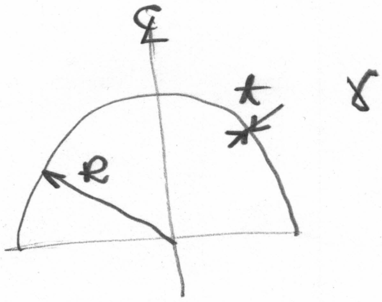
Deformaciones en Membranas.

Aplicaciones a Cubiertas Esfericas

1. Cupula semiesferica con cargas simetricas pprop.



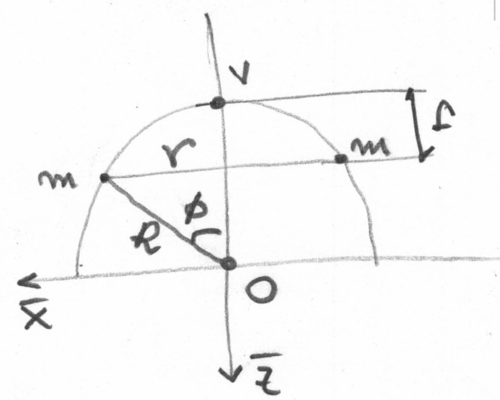
- $\sum F_x =$

$$\frac{\partial S_1}{\partial \phi} R_\theta \operatorname{sen}\phi - S_2 R_\phi \cos\phi + p_x R_\phi R_\theta \operatorname{sen}\phi = 0$$

- $\sum F_y = 0$ por simetria

- $\sum F_z$

$$S_1 R_\theta + S_2 R_\phi + p_z R_\phi R_\theta = 0$$



$r = R \operatorname{sen}\phi$

$$Q = 2\pi R \gamma t f$$

$$f = R(1-\cos\phi)$$

$$Q = 2\pi R \gamma t R (1-\cos\phi)$$

$$Q = 2\pi R^2 \gamma t (1-\cos\phi)$$

$$S_1 2\pi r \operatorname{sen}\phi + Q = 0$$

$$S_1 \cancel{2\pi} \cancel{R} \operatorname{sen}^2\phi = - \cancel{2\pi} R^{\cancel{2}} \gamma t (1-\cos\phi)$$

$$\boxed{S_1 = - \gamma t R \frac{(1-\cos\phi)}{\operatorname{sen}^2\phi}}$$

$$\frac{S_1}{R_\phi} + \frac{S_2}{R_\theta} = -p_z$$

$$\boxed{S_2 = R_\theta \left(-p_z - \frac{S_1}{R_\phi}\right)}$$

Reemp y ordenando

$$S_1 = -\gamma t R \left(\frac{1}{1+\cos\phi} \right) \qquad (a)$$

$$S_2 = -\gamma t R \left(c - \frac{1}{1+\cos\phi} \right) \qquad (b)$$

$$\boxed{\bar{u} = \frac{r}{Et}\left(S_2 - \mu S_1\right)} \qquad (c)$$

P/ $\phi = 90°$

$$S_1 = -\gamma t R \quad ; \quad S_2 = \gamma t R$$

$$\bar{u} = \frac{R^2 \gamma \cancel{t}}{E\cancel{t}}\left(1+\mu\right)$$

$$\boxed{\bar{u} = \frac{\gamma R^2}{E}\left(1+\mu\right)}$$

$$f(\phi) = \frac{1}{Et}\left(S_1\left(R_\phi + \mu R_\theta\right) - S_2\left(R_\theta + \mu R_\phi\right)\right)$$

$$R_\phi = R_\theta$$

$$f(\phi) = \frac{1}{Et}\left[S_1 R\left(1+\mu\right) - S_2 R\left(1+\mu\right)\right]$$

$$\boxed{f(\phi) = \frac{1}{Et}\left[\left(S_1 - S_2\right) R\left(1+\mu\right)\right]}$$

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$$\bar{\mu} = \frac{r}{Et}\left(S_2 - \mu S_1\right)$$

$$\bar{\mu} = \frac{r}{Et}\left[-\gamma t R\left(c - \frac{1}{1+c}\right) - \mu\left(-\gamma t R\right)\left(\frac{1}{1+c}\right)\right]$$

$$= \frac{r}{E\cancel{t}}\,\gamma \cancel{t} R\left[-\left(c - \frac{1}{1+c}\right) + \frac{\mu}{1+c}\right]$$

$$= \frac{\gamma R^2 \operatorname{sen}\phi}{E}\left[-\left(\frac{c + c^2 - 1}{1+c}\right) + \frac{\mu}{1+c}\right]$$

$$= \frac{\gamma R^2 \operatorname{sen}\phi}{E}\left[\frac{-c - c^2 + 1 + \mu}{1+c}\right]$$

$$\boxed{\bar{\mu} = \frac{\gamma R^2}{E}\,\frac{s}{1+c}\left(1 - c - c^2 + \mu\right)}$$

Giro del meridiano de 18

$$\varphi = \frac{1}{R\phi}\left[\frac{d}{d\phi}\left(\frac{\bar{\mu}}{\operatorname{sen}\phi}\right) - \frac{f(\phi)}{tg\,\phi}\right]$$

$$\frac{d}{d\phi}\left(\frac{\bar{\mu}}{\operatorname{sen}\phi}\right) = \frac{d}{d\phi}\left[\frac{\gamma R^2}{E}\,\frac{1}{1+c}\left(1 - c - c^2 + \mu\right)\right]$$

$$= \frac{\gamma R^2}{E}\left[-\frac{-s}{(1+c)^2}\left(1 - c - c^2 + \mu\right) + \frac{1}{1+c}\left(s + 2cs\right)\right]$$

$$\boxed{\varphi = \frac{\gamma R}{E}\operatorname{sen}\phi\,(2 + \mu)}$$

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$$\bar{w} = \int \frac{f(\phi)}{\operatorname{sen}\phi}\, d\phi + C - \frac{\bar{\mu}}{tg\,\phi}$$

$$f(\phi) = \frac{R(1+\mu)}{Et}\left(s_1 - s_2\right)$$

$$= -\frac{R(1+\mu)}{E\cancel{t}}\, \gamma \cancel{t} R \left[\frac{1}{1+\cos\phi} - \frac{c+c^2-1}{1+\cos\phi}\right]$$

$$= -\frac{\gamma R^2(1+\mu)}{E}\, \frac{1}{1+c}\left(1 - c - c^2 + 1\right)$$

$$= -\frac{\gamma R^2(1+\mu)}{E}\, \frac{(-c^2 - c + 2)}{1+c}$$

$$f(\phi) = \frac{\gamma R^2(1+\mu)}{E}\, \frac{(c+c^2-2)}{1+c}$$

$$\int \frac{f(\phi)}{\operatorname{sen}\phi}\, d\phi = \frac{\gamma R^2(1+\mu)}{E}\left[\log(1+\cos\phi) - \frac{1}{1+\cos\phi}\right]$$

$$\boxed{\bar{w} = \frac{\gamma R^2(1+\mu)}{E}\left[\log(1+\cos\phi) - \frac{1}{1+\cos\phi}\right] + C + {} - \frac{\gamma R^2}{E}\, \frac{c}{1+c}\left(1 - c - c^2 + \mu\right)}$$

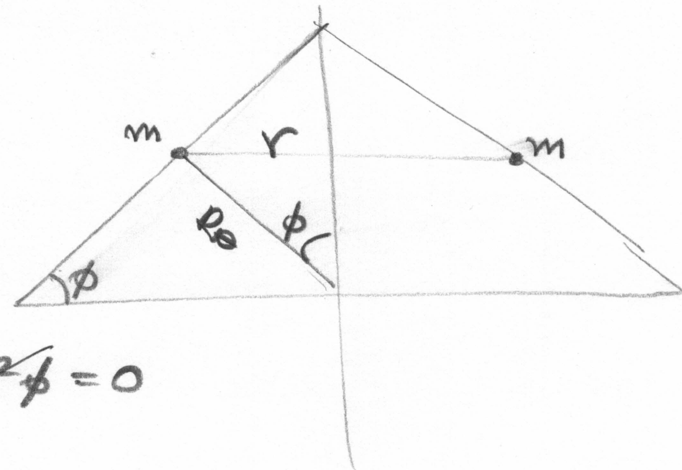
En $\phi = 90^\circ \rightarrow \bar{w} = 0 \rightarrow C$

$$\boxed{C = (1+\mu)\frac{\gamma R^2}{E}}$$

Aplicación Cubierta Cónica. Peso nieve

$$P = p_0 \pi r^2 = p_0 \pi R_\theta^2 \operatorname{sen}^2\phi$$

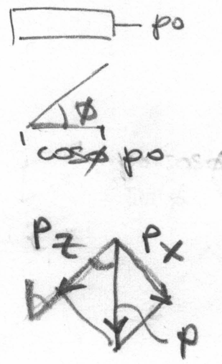
$$p_z = p\cos\phi = p_0 \cos^2\phi$$

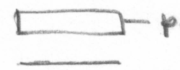


$$S_1 \operatorname{sen}\phi \, 2\pi r + p_0 \pi R_\theta^2 \operatorname{sen}^2\phi = 0$$

$$S_1 \cancel{\operatorname{sen}\phi} \, 2\cancel{\pi} \cancel{R_\theta} \operatorname{sen}\phi + p_0 \cancel{\pi} R_\theta^{\cancel{2}} \cancel{\operatorname{sen}^2}\phi = 0$$

$$\boxed{S_1 = \frac{p_0 R_\theta}{2}}$$





$p_z = p\cos\phi$

$$S_2 = R_\theta \left(-p_z - \frac{S_1}{R_\phi} \right)$$

$$S_2 = -p_0 R_\theta \cos^2\phi$$