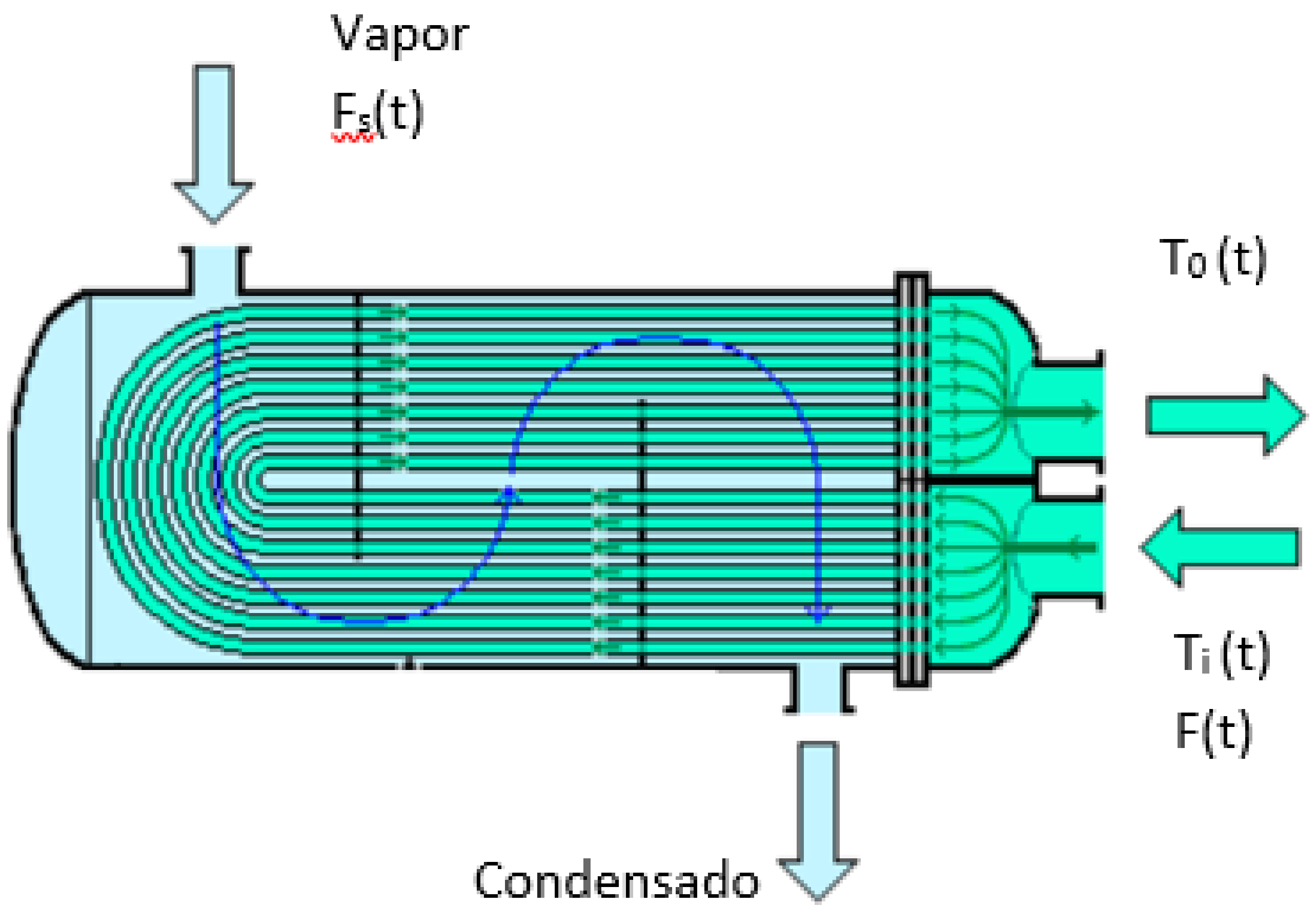
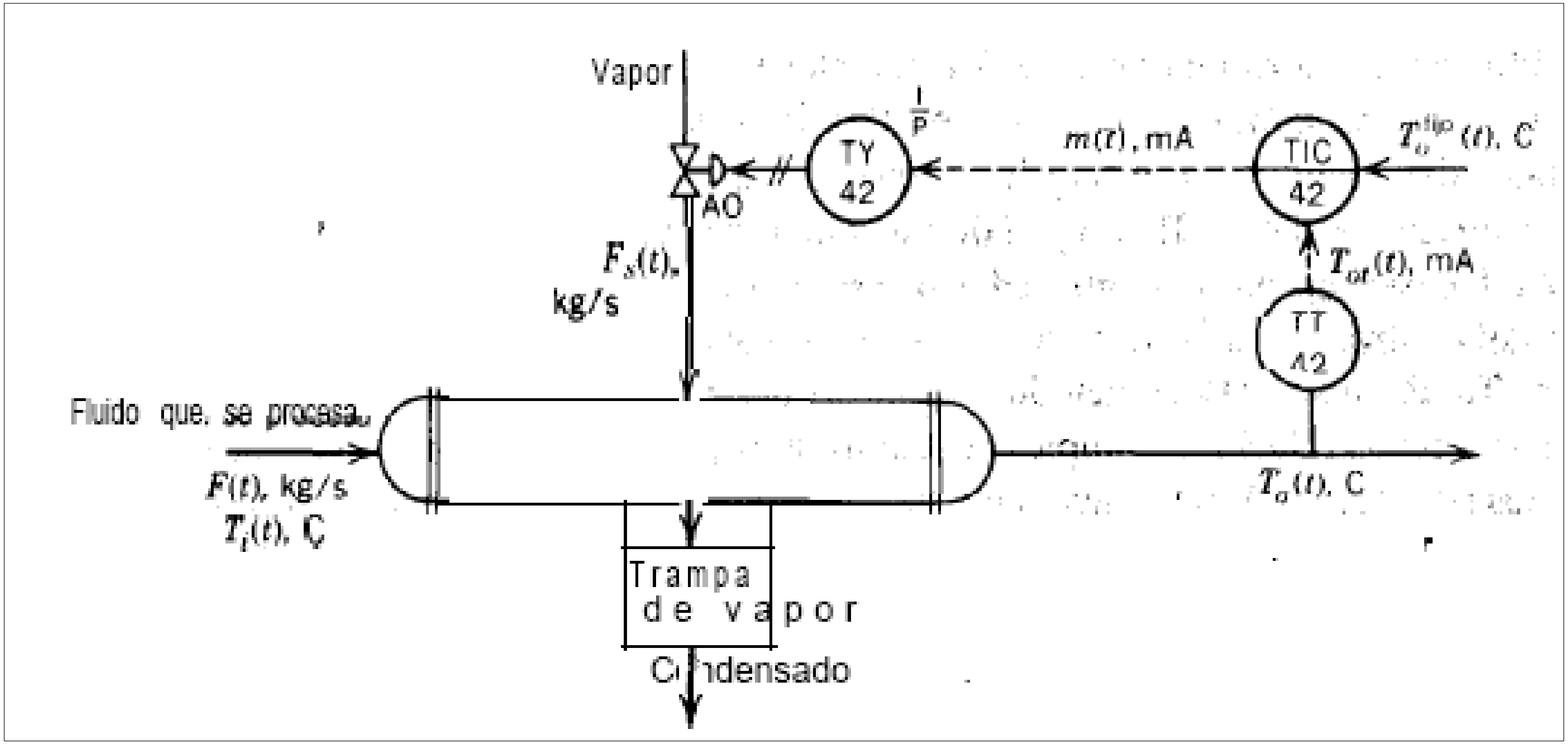
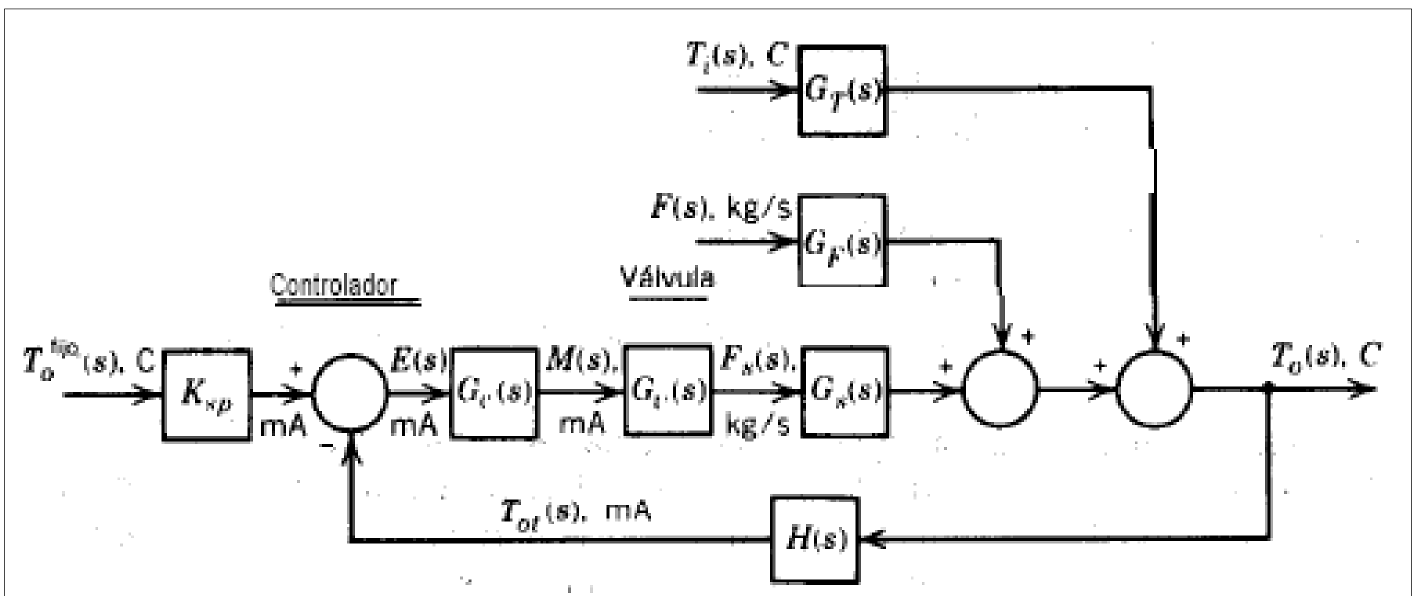
UNIDAD 2

ESTABILIDAD

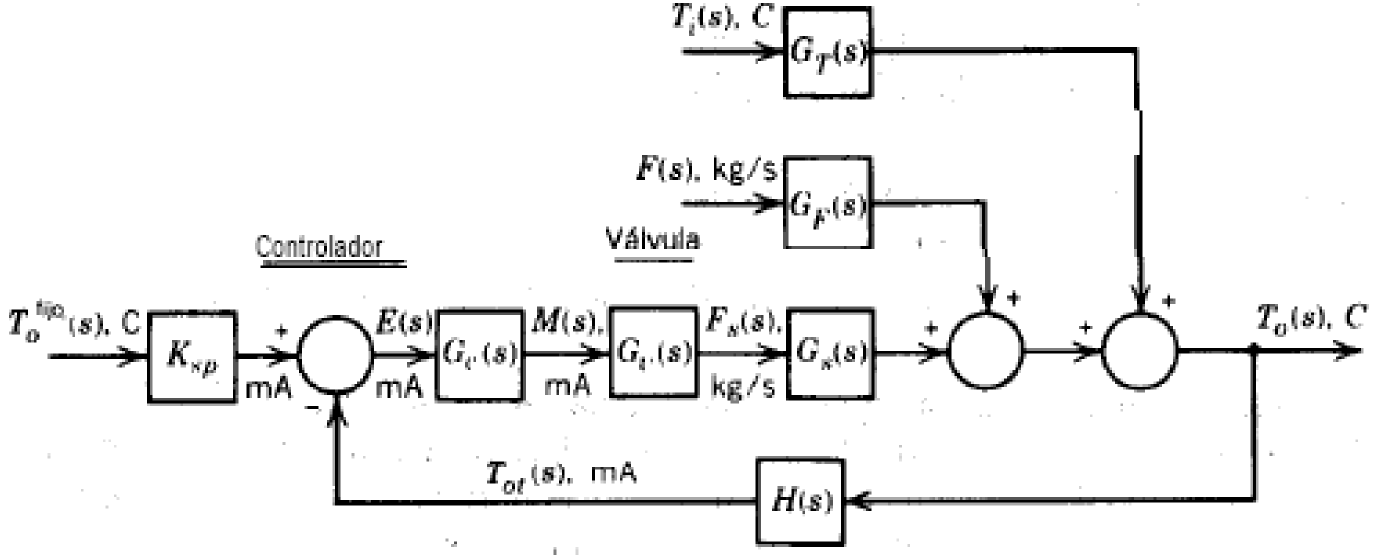

Vapor
Fs(t)
T0 (t)
Ti (t)
F(t)
Condensado



Vapor
I/P
TY 42
AO
m(t), mA
TIC 42
$T_o^{fijo}(t)$, C
$F_s(t)$, kg/s
$T_{ot}(t)$, mA
TT 42
Fluido que se procesa
$F(t)$, kg/s
$T_i(t)$, C
$T_o(t)$, C
Trampa de vapor
Condensado



$T_i(s)$, C
$G_T(s)$
$F(s)$, kg/s
$G_F(s)$
Controlador
Válvula
$T_o^{fijo}(s)$, C
K_{sp}
mA
$E(s)$
mA
$G_c(s)$
$M(s)$, mA
$G_v(s)$
$F_s(s)$, kg/s
$G_s(s)$
$T_o(s)$, C
$T_{ot}(s)$, mA
$H(s)$



Señal de error: $E(s) = K_{sp}T_o^{fijo}(s) - T_{ot}(s)$

Variable manipulada: $M(s) = G_c(s)E(s)$

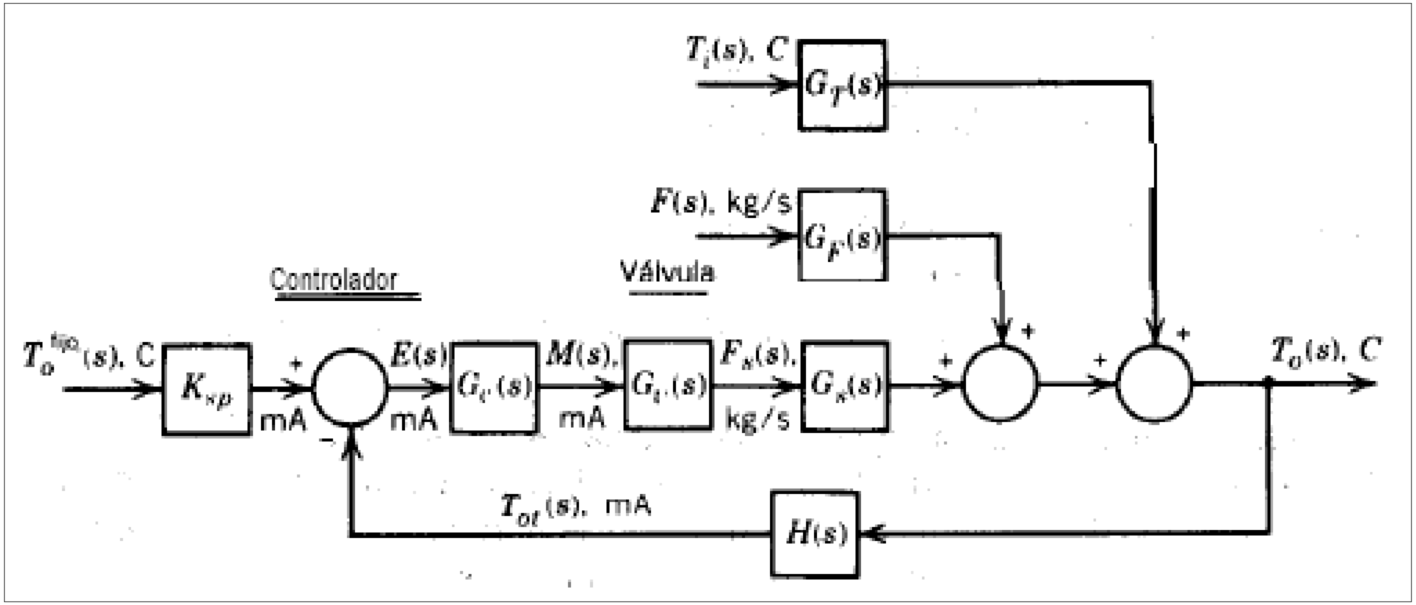
Flujo de vapor: $F_s(s) = G_v(s)M(s)$

Temperatura de salida: $T_o(s) = G_s(s)F_s(s) + G_F(s)F(s) + G_T(s)T_i(s)$

Señal del transmisor: $T_{ot}(s) = H(s)T_o(s)$

$$\left.\begin{aligned} F(s) &= 0 \\ T_o^{fijo}(s) &= 0 \end{aligned}\right\} \Rightarrow T_o(s) = G_s(s)\,G_v(s)\,G_c(s)\,[-H(s)T_o(s)] + G_T(s)\,T_i(s)$$

$$\frac{T_o(s)}{T_i(s)} = \frac{G_T(s)}{1 + H(s)\,G_s(s)\,G_v(s)\,G_c(s)}$$



$T_o^{fijo}(s) = 0$, $T_i(s) = 0$ ⇒

$$\frac{T_o(s)}{F(s)} \quad \frac{G_F(s)}{1 + H(s)\,G_s(s)\,G_v(s)\,G_c(s)}$$

$T_i(s) = 0$, $F(s) = 0$ ⇒

$$\frac{T_o(s)}{T_o^{set}(s)} = \frac{G_s(s)G_v(s)G_c(s)K_{sp}}{1 + H(s)\,G_s(s)\,G_v(s)\,G_c(s)}$$

Ecuación característica:

$$1 + H(s)G_s(s)G_v(s)G_c(s) = 0$$

$$T_o(t) = b_1 e^{r_1 t} + b\ e^{r_2 t} + \ldots + b_n e^{r_n t} + (\text{términos de entrada})$$

Respuesta sin forzamiento — Respuesta forzada

$$\left.\begin{matrix} F(s) = 0 \\ T_o^{\text{fijo}}(s) = 0 \end{matrix}\right\} \Rightarrow \frac{T_o(s)}{T_i(s)} = \frac{G_T(s)}{1 + H(s)\,G_s(s)\,G_v(s)\,G_c(s)}$$

$$\left.\begin{matrix} T_o^{\text{fijo}}(s) = 0 \\ T_i(s) = 0 \end{matrix}\right\} \Rightarrow \frac{T_o(s)}{F(s)} \quad \frac{G_F(s)}{1 + H(s)\,G_s(s)\,G_v(s)\,G_c(s)}$$

$$\left.\begin{matrix} T_i(s) = 0 \\ F(s) = 0 \end{matrix}\right\} \Rightarrow \frac{T_o(s)}{T_o^{\text{set}}(s)} = \frac{G_s(s)\,G_v(s)\,G_c(s)\,K_{sp}}{1 + H(s)\,G_s(s)\,G_v(s)\,G_c(s)}$$

$$a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0 = a_n(s - r_1)(s - r_2) \ldots (s - r_n) = 0$$

$$T_o(s) = \frac{(\text{términos del numerador})}{a_n(s - r_1)(s - r_2) \ldots (s - r_n)(\text{términos de entrada})}$$

$$T_o(t) = \underbrace{b_1 e^{r_1 t} + b_2 e^{r_2 t} + \ldots + b_n e^{r_n t}}_{\text{Respuesta sin forzamiento}} + \underbrace{(\text{términos de entrada})}_{\text{Respuesta forzada}}$$

Criterio de estabilidad

$$c(t) = b_1 e^{r_1 t} + b_2 e^{r_2 t} + \ldots + b_n e^{r_n t} + \text{(términos de entrada)}$$

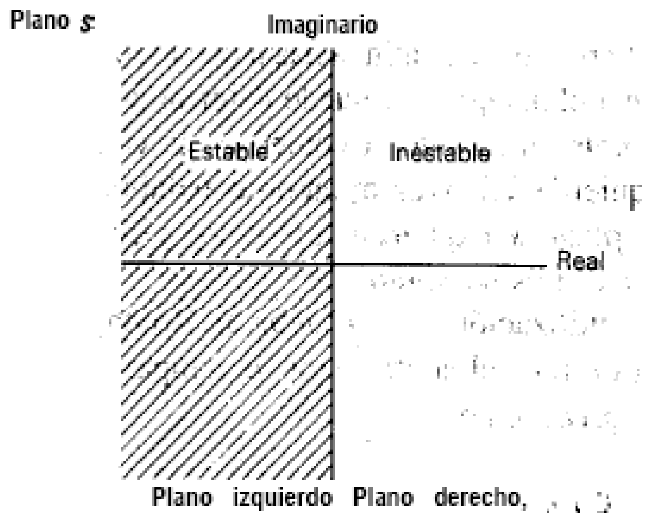
Para raíces reales: Si $r < 0$, entonces $e^{rt} \to 0$ conforme $t \to \infty$

Para raíces complejas: $r = \sigma + i\omega$ $e^{rt} = e^{\sigma t}(\cos \omega t + i \operatorname{sen} \omega t)$

Si $\sigma < 0$, entonces $e^{\sigma t}(\cos \omega t + i \operatorname{sen} \omega t) \to 0$ conforme $t \to \infty$

Para **que el circuito** de control con **retroalimentación sea estable, todas las raíces** de **su ecuación característica deben ser** números reales negativos o números **complejos** con **partes reales** negativas.

Prueba de Routh

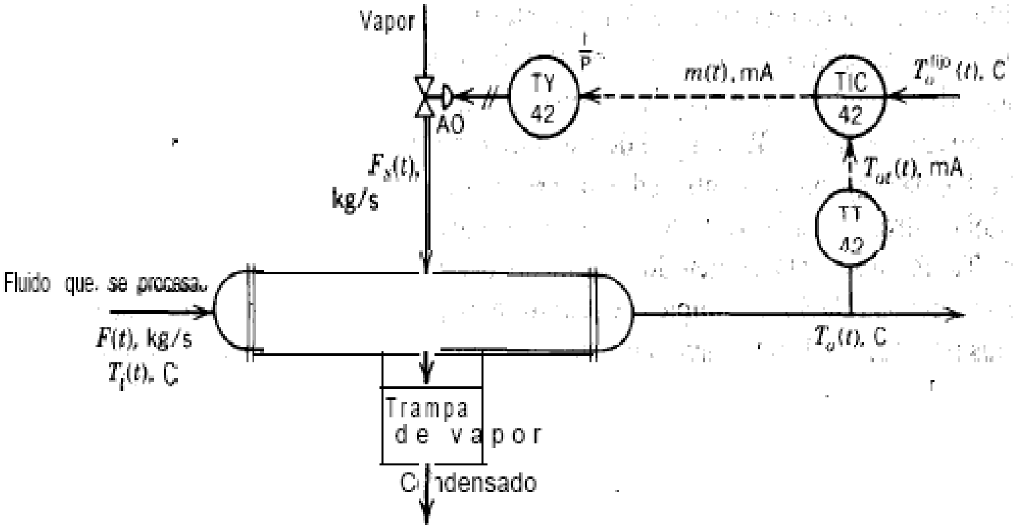


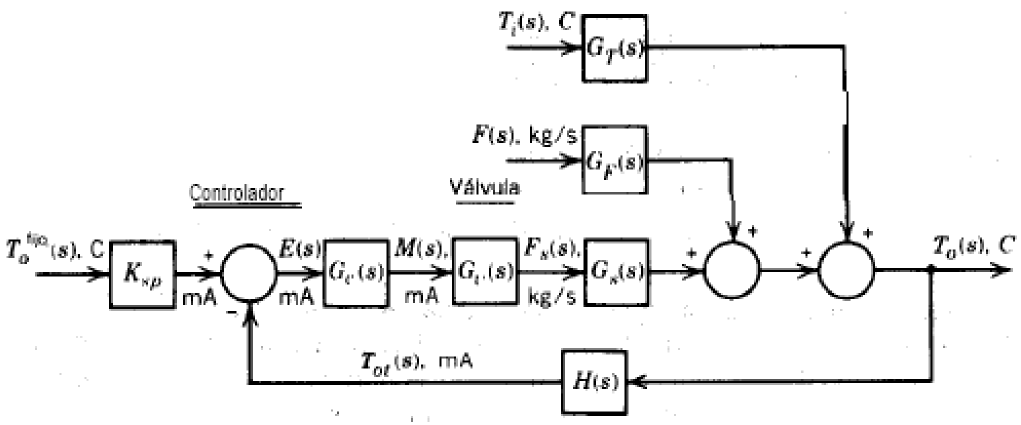
$$a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 = 0$$

fila 1	a_n	a_{n-2}	a_{n-4}	.	.	.	a_1	0
fila 2	a_{n-1}	a_{n-3}	a_{n-5}	.	.	.	a_0	0
fila 3	b_1	b_2	b_3	.	.	.	0	0
fila 4	c_1	c_2	c_3	.	.	.	0	0
$\vdots$	$\vdots$	.	.	.	.	.	.	.
fila n	d_1	d_2	0	.	.	.	0	0

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \qquad b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

$$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1} \qquad c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1}$$





Intercambiador:

$$G_s(s) = \frac{50}{30\,s + 1}\ \text{C/(kg/s)}$$

Sensor-transmisor (escala de 50 a 150 ªC)

$$H(s) = \frac{1.0}{10\,s + 1}\ \text{\%/C}$$

Válvula de control (incluye el conversor I/P):

$$\frac{1.6\ \text{(kg/s)}}{100\%} = 0.016\ \text{(kg/s)/\%}$$

$$G_v(s) = \frac{0.016}{3\,s + 1}\ \text{(kg/s)/\%}$$

Controlador (proporcional)

$$G_c(s) = K_c\ \text{\%/\%}$$

Ecuación característica: $$1+\left(\frac{50}{30.s+1}\right).\left(\frac{1}{10.s+1}\right).\left(\frac{0.016}{3.s+1}\right).(Kc)=0$$

$$(10.s+1).(30.s+1).(3.s+1)+0.80.Kc=0 \longrightarrow 900.s^3+420.s^2+43.s+(1+0.80.Kc)=0$$

Prueba de Routh

$$900.s^3 + 420.s^2 + 43.s + (1 + 0.80.Kc) = 0$$

fila 1	900	43	0
tila 2	420	$1 + 0.80K_c$	0
tila 3	b_1	0	0
fila 4	$1 + 0.80K_c$	0	0

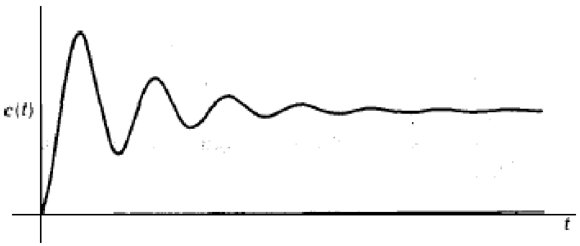
$$b_1 = \frac{(420).(43) - 900.(1 + 0.80.Kc)}{420} = \frac{17160 - 720.Kc}{420}$$

$$b_1 \geq 0 \rightarrow 17160 - 720.Kc \geq 0 \rightarrow Kc \leq 23.8$$

$$1 + 0.80.Kc \geq 0 \rightarrow 0.80.Kc \geq 0 \rightarrow Kc \geq -1.25$$

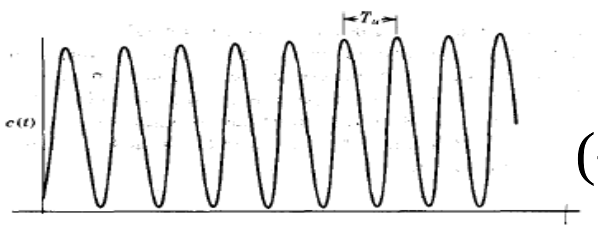
$$\boxed{Kc \leq 23.8}$$

Método de substitución directa



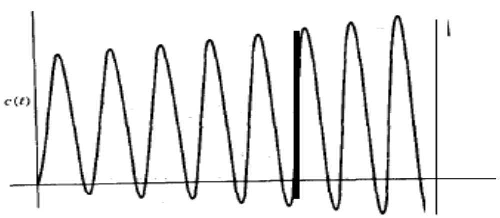
$$900.s^3 + 420.s^2 + 43.s + (1 + 0.80.Kc) = 0$$

$$s = i\omega$$



$$900.(i\omega)^3 + 420.(i\omega)^2 + 43.(i\omega) + (1 + 0.80.Kc) = 0$$

$$(-420.\omega_u{}^2 + 1 + 0.80.Kc_u) + i(-900.\omega_u{}^3 + 43\omega_u) = 0 + i0$$



$$(-420.\omega_u{}^2 + 1 + 0.80.Kc_u) = 0$$

$$(-900.\omega_u{}^3 + 43\omega_u) = 0$$

$$\omega_u = 0 \rightarrow Kc_u = -1.25 \frac{\%}{\%}$$

$$\omega_u = 0.22 rad/seg \rightarrow Kc_u = 23.8 \frac{\%}{\%}$$

Efecto del tiempo muerto:

$$e^{-t_0 s} \doteq \frac{1 - \frac{1}{2} t_0 s}{1 + \frac{1}{2} t_0 s}$$

Aproximacion de Pade:

$$G(s) = \frac{K e^{-t_0 s}}{\tau s + 1}$$

$$G_c(s) = K_c$$

$$1 + G.Kc = 0 \longrightarrow 1 + \left(\frac{K.e^{-t_0 s}}{\tau.s + 1}\right).(Kc) = 0$$

$$1 + \left(\frac{K}{\tau.s + 1}\right)\left(\frac{1 - \frac{t_0 s}{2}}{1 + \frac{t_0 s}{2}}\right).(Kc) = 0$$

$$1 + \frac{K.Kc(1 - \frac{t_0 s}{2})}{(\tau.s + 1)(1 + \frac{t_0 s}{2})} = 0$$

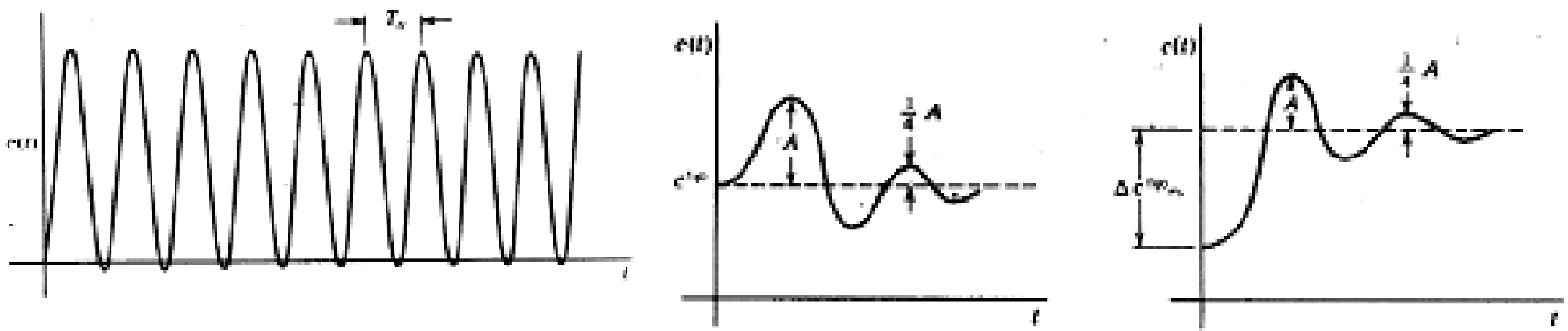
$$1/2.t_0 \tau.s^2 + (\tau + 1/2.t_0 - 1/2.K.Kc.t_0)s + 1 + K.Kc = 0$$

$$(K.Kc)_u = 1 + 2\frac{\tau}{t_0}$$

$$\omega_u = \frac{2}{t_0}\sqrt{\frac{t_0}{\tau} + 1}$$

Auto sintonía de controladores- Kcu y Tu

1. se desconectan las acciones integral y derivativo del controlador, de manera de tener un controlador proporcional. En algunos modelos no es posible desconectar la acción integral, se iguala R al valor máximo.
2. con el controlador cerrando el circuito, se incrementa la acción proporcional constante. Luego se registra el valor de Kcu. Los incrementos deben ser pequeños, en especial al acercarse al valor de oscilación permanente.
3. del registro del tiempo de la variable controlada, se registra y mide el período de oscilación como *Tu*, período último, según se muestra en la figura 3.13



Entrada de perturbación   Cambio del punto de control

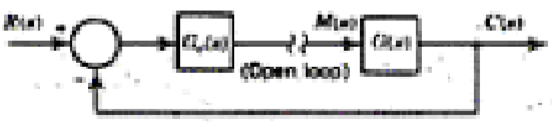
Tabla 6-1 Fórmulas para ajuste de razón de asentamiento de un cuarto.

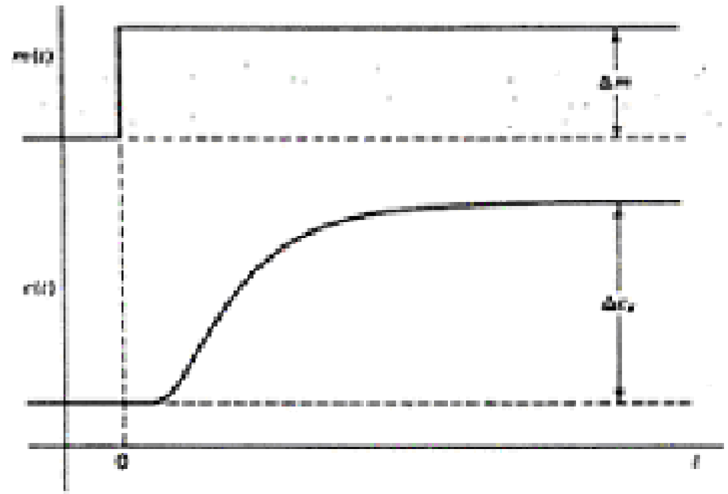
Tipo de controlador		*Ganancia proporcional* K_C	*Tiempo de integración* τ_I	*Tiempo de derivación* τ_D
Proporcional	P	$K_{cu}/2$	—	—
Proporcional-integral	PI	$K_{cu}/2.2$	$T_u/1.2$	—
Proporcional-integral-derivativo	PID	$K_{cu}/1.7$	$T_u/2$	$T_u/8$

Caracterización del proceso- Ajuste de controladores

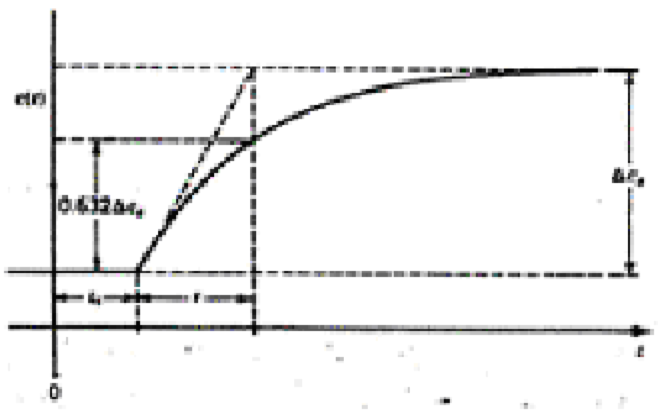
$$C(s) = G(s)\frac{\Delta m}{s}$$

$$C(s) = \frac{K.e^{-t_0 s}}{\tau.s+1}\frac{\Delta m}{s}$$



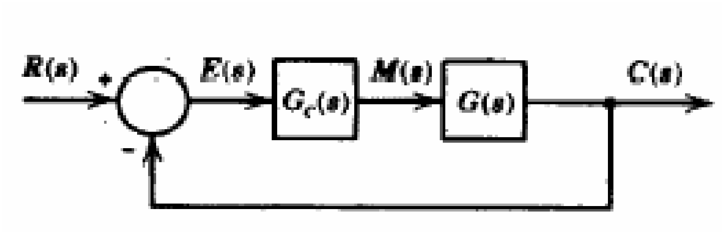


$$K = \frac{\Delta Cs}{\Delta m}$$



Controller Type		Proportional Gain K_c	Integral Time τ_I	Derivative Time τ_D
Proportional only	P	$\frac{1}{K}\left(\frac{t_0}{\tau}\right)^{-1}$	—	—
Proportional-integral	PI	$\frac{0.9}{K}\left(\frac{t_0}{\tau}\right)^{-1}$	$3.33\ t_0$	—
Proportional-integral-derivative	PID	$\frac{1.2}{K}\left(\frac{t_0}{\tau}\right)^{-1}$	$2.0\ t_0$	$\frac{1}{2}t_0$

Método de síntesis directa o ajuste de Dahlin



$$\frac{C(s)}{R(s)} = \frac{Gc(s)\,G(s)}{1 + Gc(s)\;G(s)}$$

$$Gc(s) = \frac{1}{G(s)}\;\frac{C(s)\,/\,R(s)}{1 - [C(s)\,/\,R(s)]} \longrightarrow Gc(s) = \frac{1}{G(s)}\;\frac{1}{1-1} = \frac{1}{G(s)}\;\frac{1}{0}$$

$$\frac{C(s)}{R(s)} = \frac{1}{\tau_c\;s+1} \longrightarrow \boxed{Gc(s) = \frac{1}{G(s)}\;\frac{1}{\tau_c\;s}}$$

Si $Gp = \dfrac{1}{\tau\;s+1}$ (proceso de primer orden)

$$Gc(s) = \frac{\tau}{K\tau_c}\left(1 + \frac{1}{\tau\;s}\right) \Rightarrow Kc = \frac{\tau}{K\tau_c} \qquad \tau_i = \tau$$

Si $Gp = \dfrac{K}{(\tau_1\;s+1)\,(\tau_2\;s+1)}$ (proceso de segundo orden) $\Rightarrow Gc(s) = \dfrac{\tau_1}{K\tau_c}\left(1 + \dfrac{1}{\tau_1\;s}\right)(\tau_2\;s+1)$

Modelo del proceso : $G(s) = \frac{K e^{-t_0 s}}{\tau s + 1}$

Controlador proporcional (P): $G_c(s) = K_c$

Integral del error		ICE	IAE	IAET
$K_c = \frac{a}{K}\left(\frac{t_0}{\tau}\right)^b$	$a =$	1.411	0.902	0.490
	$b =$	−0.917	−0.985	−1.084

Controlador proporcional-integral (PI)

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

Integral de error		ICE	IAE	IAET
$K_c = \frac{a_1}{K}\left(\frac{t_0}{\tau}\right)^{b_1}$	$a_1 =$	1.305	0.984	0.859
	$b_1 =$	−0.959	−0.986	−0.977
$\tau_I = \frac{\tau}{a_2}\left(\frac{t_0}{\tau}\right)^{b_2}$	$a_2 =$	0.492	0.608	0.674
	$b_2 =$	0.739	0.707	0.680

Controlador proporcional-integral-derivativo (PID):

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Integral de error		ICE	IAE	IAET
$K_c = \frac{a_1}{K}\left(\frac{t_0}{\tau}\right)^{b_1}$	$a_1 =$	1.495	1.435	1.357
	$b_1 =$	−0.945	−0.921	−0.947
$\tau_I = \frac{\tau}{a_2}\left(\frac{t_0}{\tau}\right)^{b_2}$	$a_2 =$	1.101	0.878	0.842
	$b_2 =$	0.771	0.749	0.738
$\tau_D = a_3\tau\left(\frac{t_0}{\tau}\right)^{b_3}$	$a_3 =$	0.560	0.482	0.381
	$b_3 =$	1.006	1.137	0.995

Ziegler - Nichols	Ziegler-Nichols	IAE	Dahlin
$K_c = \frac{1,2}{K}\left(\frac{\tau}{\tau_d}\right)$	$K_c = \frac{K_{cu}}{1,7}$	$K_c = \frac{a_1}{K}\left(\frac{\tau_d}{\tau}\right)^{b_1}$ $a_1 = 1,435$ $b_1 = -0,921$	$K_c = \frac{\tau}{K(\tau_d + \tau_c)}$ $\tau_c = \frac{1}{5}\tau_d$
$R = 2\tau_d$	$R = \frac{\tau_u}{2}$	$R = \frac{\tau}{a_2}\left(\frac{\tau_d}{\tau}\right)^{b_2}$ $a_2 = 0,878$ $b_2 = 0,749$	$R = \tau$
$D = \frac{\tau_d}{2}$	$D = \frac{\tau_u}{8}$	$D = a_3\tau\left(\frac{\tau_d}{\tau}\right)^{b_3}$ $a_3 = 0,482$ $b_3 = 1,137$	$D = \frac{\tau_d}{2}$

Lugar de raíces

$$G(s)=\frac{Gc.Gp}{1+Gc.Gp}$$

Función de transferencia de lazo cerrado

Si: $Gc(s)=Kc$ $\quad Gp(s)=\frac{1}{s(s+1)}$

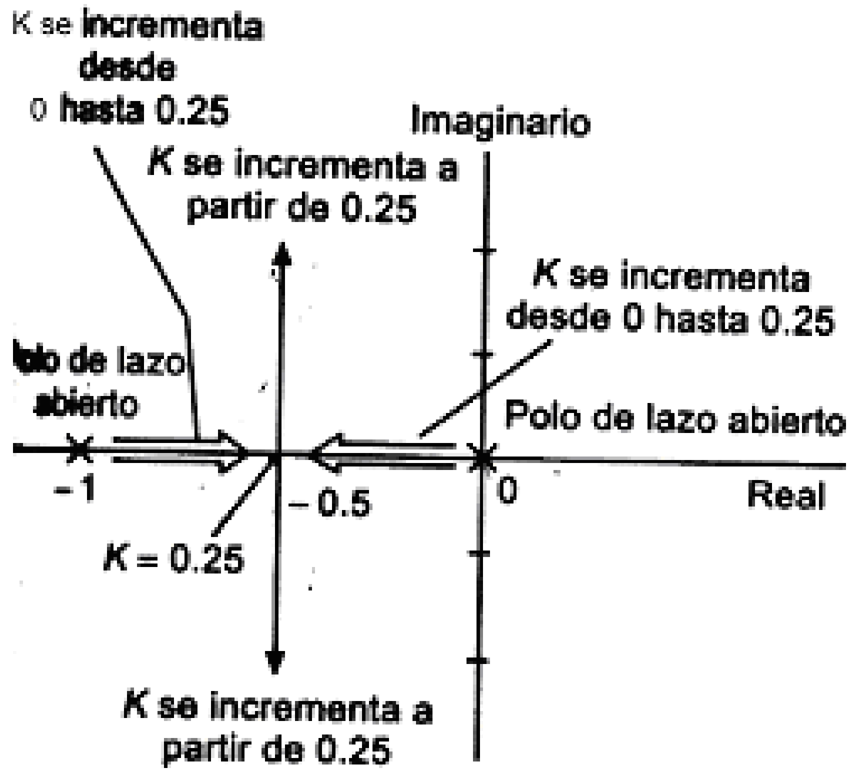
$$G(s)=\frac{Kc/[s(s+1)]}{1+Kc/[s(s+1)]}$$

$$G(s)=\frac{Kc}{s^2+s+Kc}$$

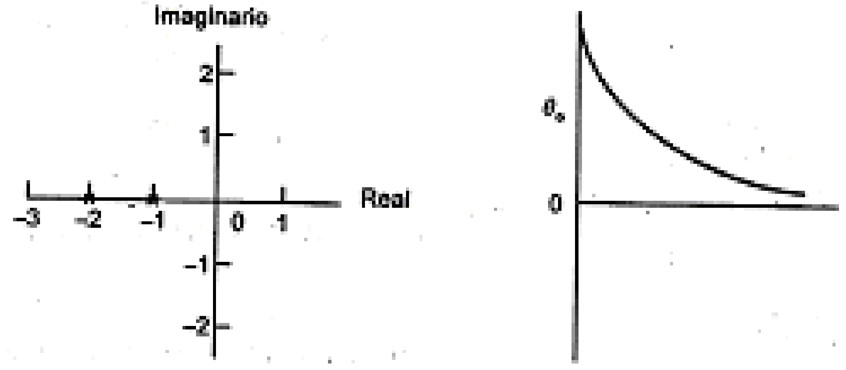
Las raíces del polinomio del denominador de la función de transferencia son:

$$p=\frac{-1\pm\sqrt{1-4Kc}}{2}$$

$$p=-\frac{1}{2}\pm\frac{1}{2}\sqrt{1-4Kc}$$

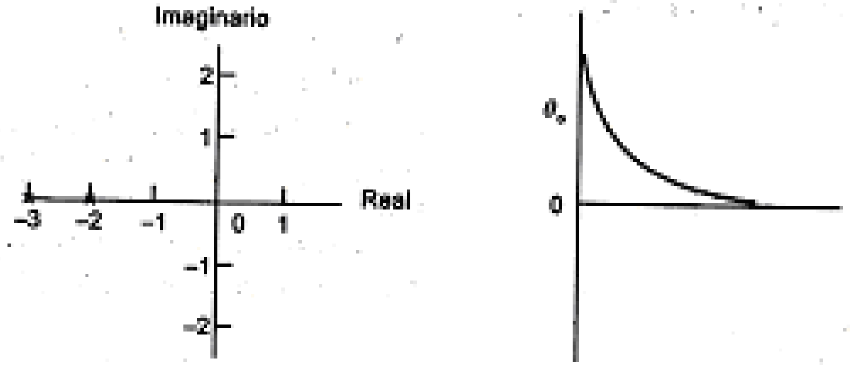


Raíz= -1



a)

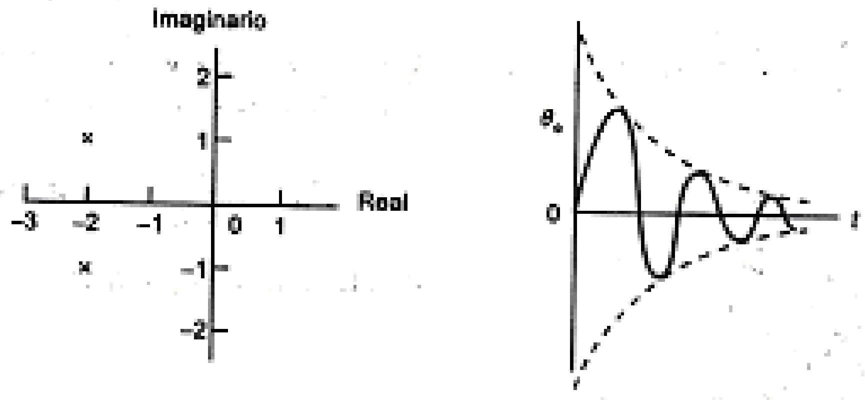
Raíz= -2

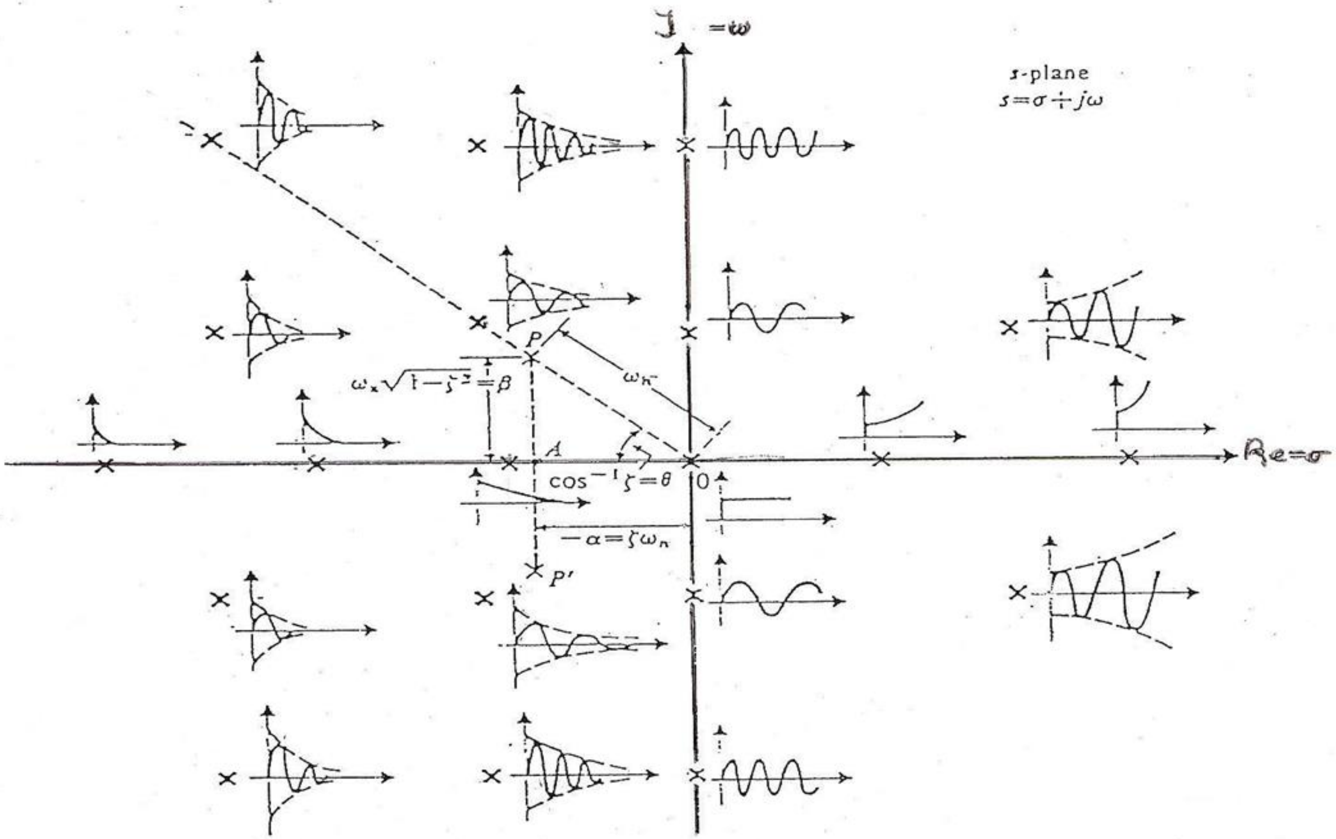


b)

Raíces= -2+1j

-2-1j





Significance of root location in the s-plane.

$$G(s)=\frac{K\,\Pi(s-z_j)\;\Pi\left[(s+\alpha_j)^2+\beta_j^{\,2}\right]}{\Pi(s-p_i)\;\Pi\left[(s+\alpha_i)^2+\beta_i^{\,2}\right]}=\frac{K\,\Pi(1+\tau_j s)\;\Pi\left[1+2\xi_j\tau_j s+(\tau_j s)^2\right]}{\Pi(1+\tau_i s)\;\Pi\left[1+2\xi_i\tau_i s+(\tau_i s)^2\right]}$$