SPE 6977

jpt forum

A Method for Extrapolation of Cut vs Recovery Curves

Introduction

For a fully developed waterflood with no major operational changes planned, a plot of fractional water cut vs total recovery is used often to obtain a quick estimate of ultimate recovery at a given economic water cut. Extrapolation of past performance on the "cut-cum" plot is a complicated task. The difficulty arises mainly because curve fitting by simple polynomial approximation does not result in satisfactory answers in most cases.

This paper presents an equation that is a fair representation of the waterflood process, based on the concepts of fractional flow and the Buckley-Leverett recovery formula,* which may be written as

$$E_R = mX + n, \quad \ldots\ldots (1)$$

E_R = over-all reservoir recovery, volume of hydrocarbon recovered divided by oil in place,

$$X = \ln\left(\frac{1}{f_w} - 1\right) - \frac{1}{f_w},$$

f_w = fractional water cut,

where m and n are constants.

Derivation of the Equation

The fractional flow equation (after neglecting the capillary pressure and gravity terms) may be written as

$$f_w = \frac{1}{1 + \frac{k_o}{k_u} \cdot \frac{\mu_w}{\mu_o}}. \quad \ldots\ldots (2)$$

The main portion of the k_o/k_w curve vs S_w plotted on semilog paper is quite linear and may be expressed as

$$\frac{k_o}{k_w} = a\, e^{bS_w}, \quad \ldots\ldots (3)$$

where a and b are constants. On substitution Eq. 2 may be written as

$$f_w = \frac{1}{1 + A\, e^{bS_w}}, \quad \ldots\ldots (4)$$

where

$$A = a \cdot \frac{\mu_w}{\mu_o}.$$

As shown by Welge,* the water saturation at the producing end (S_w) may be expressed as

$$S_w = S_{av} - \frac{1 - f_w}{f'_w}, \quad \ldots\ldots (5)$$

where

S_{av} = average water saturation throughout the system.

Since

$$S_{av} = E_R (1 - S_{w_i}) + S_{w_i},$$

then

$$S_w = E_r (1 - S_{w_i}) + S_{w_i} - \frac{1 - f_w}{f'_w}. \quad \ldots\ldots (6)$$

In these equations, S_{wi} is the initial water saturation. From Eq. 4, the first derivative can be obtained and written as

*Welge, H. J.: "Simplified Method for Computing Oil Recoveries by Gas or Water Drive," *Trans.*, AIME (1952) 195, 91-98.

0149-2136/78/0002-6977$00.25

JPT *Forum* articles are limited to 1,500 words including 250 words for each table and figure, or a maximum of two pages in JPT. A *Forum* article may present preliminary results or conclusions of an investigation that the author wishes to publish before completing a full study; it may impart general technical information that does not warrant publication as a full-length paper. All *Forum* articles are subject to approval by an editorial committee.

Letters to the editor are published under *Dialogue*, and may cover technical or nontechnical topics. SPE-AIME reserves the right to edit letters for style and content.

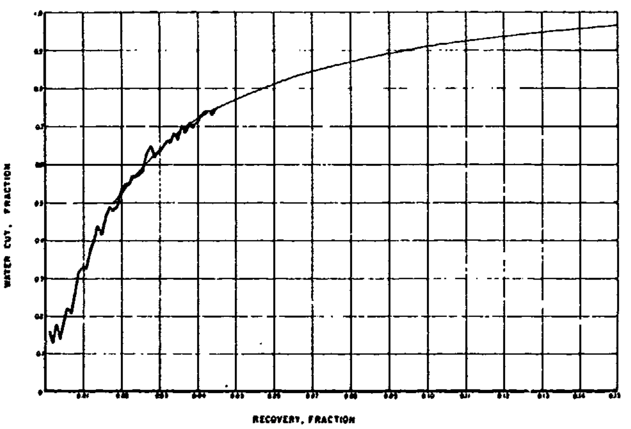

Fig. 1—Past performance of the example waterflood. The smooth line is the projection using m and n from Fig. 2.

$$f'_w = \frac{-A\,b\,e^{bS_w}}{(1 + A\,e^{bS_w})^2} = -\,b f_w\,(1 - f_w). \; \ldots\ldots. \; (7)$$

On substitution, Eq. 5 may be expressed as

$$S_w = E_R\,(1 - S_{w_i}) + S_{w_i} + \frac{1}{b \cdot f_w}. \; \ldots\ldots\ldots\ldots \; (8)$$

From Eq. 4, the following relationship becomes true.

$$f_w = 1./1 + A\,e^{b\left[E_R(1 - S_{w_i}) + S_{w_i} + \frac{1}{b \cdot f_w}\right]}. \; \ldots \; (9)$$

Solving for recovery gives

$$E_R = \frac{1}{b(1 - S_{w_i})}\left[\ln\left(\frac{1}{f_w} - 1\right) - \frac{1}{f_w}\right] - \frac{1}{1 - S_{w_i}}\left(S_{w_i} + \frac{1}{b}\ln A\right). \; \ldots\ldots\ldots \; (10)$$

This equation now may be written as

$$E_R = m X + n,$$

where

$$m = \frac{1}{b\,(1 - S_{w_i})},$$

$$X = \ln\left(\frac{1}{f_w} - 1\right) - \frac{1}{f_w},$$

$$n = -\,\frac{1}{1 - S_{w_i}}\left(S_{w_i} + \frac{1}{b}\ln A\right).$$

Method of Application

Given the past performance data, a table of cut vs fractional recovery can be constructed. From this table, a graph of fractional recovery vs $X = \ln\,(1/f_w - 1) - 1/f_w$ would result in a straight line. The straight line may be extrapolated to any desired water cut to obtain the corresponding recovery.

In applying this equation, the recovery may be used as a fraction of hydrocarbon in place (E_R), fraction of total pore volume (E'_R), or actual volume of oil produced (NP), the last two values being proportional to E_R.

The fractional flow curve has a point of inflection at $f_w = 0.5$. The proposed equation should be applied to the data point on either side of the inflection point separately. Because the objective is always to project a waterflood performance into the future, only water cuts higher than 0.5 should be used in the linear regression model.

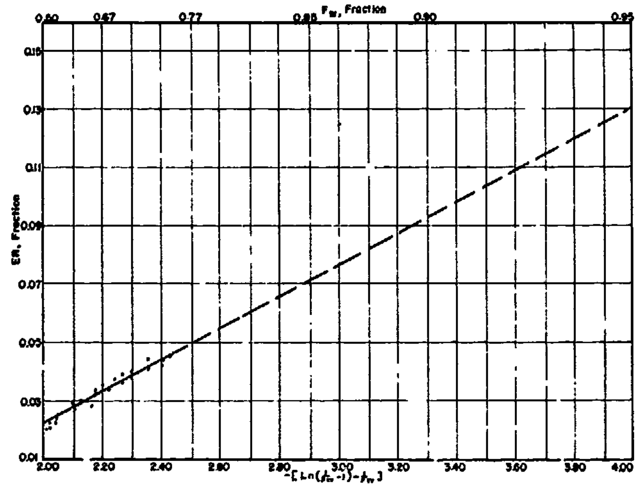

Fig. 2—Plot of recovery vs $X = -\left[\ln\left(\frac{1}{f_w} - 1\right) - \frac{1}{f_w}\right]$

Fig. 1 shows the production history of a pattern flood in a fault block. The application of this method to the flood using a linear regression computer program resulted in parameters m and n. These values then were used in Eq. 1 to obtain the extrapolations shown in Figs. 1 and 2.

Advantages and Limitations

The proposed technique is based purely on actual performance of a waterflood project. It implicitly considers reservoir configurations, heterogeneity, and displacement efficiency. One major assumption is that the operating method will remain relatively unchanged. Variations in operational procedure, resulting in a shift of actual performance, can be integrated easily in updating runs.

As an alternative, it is common to plot WOR vs fractional recovery on semilog paper and extrapolate the results. This assumption, based on the derivations shown in this paper, is equivalent to neglecting the term $1/b \cdot f_w$ in Eq. 10. Thus, the method presented here is more precise.

One interesting application of the proposed method is that from the linear plot of E_R vs X, the two constants m and n can be determined. These constants, in turn, may be used to derive a field k_w/k_o plot from Eq. 3. An estimate of S_{wi} is required to compute the constant a.

$$a = \frac{\mu_o}{\mu_w}\,e^{-b[n(1 - S_{w_i}) + S_{w_i}]},$$

$$b = \frac{1}{m\,(1 - S_{w_i})}.$$

Acknowledgment

We wish to thank L. L. Handy, who reviewed the manuscript and made helpful suggestions.

Iraj Ershaghi, SPE-AIME
Osazuwa Omoregie
U. of Southern California
Los Angeles

Original manuscript received in Society of Petroleum Engineers office Aug. 12, 1977. Paper (SPE 6977) accepted for publication Oct. 7, 1977. Revised manuscript received Dec. 6, 1977.