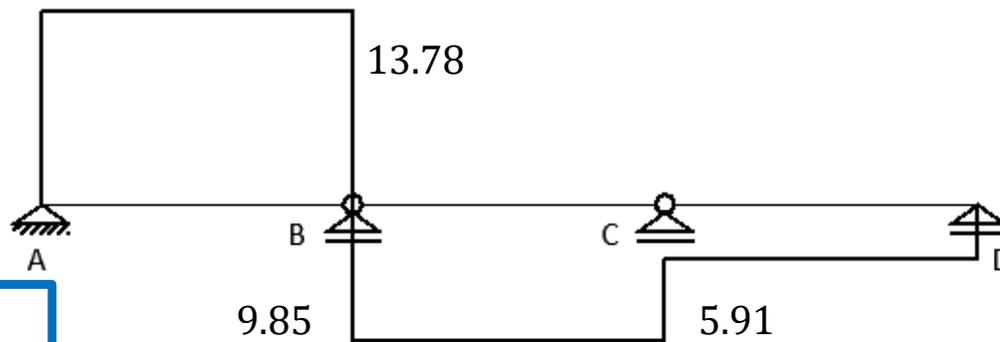
M



$$X_1 = 55.12 \text{ kNm}$$

$$X_2 = 15.75 \text{ kNm}$$

Q



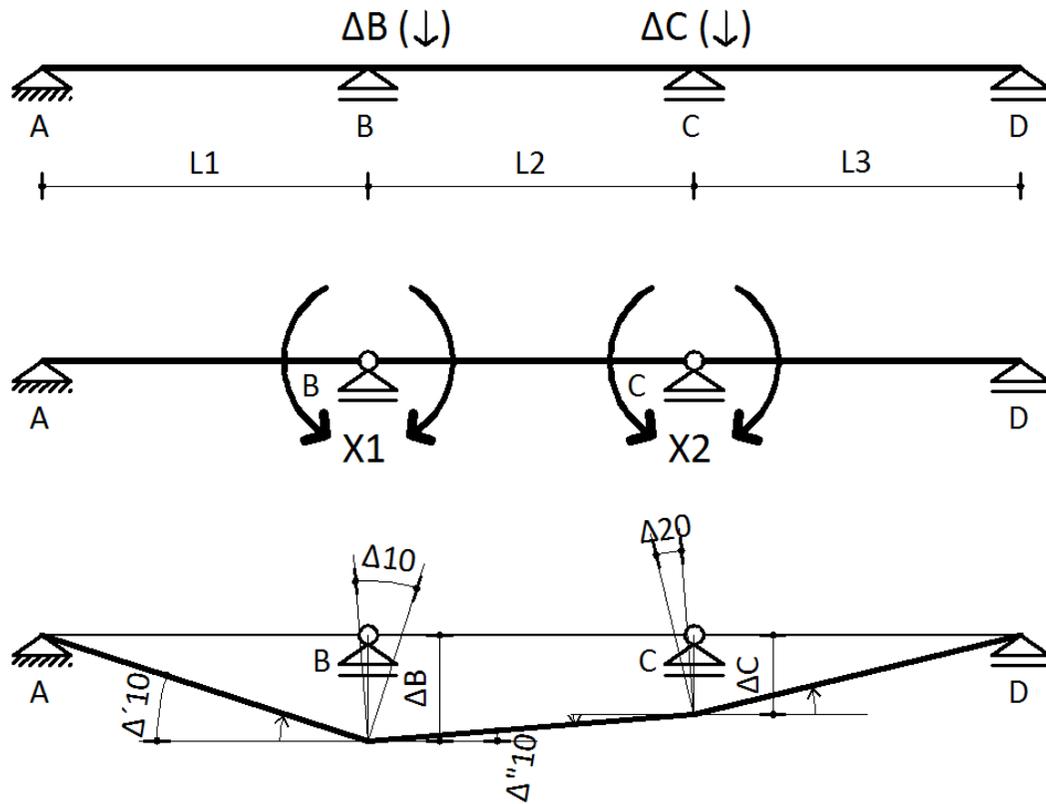
$$R_A = 13.78 \text{ kN } (\uparrow)$$

$$R_B = 23.63 \text{ kN } (\downarrow)$$

$$R_C = 5.91 \text{ kN } (\uparrow)$$

$$R_D = 3.94 \text{ kN } (\downarrow)$$

DESCENSO DE APOYOS



$$\Delta 10 = \Delta' 10 + \Delta'' 10 = - \left(\frac{\Delta B}{L1} + \frac{\Delta B - \Delta C}{L2} \right)$$

$$\Delta 20 = + \left(\frac{\Delta B - \Delta C}{L2} - \frac{\Delta C}{L3} \right)$$

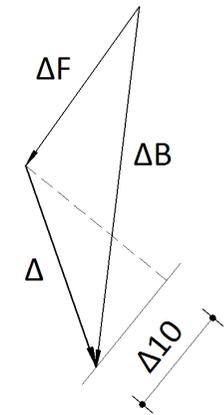
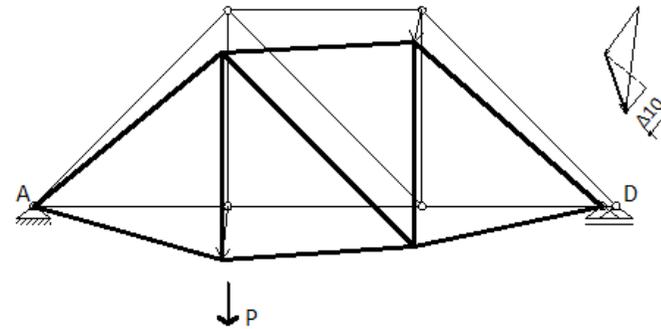
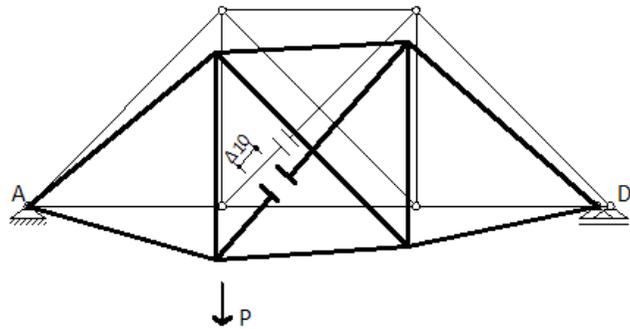
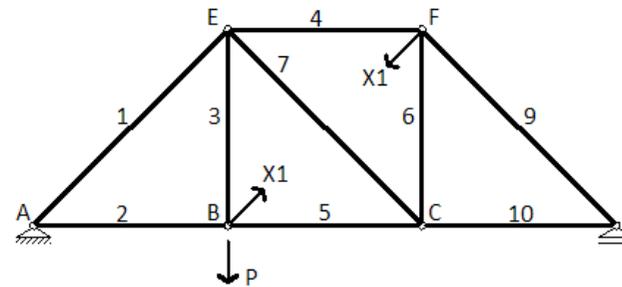
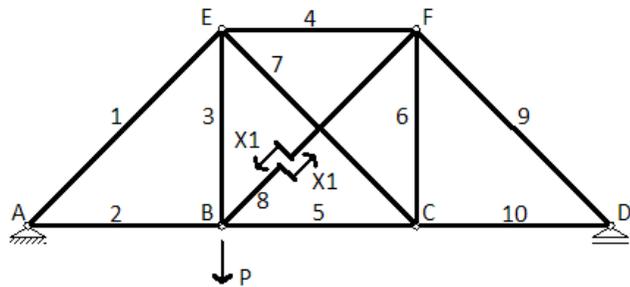
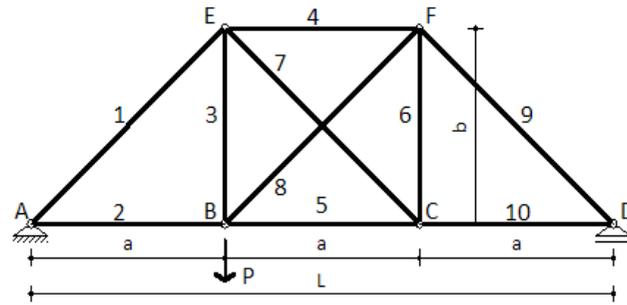
$$\delta 11 = \frac{8}{3EJ}$$

$$\delta 12 = \frac{2}{3EJ}$$

$$\delta 21 = \frac{2}{3EJ}$$

$$\delta 22 = \frac{8}{3EJ}$$

RETICULADOS



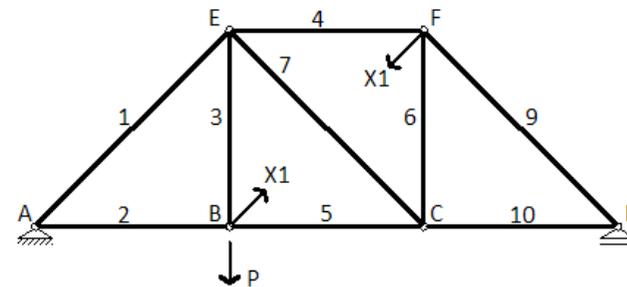
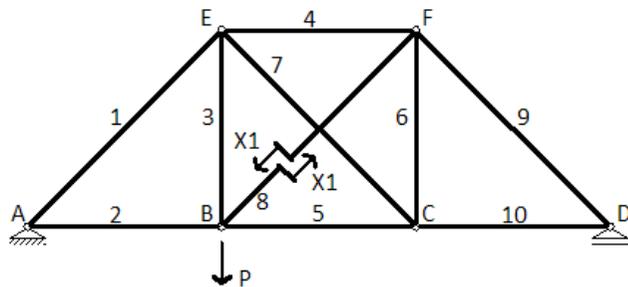
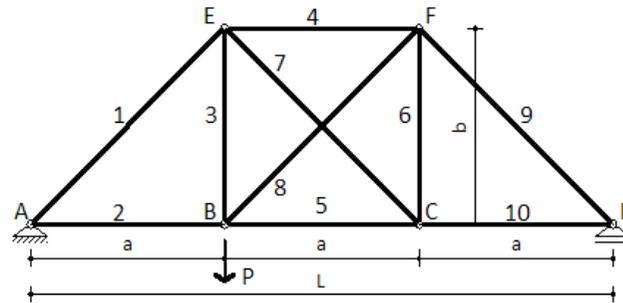
$$\Delta_{10} + \Delta_{11} = 0$$

$$\Delta_{10} + \Delta_{11} = X_1 L / (EA)$$

$$\Delta_{10} + \delta_{11} X_1 = 0$$

$$\Delta_{10} + \delta_{11} X_1 = X_1 L / (EA)$$

RETICULADOS



$$\Delta_{10} + \Delta_{11} = 0$$

$$\Delta_{10} + \Delta_{11} = X_1 L / (EA)$$

$$\Delta_{10} + \delta_{11} X_1 = 0$$

$$\Delta_{10} + [\delta_{11} + L / (EA)] X_1 = 0$$

$$\Delta_{k0} = \sum \frac{S_{ik} S_{i0}}{EA_i} L_i$$

$$i = 1, \dots, n$$

$$k = j = 1, \dots, h$$

$$\delta_{kj} = \sum \frac{S_{ik} S_{ij}}{EA_i} L_i$$

$$\delta_{kk} = \sum \frac{S_{ik}^2}{EA_i} L_i$$

Barras

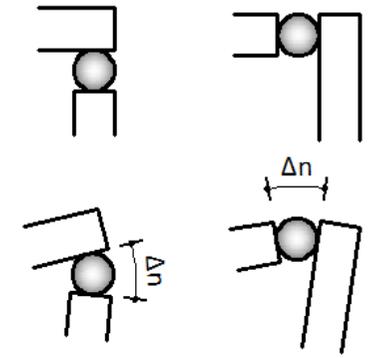
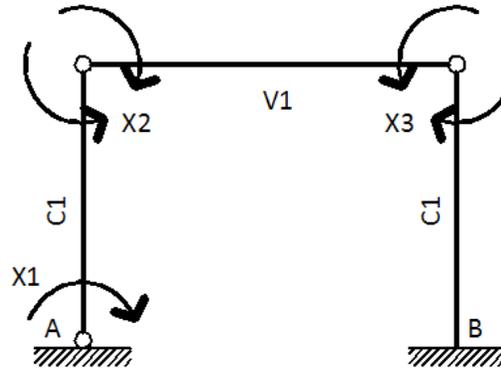
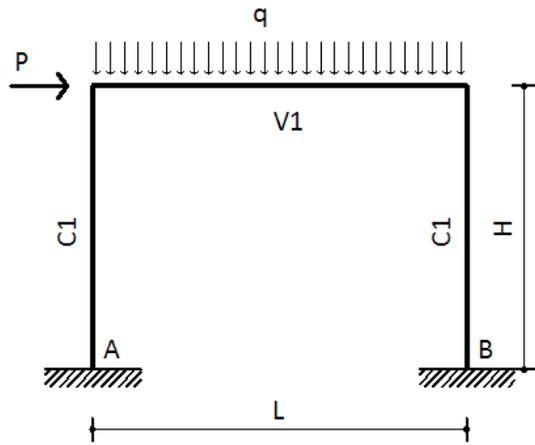
Incógnitas

$$S_i = S_{i0} + S_{i1} X_1 + \dots + S_{in} X_n$$

Temperatura

$$\Delta_{kt} = \alpha \Delta t \sum S_{ik} L_i$$

PÓRTICOS

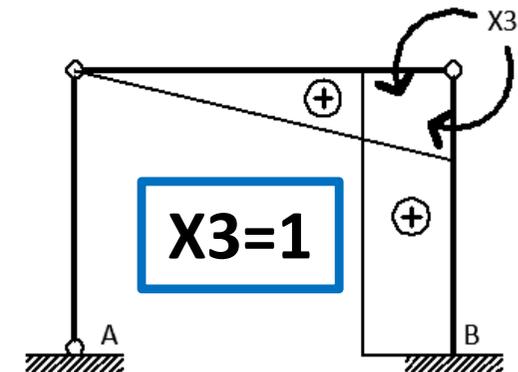
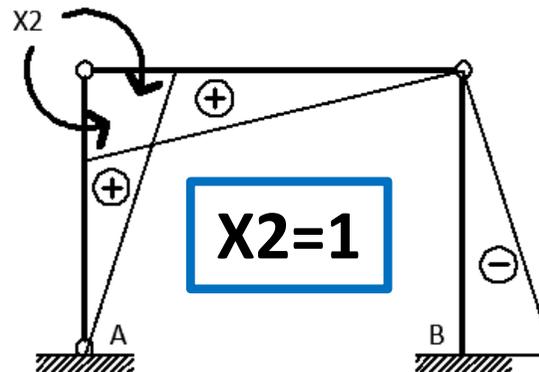
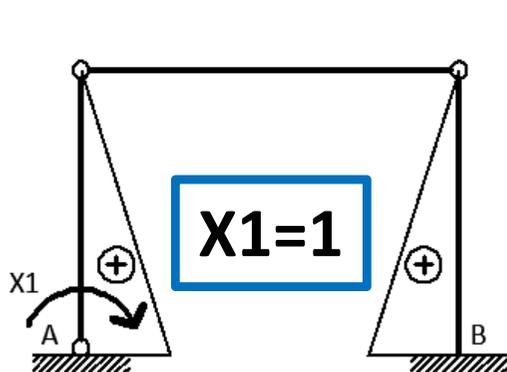
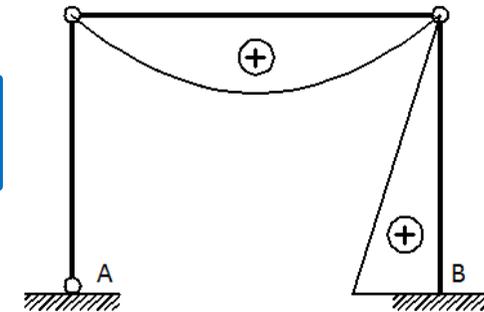


$$\Delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 + \delta_{13} X_3 = 0$$

$$\Delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 + \delta_{13} X_3 = 0$$

$$\Delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 + \delta_{13} X_3 = 0$$

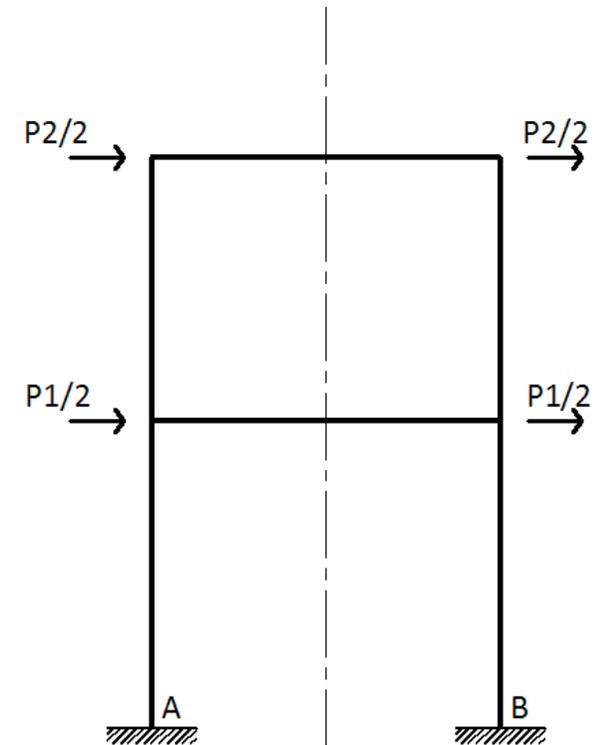
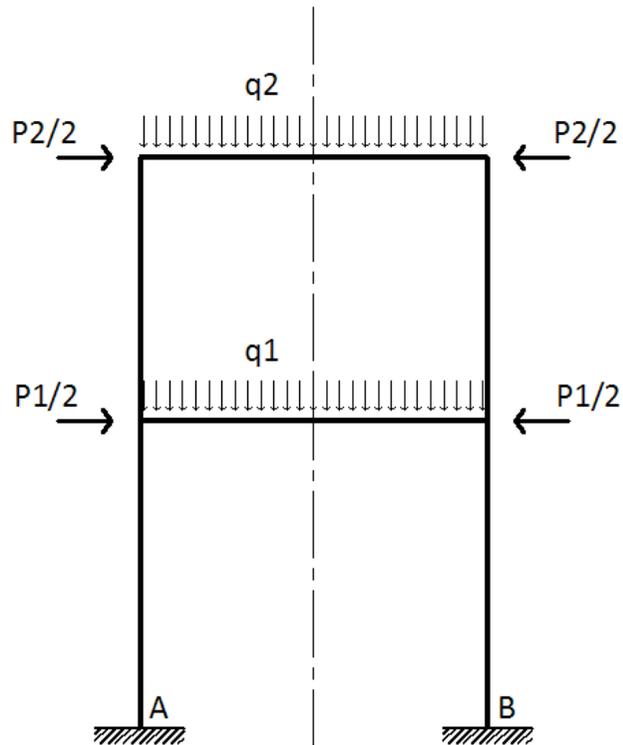
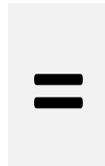
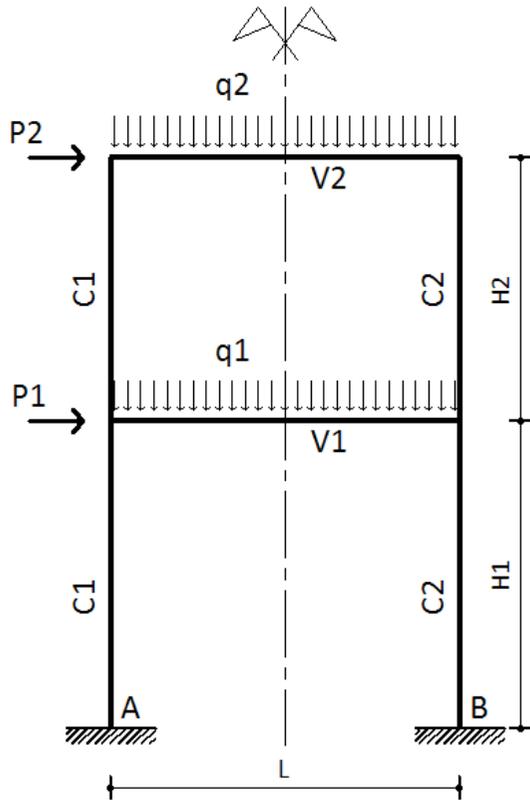
P0



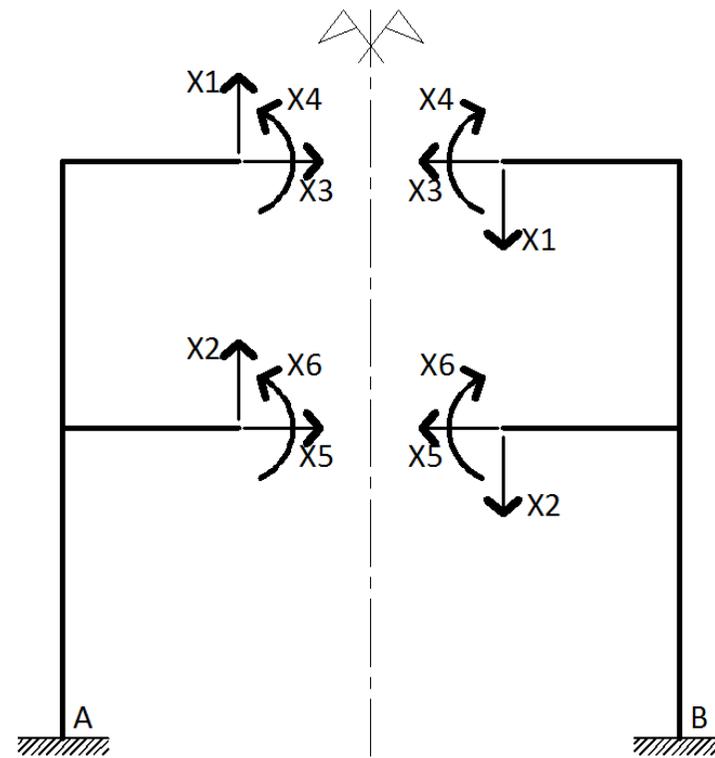
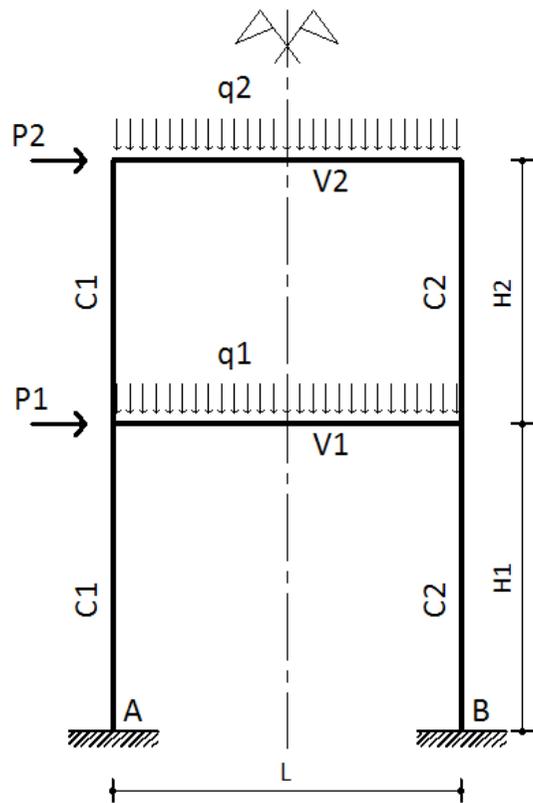
PÓRTICOS. SIMETRÍA y ANTISIMETRÍA

Simétrica

Antisimétrica



PÓRTICOS. SIMETRÍA y ANTISIMETRÍA



En una estructura simétrica con cargas simétricas los desplazamientos relativos antisimétricos en el plano de simetría son nulos.

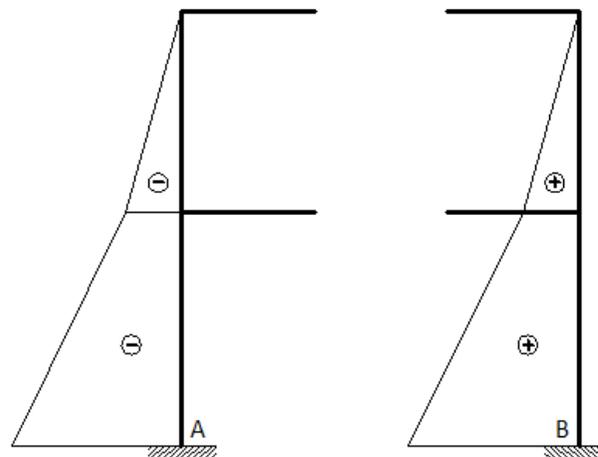
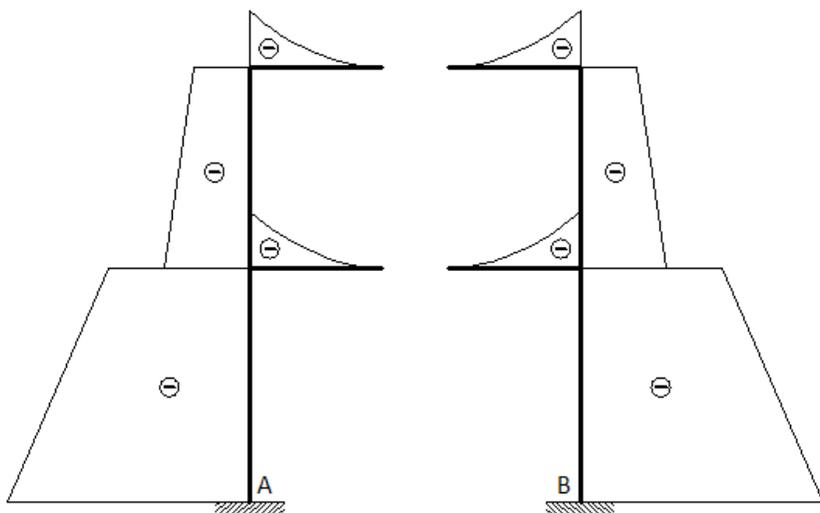
En una estructura simétrica con cargas simétricas las incógnitas antisimétricas en el plano de simetría son nulas.

PÓRTICOS. SIMETRÍA y ANTISIMETRÍA

Simétrica

Antisimétrica

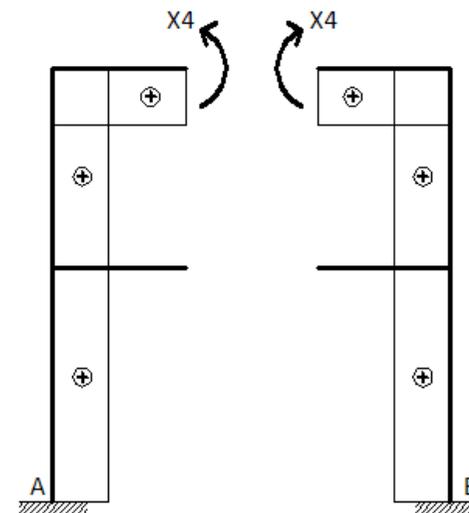
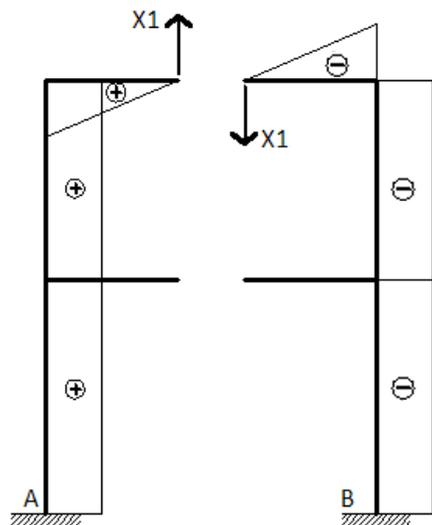
P0



Antisimétrica

Simétrica

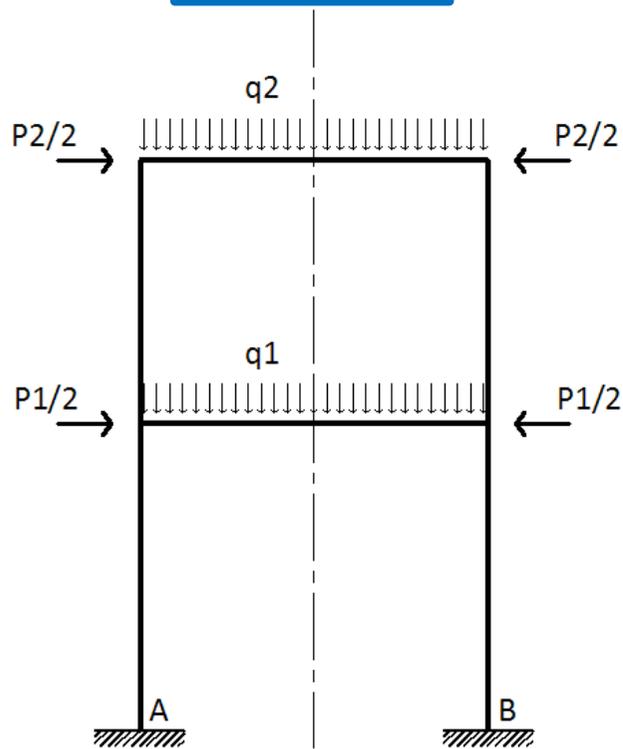
X1



X4

PÓRTICOS. SIMETRÍA y ANTISIMETRÍA

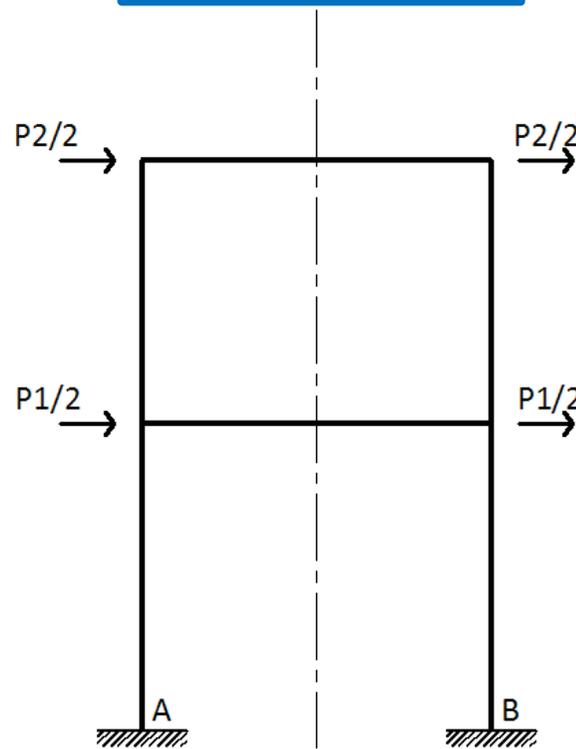
Simétrica



$$\Delta_{10} = \Delta_{20} = 0$$

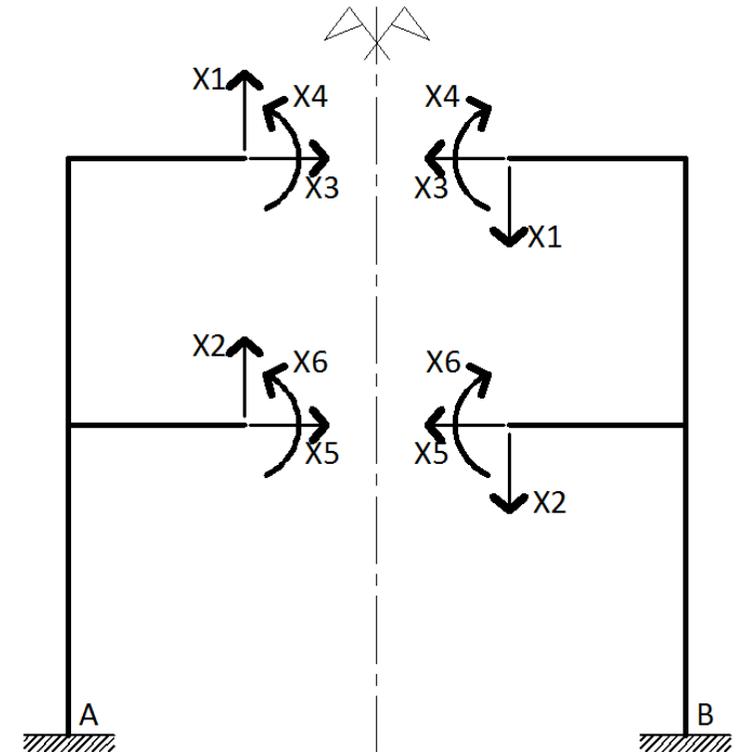
$$\Delta_{30}, \dots, \Delta_{60} \neq 0$$

Antisimétrica



$$\Delta_{10}, \Delta_{20} \neq 0$$

$$\Delta_{30} = \dots = \Delta_{60} = 0$$



$$\delta_{11}, \delta_{12}, \delta_{22} \neq 0$$

$$\delta_{13}, \dots, \delta_{16} = 0$$

$$\delta_{23}, \dots, \delta_{26} = 0$$

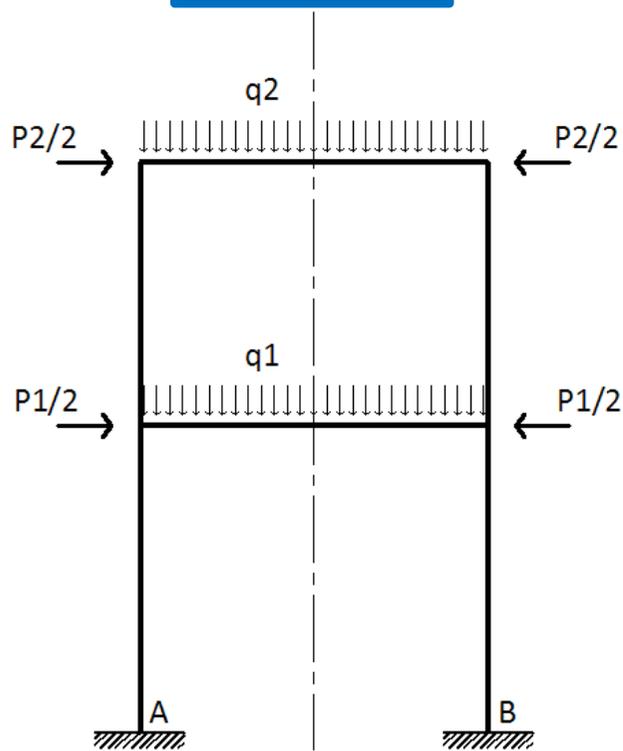
$$\delta_{33}, \dots, \delta_{36} \neq 0$$

$$\vdots, \dots, \vdots \neq 0$$

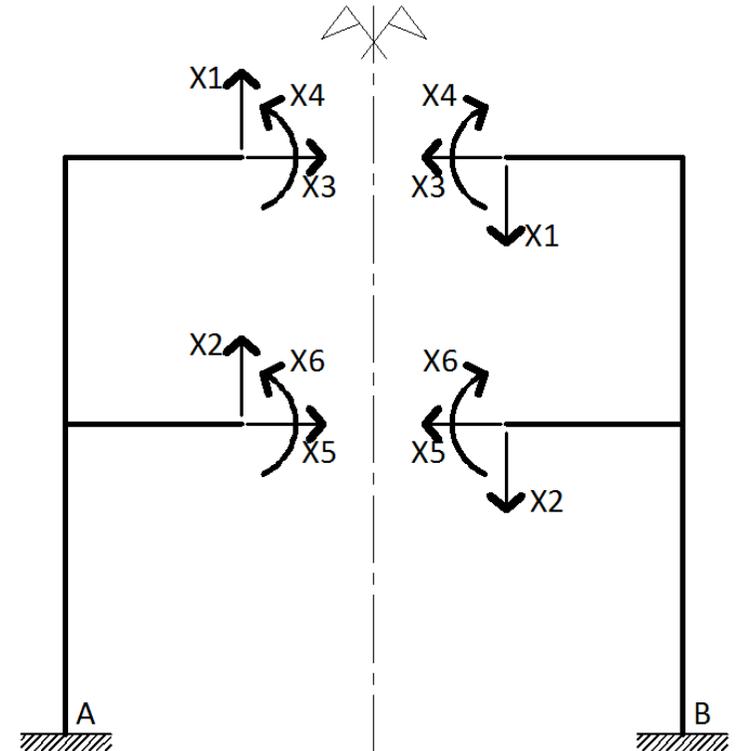
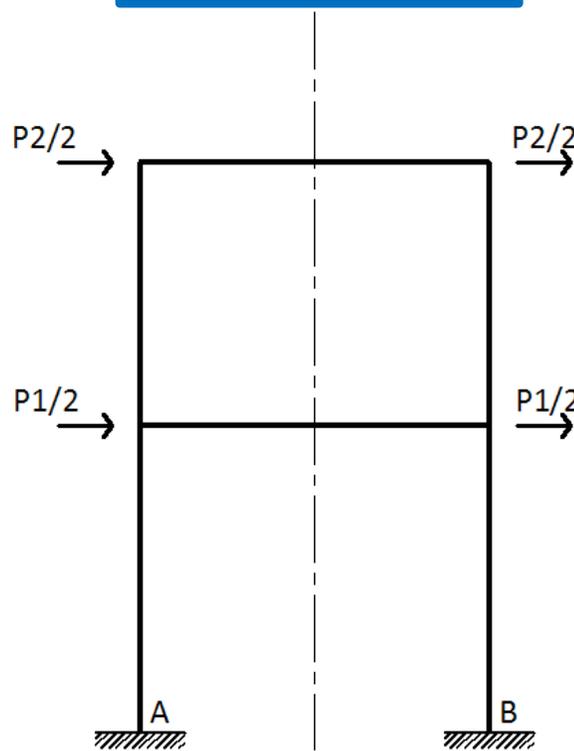
$$\delta_{63}, \dots, \delta_{66} \neq 0$$

PÓRTICOS. SIMETRÍA y ANTISIMETRÍA

Simétrica



Antisimétrica



$$\Delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0$$

$$\Delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0$$

$$\Delta_{30} + \delta_{33}X_3 + \delta_{34}X_4 + \delta_{35}X_5 + \delta_{36}X_6 = 0$$

$$\Delta_{40} + \delta_{43}X_3 + \delta_{44}X_4 + \delta_{45}X_5 + \delta_{46}X_6 = 0$$

$$\Delta_{50} + \delta_{53}X_3 + \delta_{54}X_4 + \delta_{55}X_5 + \delta_{56}X_6 = 0$$

$$\Delta_{60} + \delta_{63}X_3 + \delta_{64}X_4 + \delta_{65}X_5 + \delta_{66}X_6 = 0$$

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