TECHNICAL NOTE

Sloshing Loads in Liquid-Storage Tanks with Insufficient Freeboard

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Seismic ground motions excite long-period sloshing response in liquid-storage tanks. A minimum freeboard is needed to prevent the sloshing waves from impacting the roof of tanks. Since freeboard results in unused storage capacity, many tanks are not provided with the sufficient freeboard. As a result, sloshing waves impact the roof, generating additional forces on the roof and tank wall. This article presents a simple method of estimating these forces. [DOI: 10.1193/1.2085188]

INTRODUCTION

Seismic response of cylindrical liquid-storage tanks is reasonably well understood (e.g., Jacobsen 1949; Housner 1963, 1982; Haroun and Housner 1981; Veletsos et al. 1974, 1977, 1984, 1997; Malhotra 2000; Malhotra et al. 2000). The liquid mass is assumed divided into two parts: (1) the impulsive mass near the base of the tank moves with the tank wall, and (2) the convective mass near the top experiences free-surface sloshing motion. The natural period of vibration of the impulsive mass ranges from 0.1 s to 0.3 s and that of the convective mass ranges from 2 s to 6 s. The response of the impulsive mass controls the base shear and overturning moment in the tank, whereas the response of the convective mass controls the height of sloshing wave.

It is desirable to provide sufficient clearance (freeboard) between the liquid surface and the tank roof to prevent sloshing waves from impacting the roof during earthquakes. However, it is not always practical to do so. For large diameter tanks, the required freeboard can be quite high. If provided, it results in unused storage capacity, which can be quite expensive. For tanks located on deep soils or those subjected to near-field motions (e.g., Somerville 1993, Malhotra 1999), the abundance of low frequencies in the ground motion can result in very large freeboard requirement. Also, for tanks located on the roofs of buildings, the freeboard requirement can be quite high. In such cases, it is common to compromise on the freeboard requirement.

Insufficient freeboard causes (1) upward load on the roof due to impacts from the sloshing wave, and (2) increased impulsive mass due to constraining action of the roof. The upward force could break the connection between the roof and shell and tear the

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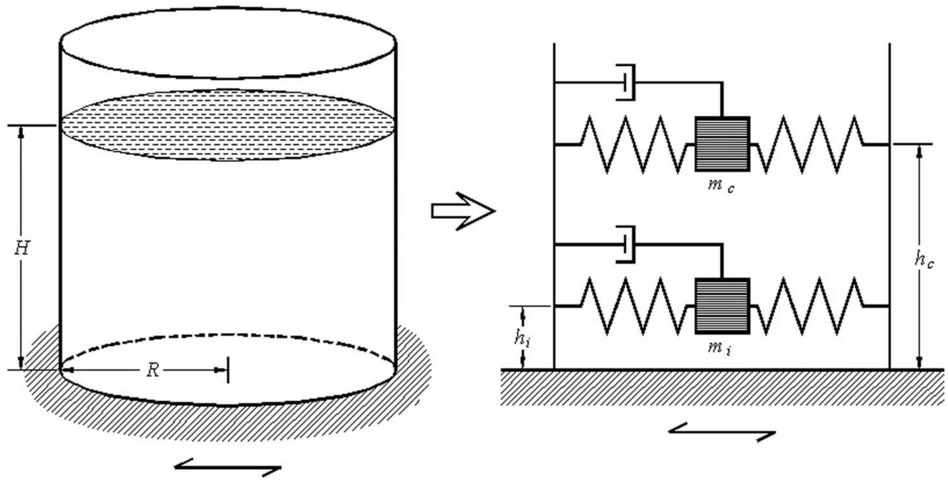

Figure 1. Simple model of liquid-filled tank.

shell if not considered in the design of the tank. Also, the tank shell could buckle or tear at the base if not designed for the loads resulting from additional impulsive mass.

The objective of this paper is to estimate the roof, shell, and foundation loads arising from insufficient freeboard. An exact solution of nonlinear sloshing response from fluid dynamics is quite complex. Instead, an approximate solution with engineering accuracy is presented.

MODEL OF TANK-LIQUID SYSTEM

IMPULSIVE AND CONVECTIVE MASSES

A sufficiently accurate model of tank of radius R filled with liquid to height H is shown in Figure 1. The impulsive mass m_{i} and the convective mass m_{c} add up to the total liquid mass m_{l}. In columns 4 and 5 of Table 1, $m_{\mathrm{i}}/m_{\mathrm{l}}$ and $m_{\mathrm{c}}/m_{\mathrm{l}}$ are presented for various H/R ratios. The higher the H/R, the higher the constraining action of the tank wall, therefore the greater the impulsive mass. The heights of impulsive and convective masses h_{i} and h_{c}, as fractions of the total liquid height H, are presented in columns 6 and 7 of Table 1, for various H/R ratios.

IMPULSIVE AND CONVECTIVE PERIODS

The natural periods of the impulsive and the convective modes, in seconds, are

$$T_{imp} = C_i \times \frac{\sqrt{\rho} \times H}{\sqrt{t_{\mathrm{eq}}/R} \times \sqrt{E}} \tag{1}$$

$$T_{con} = C_c \times \sqrt{R} \tag{2}$$

Table 1. Recommended design values for the impulsive and convective modes of vibration as a function of the tank height to radius ratio H/R (Malhotra et al. 2000)

H/R (1)	C_i (2)	$C_c[\mathrm{s/m}^{1/2}]$ (3)	m_i/m_l (4)	m_c/m_l (5)	h_i/H (6)	h_c/H (7)	h_i'/H (8)	h_c'/H (9)
0.3	9.28	2.09	0.176	0.824	0.400	0.521	2.640	3.414
0.5	7.74	1.74	0.300	0.700	0.400	0.543	1.460	1.517
0.7	6.97	1.60	0.414	0.586	0.401	0.571	1.009	1.011
1.0	6.36	1.52	0.548	0.452	0.419	0.616	0.721	0.785
1.5	6.06	1.48	0.686	0.314	0.439	0.690	0.555	0.734
2.0	6.21	1.48	0.763	0.237	0.448	0.751	0.500	0.764
2.5	6.56	1.48	0.810	0.190	0.452	0.794	0.480	0.796
3.0	7.03	1.48	0.842	0.158	0.453	0.825	0.472	0.825

where t_{eq}=equivalent uniform thickness of the tank wall, ρ=mass density of liquid, and E=modulus of elasticity of tank material. The coefficients C_i and C_c are presented in columns 2 and 3 of Table 1. The coefficient C_i is dimensionless, whereas C_c is expressed in $\mathrm{s/m}^{1/2}$; therefore, substituting R in meters in Equation 2 yields the correct value of the convective period in seconds. For tanks with non-uniform wall thickness, t_{eq} may be computed by taking a weighted average over the wetted height of the tank wall, assigning highest weight to the thickness near the base of the tank where the strain is maximum.

IMPULSIVE AND CONVECTIVE DAMPING RATIOS

The damping ratio for the impulsive mode of vibration may be assumed to be 2 percent of critical for steel and pre-stressed concrete tanks and 5 percent of critical for reinforced concrete tanks. The damping ratio for the convective mode of vibration may be assumed to be 0.5 percent of critical.

SEISMIC BASE SHEAR AND OVERTURNING MOMENT

BASE SHEAR

The impulsive and convective base shears are

$$Q_i = (m_i + m_w + m_r + m_b) \times SA(T_{imp}) \tag{3}$$

$$Q_c = m_c \times SA(T_{con}) \tag{4}$$

where m_w=the mass of tank wall, m_r=the mass of tank roof; m_b=the mass of tank base; $SA(T_{imp})$=the impulsive spectral acceleration, obtained from a 2 percent damping elastic response spectrum for steel and pre-stressed concrete tanks, and a 5 percent damping elastic response spectrum for concrete tanks; and $SA(T_{con})$=the convective spectral acceleration, obtained from a 0.5 percent damping elastic response spectrum.

OVERTURNING MOMENT ABOVE BASE PLATE

The impulsive and convective overturning moments immediately above the base plate are

$$M_i = (m_i \times h_i + m_w \times h_w + m_r \times h_r) \times SA(T_{imp}) \tag{5}$$

$$M_c = m_c \times h_c \times SA(T_{con}) \tag{6}$$

where h_i and h_c are the heights of the centroid of the impulsive and convective hydrodynamic wall pressures; they are provided in columns 6 and 7 of Table 1 as fractions of liquid height H; h_w and h_r are the heights of the centers of gravity of the tank wall and roof, respectively.

OVERTURNING MOMENT BELOW BASE PLATE

Additional overturning moment is generated by hydrodynamic pressure on the tank base. The net impulsive overturning moment, immediately below the base plate, is given by

$$M_i' = (m_i \times h_i' + m_w \times h_w + m_r \times h_r) \times SA(T_{imp}) \tag{7}$$

$$M_c' = m_c \times h_c' \times SA(T_{con}) \tag{8}$$

where the heights h_i' and h_c' are provided in columns 8 and 9 of Table 1 as fractions of liquid height H.

If the tank is supported on a ring foundation, the moments M_i and M_c are used to design the tank wall and the foundation. If the tank is supported on mat or pile foundations, moments M_i and M_c are used to design the tank wall and anchors, while M_i' and M_c' are used to design the foundation.

FREE-SURFACE WAVE HEIGHT

The vertical displacement of liquid surface due to sloshing is

$$d = R \times \frac{SA(T_{\mathrm{con}})}{g} \tag{9}$$

where g = acceleration due to gravity. A simple way to understand Equation 9 is to imagine that the liquid-filled tank moves horizontally with an acceleration $SA(T_{\mathrm{con}})$, as shown in Figure 2a. Under equilibrium, the free-surface would be at an angle θ with respect to the horizontal, where

$$\theta = \tan^{-1}\left(\frac{SA(T_{\mathrm{con}})}{g}\right) \tag{10}$$

This gives the height of the sloshing wave as $d = R \cdot \tan\theta = R \cdot SA(T_{\mathrm{con}})/\mathrm{g}$, thus, the proof of Equation 9.

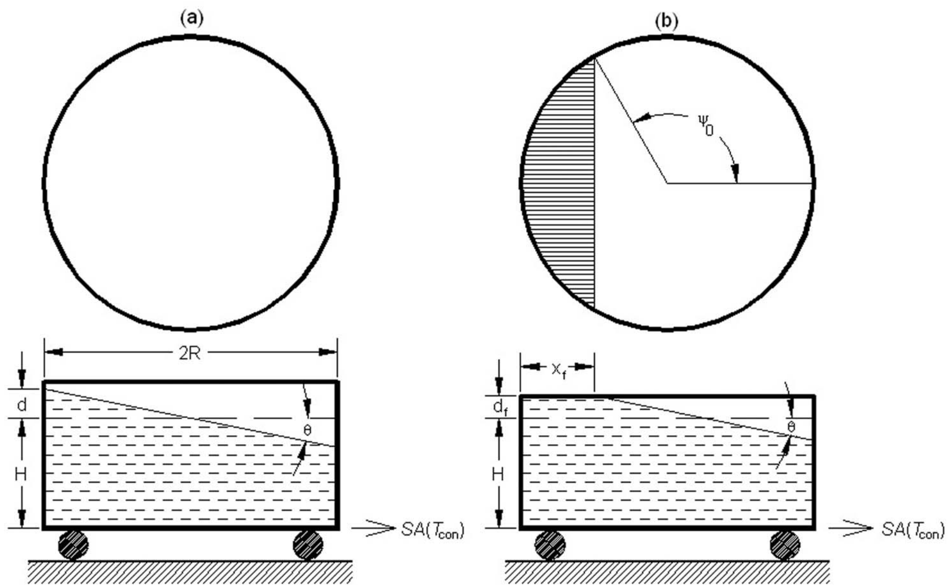

Figure 2. Liquid-filled tank translating with an acceleration $SA(T_{con})$: (a) sufficient freeboard, and (b) insufficient freeboard.

EFFECTS OF INSUFFICIENT FREEBOARD

ROOF LOAD

Next, consider the case of insufficient freeboard, i.e., actual freeboard d_f is less than the required freeboard d obtained from Equation 9. For a horizontal acceleration of $SA(T_{con})$, the free-surface of the liquid is still at an angle θ from the horizontal. However, a portion of the tank roof is wetted, as seen in Figure 2b. We assume that the tank roof is flat. This provides a conservative estimate of the effect of sloshing wave, because a non-flat roof provides extra room to accommodate the sloshing wave. From $SA(T_{con})/g$ we know θ (Equation 10). We can then determine the wetted width x_f of the tank roof by equating the volume of the empty space in the tank to $\pi R^2 d_f$. This gives the following relationship between x_f and d_f:

$$\frac{d_f}{d} = \frac{1}{\pi}\left(1 - \frac{x_f}{R}\right) \cdot \left(\psi_0 - \frac{\sin 2\psi_0}{2}\right) + \frac{2}{3\pi}\sin^3 \psi_0 \tag{11}$$

where $\psi_0 = \cos^{-1}(x_f/R - 1)$.

Figure 3 shows a plot between the normalized freeboard d_f/d and the normalized wetted width x_f/R.

The amplification of roof pressure due to dynamic response of the tank roof has not been considered. This is because the sloshing loads on the roof are applied slowly com-

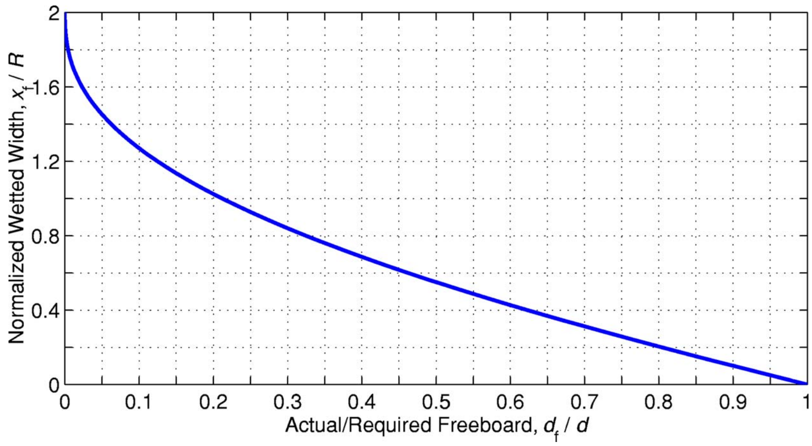

Figure 3. Normalized wetted width of tank roof, x_f/R as a function of actual/required freeboard, d_f/d.

pared to the expected natural period of vibration of the tank roof. Typically, the period of the sloshing wave is longer than 3 s and because it is applied near the circumference of the roof, it excites higher modes of vibration of the roof, which are generally of much shorter period (stiff).

SHELL AND FOUNDATION LOADS

The maximum upward pressure on the tank roof due to sloshing wave is (Figure 4)

$$P_{\max} = \rho \cdot g \cdot x_f \tan\theta \tag{12}$$

The upward force on the roof is resisted by the vertical tensile force in the shell. The connection between the shell and the roof should be designed to transfer this force. If x_f is small compared to R, the force per unit circumference of the tank shell may be approximated as follows:

$$F_{\max} \approx \frac{1}{2} P_{\max} \cdot x_f = \frac{1}{2} \rho \cdot g \cdot x_f^2 \cdot \tan\theta \tag{13}$$

Substituting, $\tan\theta = SA(T_{con})/\mathrm{g}$ (Equation 10) gives

$$F_{\max} \approx \frac{1}{2} \rho \cdot x_f^2 \cdot SA(T_{con}) \tag{14}$$

Equation 14 assumes that the upward force is resisted by the wet side of the tank shell only. This is not a good assumption when x_f/R is greater than, say, 0.5. $F_{\max}$ should then be estimated from more accurate static force-equilibrium analysis of the tank roof.

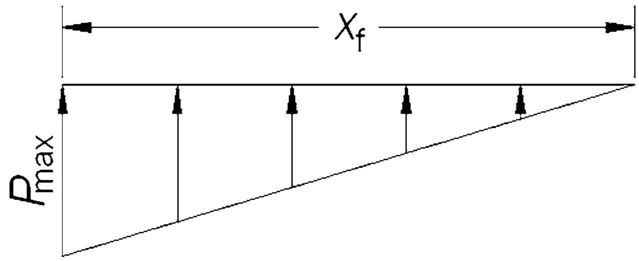

Figure 4. Radial variation of pressure on tank roof.

The constraint on the sloshing motion increases the mass participation in the impulsive mode. In the limiting case, if the freeboard is reduced to zero, the entire liquid in the tank becomes impulsive. Therefore, the smaller the actual/required freeboard d_f/d, the smaller the convective mass and the larger the impulsive mass. Assuming that the convective mass reduces linearly from m_c to 0 as d_f/d reduces from 1 to 0, the adjusted values of the impulsive and convective masses are

$$\overline{m_i} = \begin{cases} m_i + m_c \times \left(1 - \dfrac{d_f}{d}\right) & \text{for } d_f < d \\ m_i & \text{for } d_f \geqslant d \end{cases} \tag{15}$$

$$\overline{m_c} = \begin{cases} m_c \times \dfrac{d_f}{d} & \text{for } d_f < d \\ m_c & \text{for } d_f \geqslant d \end{cases} \tag{16}$$

For tanks with insufficient freeboard, masses $\overline{m_i}$ and $\overline{m_c}$ should be used instead of m_i and m_c to compute the base shears and moments (Equations 3–8). We assume that the effect of insufficient freeboard on impulsive and convective periods, hence $SA(T_{imp})$ and $SA(T_{con})$ can be ignored.

CONCLUSION

A simple method has been presented to estimate additional loads on a tank's roof, wall, and foundation due to impacts from sloshing waves. In many cases, it may be economical to design a tank for these additional loads than to build a taller tank with sufficient freeboard.

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