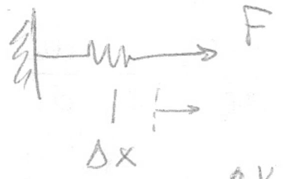
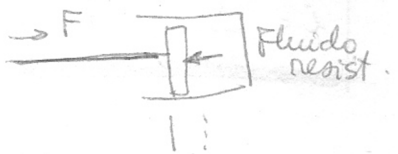
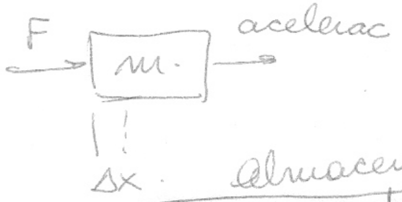






$\boxed{F = kx}$ ↑K → ↑rigidez

almacena E

$\boxed{F = cv = c\frac{dx}{dt}}$

Disipa E.

almacena E.

$\boxed{F = ma = m\frac{d^2x}{dt^2}}$

La Energía aplicada al resorte y la m. se recuperan.

La E aplicada al amortiguador no.

Resorte
$$E = \frac{1}{2}kx^2 = \frac{1}{2}\frac{F^2}{k}$$

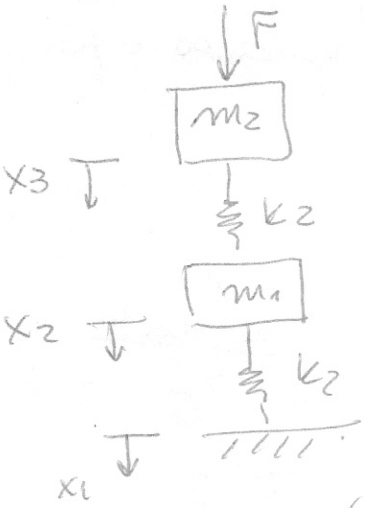
$$\boxed{E = \frac{1}{2}\frac{F^2}{k}}$$

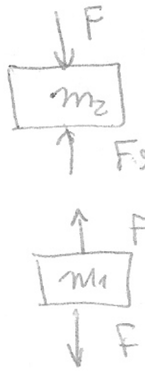
amortig. la disipa
$$\boxed{P = cv^2}$$

masa
$$\boxed{E = \frac{1}{2}mv^2}$$

En los sist rotacionales
$$T = k\theta$$
$$T = c\omega = c\frac{d\theta}{dt}.$$
$$T = I\alpha. = I\frac{d\omega}{dt} = I\frac{d\theta^2}{dt^2}$$

$$E = \frac{1}{2}\frac{T^2}{k} \qquad E = \frac{1}{2}I\omega^2 \qquad P = c\omega^2$$





F resorte sup $= k_2(x_3 - x_2)$

F resorte sup.

F resorte inf $= k_1(x_2 - x_1)$

$$\left[F_{neta} = F - k_2(x_3 - x_2) = m_2\frac{d^2x_3}{dt^2} \right.$$

$$\left[F_{neta} = k_1(x_2 - x_1) - k_2(x_3 - x_2) = m_1\frac{d^2x_2}{dt^2} \right.$$

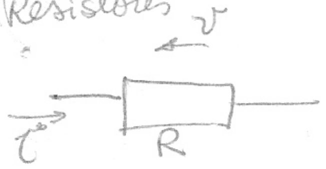
$$\begin{cases} m_1\frac{d^2x_2}{dt^2} = k_1(x_2 - x_1) - k_2(x_3 - x_2) \\ \qquad = (k_1 + k_2)x_2 - k_1x_1 - k_2x_3 \\ m_2\frac{d^2x_3}{dt^2} = F + k_2x_2 - k_2x_3. \end{cases}$$

$$\frac{d^2x_1}{dt} = 0$$

$$\frac{d^2x_2}{dt^2} = -\frac{k_1}{m_1}x_1 + \left(\frac{k_1 + k_2}{m_1}\right)x_2 - \left(\frac{k_2}{m_1}\right)x_3$$

$$\frac{d^2x_3}{dt^2} = \frac{F}{m_2} + \frac{k_2}{m_2}x_2 - \frac{k_2}{m_2}x_3$$

- Resistores

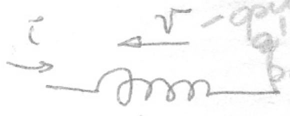


$\boxed{V = Ri}$

Disipa energía

$$\boxed{P = iv = \frac{V^2}{R}}$$

Inductores



- opuesta a la q' provoca el paso de corriente

inductancia

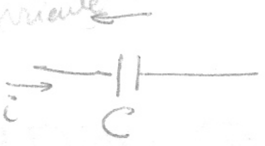
$$V = L\frac{di}{dt}$$

$$\boxed{i = \frac{1}{L}\int V\,dt}$$

→ almacenan E

$$\boxed{E = \frac{1}{2}Li^2}$$

Capacitores



$$V = \frac{q}{C} = \frac{\int i\,dt}{C}$$

$$\frac{dV}{dt} = \frac{1}{C}\frac{dq}{dt} = \frac{1}{C}i$$

$$\boxed{i = C\frac{dV}{dt}}$$

$$\boxed{E = \frac{1}{2}CV^2}$$

Modelos eléctricos: Leyes de Kirchoff.

1° la corriente total q' entra a un nodo = suma de las corrientes que salen del nodo

2° En un circuito cerrado o malla, la Σ algebraica de las dif de potencial de c/parte del circuito = al voltaje aplicado o fuerza electromotriz (fem).

Analogías eléctricas y mecánicas

$F = \dot{i}$

Resistor

$V = iR \rightarrow i = \frac{V}{R}$ $\quad P = \frac{V^2}{R}$

Amortiguador

$F = cv =$ $\quad P = cv^2$

c anál. $\frac{1}{R}$

V anál veloc.

Disipan E.

Inductor

$V = L\frac{di}{dt}$ $\quad i = \frac{1}{L}\int V dt$ $\quad E = \frac{1}{2}Li^2$

Resorte

$F = kx$ $\quad E = \frac{1}{2}\frac{F^2}{k}$

$V = \frac{dx}{dt} = v$

L anál. $\frac{1}{k}$

Acumula E

Capacitor

$i = C\frac{dV}{dt}$ $\quad E = \frac{1}{2}cV^2$

Masa

$F = m\frac{d^2x}{dt} = m\frac{dv}{dt}$ $\quad E = \frac{1}{2}mv^2$

C anál m

V anál v

Acumula E

$$V = RC\frac{dV_c}{dt} + LC\frac{d^2V_c}{dt^2} + V_c.$$

$$F = c\frac{dx}{dt} + m\frac{d^2x}{dt^2} + kx.$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m}} \qquad \xi = \frac{c}{2\sqrt{mk}}}$$

⑥ ⓐ

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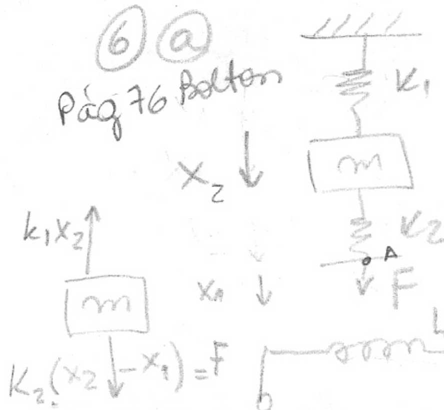
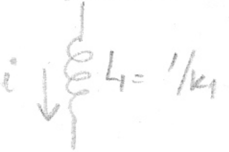
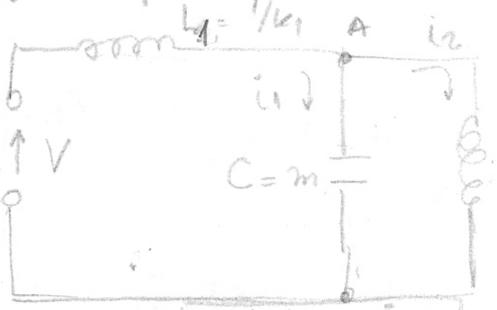




K_1 es equivalente a una inducción $L_1 = 1/k_1$

$$F_{neta} = k_1x_2 - \underbrace{k_2(x_2-x_1)}_{F} = m\frac{dx^2}{dt^2}$$

$$\boxed{F = m\frac{d^2x}{dt^2} - k_1x}$$



$$i_1 = C\frac{dV_A}{dt} \longrightarrow V_A = \frac{1}{C}\int i_1 dt$$

$$V_A = L_2\frac{di_2}{dt} \Rightarrow i_2 = \frac{1}{L_2}\int V_A dt$$

$$V = L_1\frac{di}{dt} + V_A \Rightarrow i = \int\frac{V - V_A}{L_1}dt$$

$$\boxed{i = i_1 + i_2}$$

$$\overbrace{\frac{1}{L_1}\int(V-V_A)dt}^{i} = \overbrace{C\frac{dV_A}{dt}}^{i_1} + \overbrace{\frac{1}{L_2}\int V_A dt}^{i_2}$$

$$\boxed{V = CL_1\frac{d^2V_A}{dt^2} + \frac{L_1}{L_2}V_A + V_A}$$

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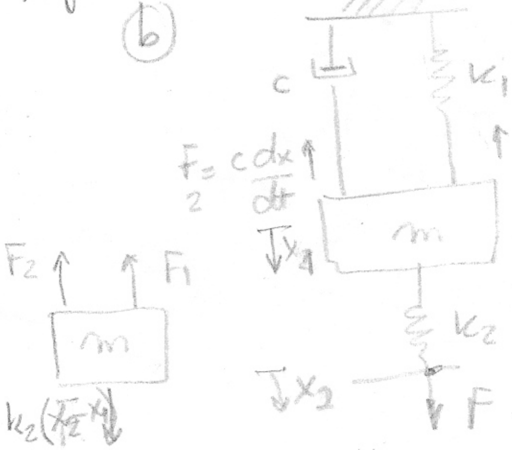
ⓑ

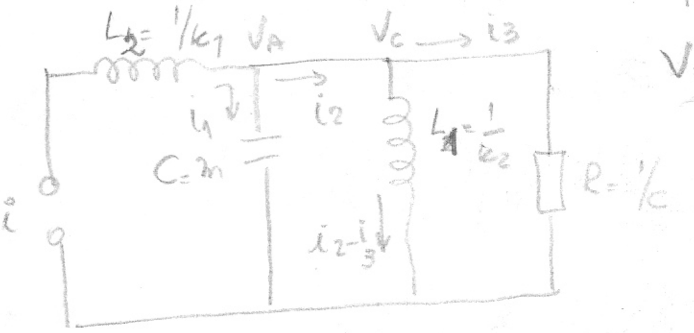


$$F_{meta} = F - F_1 - F_2 = m\,dx_1/dt$$

$$F - k_1x_1 - C\frac{dx_1}{dt} = m\frac{d^2x_1}{dt^2}$$

$k_2(x_2 - x_1)$

$$\boxed{F = m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + k_1x_1}$$



$V = iR$

$i = i_1 + i_2$

$i_1 = C\frac{dV_A}{dt}$

$i_2 = \frac{V_C}{R} + V_C = V_C(\frac{1}{R}+1)$

V_C

$V = L_2\frac{di}{dt} + V_A$

$0 = -\frac{1}{C}\int i_1 dt + L_1\frac{d}{dt}(i_2 - i_3)$

$V_C = i_3R$

ⓐ $$V = L_1\frac{di}{dt} + V_A = L_1\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right) + V_A = L_1C\frac{d^2V_A}{dt^2} + \frac{L_1}{L_2}V_A + V_A$$

$$\boxed{V = L_1C\frac{d^2V_A}{dt^2} + \left(\frac{L_1}{L_2} + 1\right)V_A}$$

ⓑ $$V = L_2\frac{d}{dt}\left(C\frac{dV_A}{dt} + V_A\left(\frac{1}{R}+1\right)\right) + V_A$$

$$\boxed{V = L_2C\frac{d^2V_A}{dt} + \left(\frac{R+1}{R}\right)\frac{dV_A}{dt} + V_A}$$

⑤

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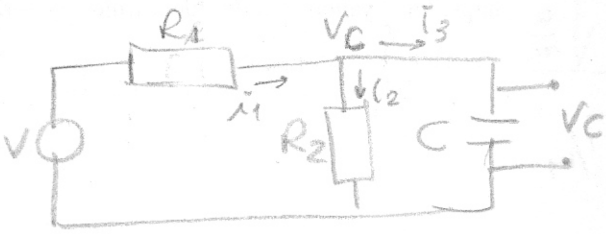

$$\boxed{V=(R_1C)\frac{dV_c}{dt}+\left(\frac{R_1}{R_2}+1\right)V_c}$$

$$i_1=i_2+i_3=\frac{V_c}{R_2}+C\frac{dV_c}{dt}=\frac{(V-V_c)}{R_1}$$

⑪

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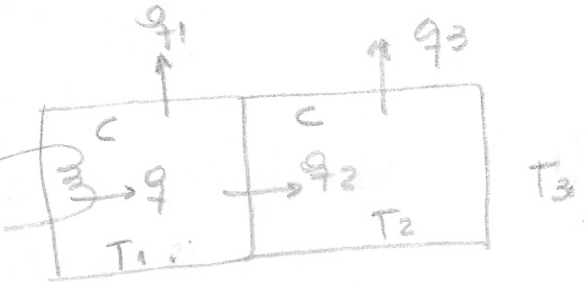

Resist de todas las paredes =

$$q_2=\frac{(T_1-T_2)}{R}$$

$$C\frac{dT_1}{dt}=q-q_1-q_2$$

$$q_3=\frac{(T_2-T_3)}{R}$$

$$C\frac{dT_2}{dt}=q_2-q_3$$

$$q_1=\frac{(T_1-T_3)}{R}$$

$$C\frac{dT_1}{dt}=q-\frac{(T_1-T_3)}{R}-\frac{(T_1-T_2)}{R}=q-\frac{(2T_1-T_2-T_3)}{R}$$

$$C\frac{dT_1}{dt}=q+\frac{(-T_1+T_3-T_1+T_2)}{R}$$

$$C\frac{dT_2}{dt}=\frac{(T_1-T_2)}{R}-\frac{(T_2-T_3)}{R}=\frac{T_1-T_2-T_2+T_3}{R}$$

$$C\frac{dT_2}{dt}=\frac{(T_1-2T_2+T_3)}{R}\rightarrow\boxed{RC\frac{dT_2}{dt}=T_1-2T_2+T_3}$$

$$\boxed{RC\frac{dT_1}{dt}=Rq-2T_1+T_2+T_3}$$

⑧

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$$A_1\frac{dh_1}{dt}=q-\left(\frac{h_1-h_2}{R}\right)\rho g\rightarrow\boxed{A_1\frac{dh_1}{dt}+h_1\frac{\rho g}{R}=q+h_2\frac{\rho g}{R}}$$

Presion: $h\rho g$

$$A_2\frac{dh_2}{dt}=\frac{(h_1-h_2)}{R}\rho g\rightarrow\boxed{\frac{RA_2}{\rho g}\frac{dh_2}{dt}+h_2=h_1}$$

$R=\frac{P}{Q}$

$C=\frac{A}{\rho g}$

Resist torsional

2.6 b

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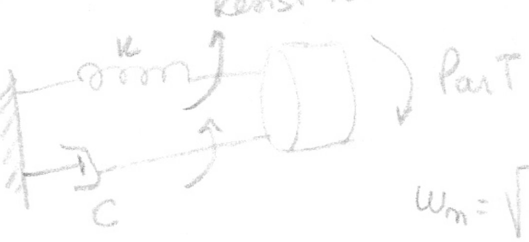


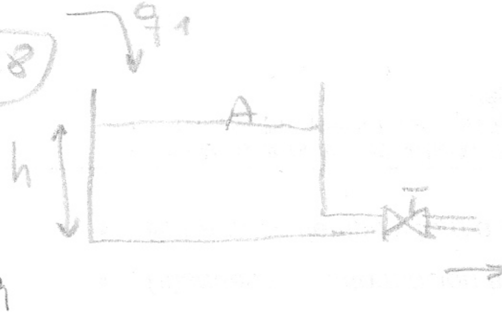
$$\frac{T}{k}=\underbrace{\frac{I}{k}}_{\frac{1}{w_n^2}}\frac{d^2\theta}{dt^2}+\underbrace{\frac{c}{k}}_{\frac{2\xi}{w_n}}\frac{d\theta}{dt}+\frac{k\theta}{k}$$

$w_n=\sqrt{\frac{k}{I}}$

$\frac{2\xi}{w_n}=\frac{2C}{2\sqrt{Ik}}\sqrt{\frac{k}{I}}$

$$\boxed{\xi=\frac{C}{2\sqrt{Ik}}}$$

(2.28)



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$$q_1 - q_2 = C\frac{dp}{dt}$$

C = capacitancia hidraúlica

$$\left\{ q_2 = \frac{1}{R} p\right.$$

$$\left\{ p = \rho g h\right.$$

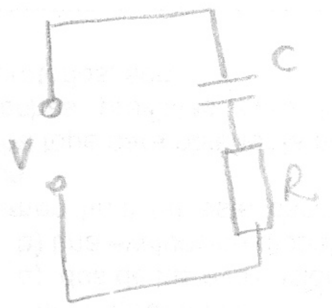
$$\left\{ C = \frac{A}{\rho g}\right.$$

$$q_1 - \frac{p}{R} = C\frac{dp}{dt}$$

$$q_1 - \frac{\rho g h}{R} = C\rho g\frac{dh}{dt}$$

$$\boxed{q_1 = A\frac{dh}{dt} + \frac{\rho g h}{R}}$$

(3-4)



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$$V = V_c + iR$$

$$\frac{dV_c}{dt} = \frac{1}{RC}(V - V_c)$$

Potenciómetro

Fig 2-36

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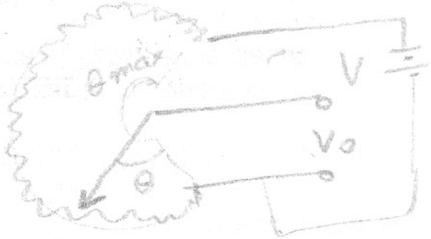

$$\frac{V_o}{V} = \frac{\theta}{\theta_{máx}}$$

$$\frac{V_o}{\theta} = \frac{V}{\theta_{máx}} = \text{cte}$$

Probl 13 → Termopar

Fig. 2.53. Probl 14 Péndulo simple } linealizar

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$$\Delta T = mgl\,\Delta\theta$$

Probl. 13 La relación entre E y la temperat (Termopar)

$$E = aT + bT^2 \qquad \Delta E = (a + 2bT_0)\,\Delta T.$$

$$f(x) = f(\bar{x}) + \left.\frac{df(x)}{dx}\right|_{x_0} \cdot (x - x_0) \rightarrow y = a + bx.$$

$$\Delta(bT^2) = 2bT_0\,(\Delta T)$$

(6) (a)

$$F = m\frac{d^2x}{dt^2} + k_1 x \rightarrow$$

$$\overset{\tau^2}{\boxed{\frac{m}{k}}}\frac{d^2x}{dt^2} + x = \overset{K}{\boxed{\frac{1F}{k}}}$$

$$\tau^2 = \frac{m}{k_1} = \frac{kg\ s^2}{kg}$$

$$K = \frac{1}{k} = \frac{s^2}{kg} = \frac{[x]}{[F]} = \frac{m}{kg\frac{m}{s^2}}$$

$$V = CL_1\frac{d^2V_A}{dt^2} + \left(\frac{L_1}{L_2}+1\right)V_A.$$

$$\tau^2 = \frac{CL_1L_2}{L_1+L_2} = \frac{F\cancel{H}^2}{\cancel{H}} = \frac{s^4A^2}{kg\,m^2}\frac{m^2kg}{s^2A^2} = seg^2$$

$$K = \frac{L_2}{L_1+L_2} = \text{adimensional.}$$

(b)

$$\overset{\tau^2}{\boxed{\frac{m}{k}}}\frac{d^2x}{dt^2} + \overset{2\tau\xi}{\boxed{\frac{C}{k}}}\frac{dx}{dt} + \overset{1}{\boxed{\frac{k}{k}}}x = \frac{F}{k}$$

$$V = \overset{\tau^2}{\boxed{CL_1}}\frac{d^2V_A}{dt^2} + \left(\frac{1}{R}+1\right)\frac{dV_A}{dt} + V_A.$$

$$\tau^2 = \frac{m}{k_1}$$

$$2\xi\tau = \frac{C}{k}$$

$$\frac{1}{k} = K.$$

(5)

$$V = R_1C\frac{dV_C}{dt} + \left(\frac{R_1}{R_2}+1\right)V_C$$

$$\tau = \frac{R_1CR_2}{R_1+R_2} = [C]\cdot[R] = \frac{V}{A}\cdot\frac{C}{V} = \frac{sA}{A}$$

$$K = \frac{R_2}{R_1+R_2} = \text{adim}$$

(11)

SIST MIMO

$$\overset{\tau}{\boxed{\frac{RC}{2}}}\frac{dT_1}{dt} + \frac{2}{2}T_1 = \overset{K_1}{\boxed{\frac{R}{2}}}q + T_2 + T_3$$

$$\overset{\tau}{\boxed{\frac{RC}{2}}}\frac{dT_2}{dt} + \frac{2}{2}T_2 = \overset{K_2}{\boxed{\frac{1}{2}}}T_1 + T_3$$

$$\tau = RC = \left(\frac{°K}{W}\right)\left(\frac{J}{°K}\right) = \frac{J}{J/s}$$

$$K_1 = R = \frac{°K}{W}\left(\frac{\text{Temp}}{\text{calor}}\right)$$

$$K_2 = \frac{1}{2}\ \text{adim.}\left(\frac{\text{Temp}}{\text{Temp}}\right)$$

(8)

SIST MIMO

$$\overset{\tau_1}{\boxed{\frac{RA_2}{\rho g}}}\frac{dh_2}{dt} + h_2 = \overset{K_1}{\boxed{1}}h_1$$

$$\underset{\tau_2}{\boxed{\frac{RA_1}{\rho g}}}\frac{dh_1}{dt} + h_1 = \overset{K_2}{\boxed{\frac{R}{\rho g}}}q + h_2$$

$$\tau_1 = \frac{RA_2}{\rho g} = \frac{\frac{kg}{m^4s}\cdot m^2}{\frac{kg}{m^3}\frac{m}{s^2}} = s$$

$$K_1 = 1\ \text{adimensional}\left(\frac{\text{Temp}}{\text{Temp}}\right)$$

$$K_2 = \frac{R}{\rho g}\ \frac{\frac{kg}{m^4s}}{\frac{kg}{m^3}\cdot\frac{m}{s^2}} = \frac{s}{m^2}\left(\frac{h}{q}\right)$$

(2.6)

$$\frac{I}{k}\frac{d^2\theta}{dt^2} + \frac{C}{k}\frac{d\theta}{dt} + \theta = \frac{I}{k}.$$

$$\frac{I}{k} = \frac{1}{\omega_n^2}\frac{d^2\theta}{dt^2} + \frac{2\xi}{\omega_n}\frac{d\theta}{dt} + \theta$$

$$\omega_n = \sqrt{\frac{k}{I}}\quad \frac{rad}{seg}$$

$$\xi = \frac{C}{2\sqrt{Ik}}$$

$$K = \frac{1}{k} \rightarrow \text{cte torsional}$$

(2-28) $q_1 = A\frac{dh}{dt} + \frac{\rho g h}{R}$

(3-4) $\frac{dV_c}{dt} = \frac{1}{RC}(V - V_c)$

Capacit. térmica $= \frac{J}{°K}$

Tensión $V = \frac{W}{A} = \frac{J}{sA} = \frac{Nm}{sA} = \frac{kg\,m^2}{s^3 A}$.

Capacitancia eléctrica $\frac{\text{Coul.}}{\text{Voltio}} = F$ (Faradio) $= \frac{q}{V} = \frac{s^4 A^2}{kg\,m^2}$

Inductancia = (Henrio) $H = \frac{m^2 kg}{s^2 A^2}$

δ flujo magnético = T (Tesla)

Resist eléctrica $= \Omega$ ohmio $= \frac{V}{A}$.

Conductancia eléctrica = S (siemens) $= \frac{A}{V}$

Carga eléctrica = C (coulombio) = sA.

Ctte del resorte $= \frac{F}{x} = \frac{N}{m} = \frac{kg\,\cancel{m}}{s^2\,\cancel{m}}$

Calor = J = Nm. $= \frac{m^2 kg}{s^2}$

Resistencia hidráulica $= \frac{\text{Pres}}{\text{caudal}} = \frac{h\rho g}{Q} = \frac{m\frac{kg}{m^3}\frac{m}{s^2}}{\frac{m^3}{s}} = \frac{kg}{m^4 s}$

Capacitancia hidráulica $= \frac{A}{\rho g} = \cancel{\frac{J}{°K}} = \cancel{\frac{m^2 kg}{s^2 °K}}\ \frac{m^2}{kg/m^3 \cdot m/s^2}$

Densidad $\frac{kg}{m^3}$

Inertancia hidráulica $= \frac{\text{Long.}}{\text{Secc.}}\rho =$

Resistencia térmica $= \frac{\Delta T}{Q} = \frac{°K}{W} = \frac{L}{Ak}$ ↘ conduct térmica $\frac{m\,kg}{s^2\,°K}$

Capacitancia térmica $= mc = C$

↘ capacidad calorífica $\frac{J}{kg\,°K}$.