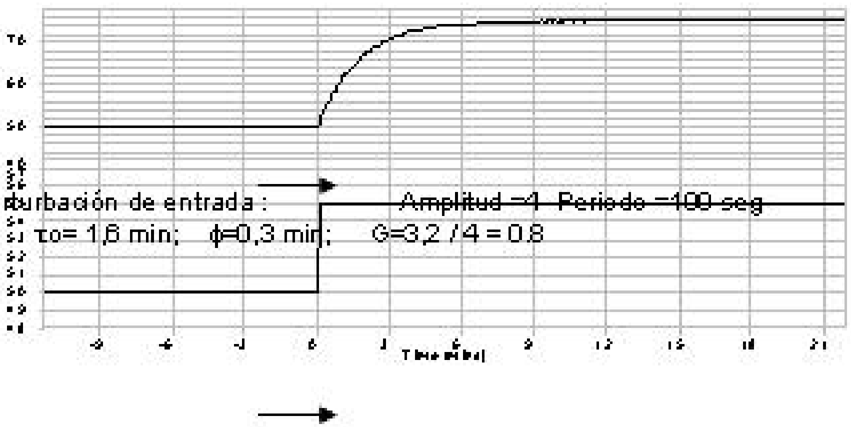
UNIDAD 3

ANALISIS EN FRECUENCIA

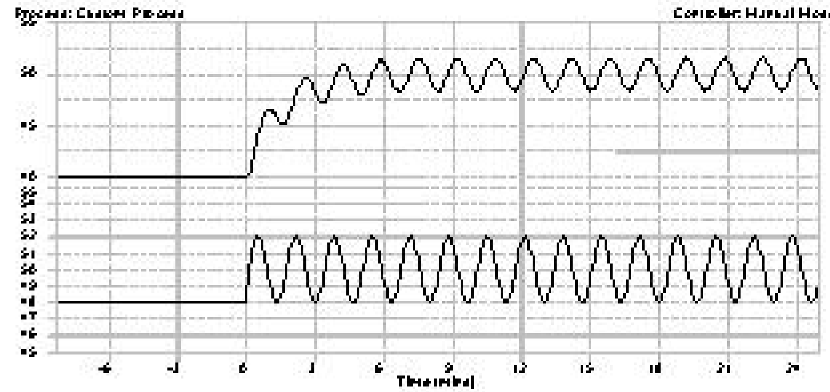
Primer orden

$\tau 1 = 100$ seg   K=5

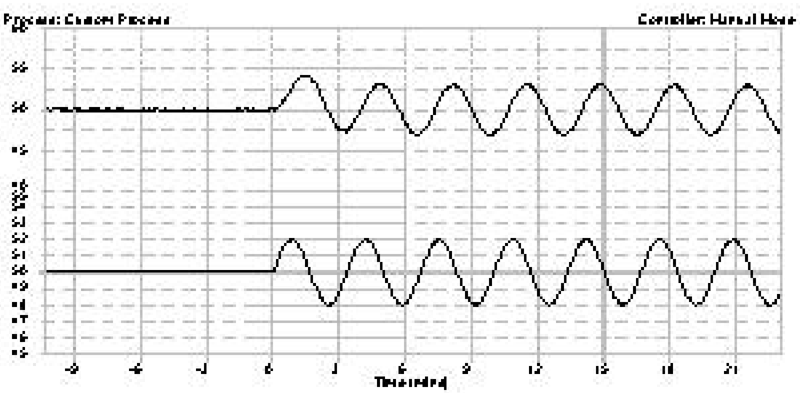
Perturbación de entrada : Amplitud =4 Periodo =100 seg
τo= 1,6 min; ϕ=0,3 min; G=3,2 / 4 = 0.8



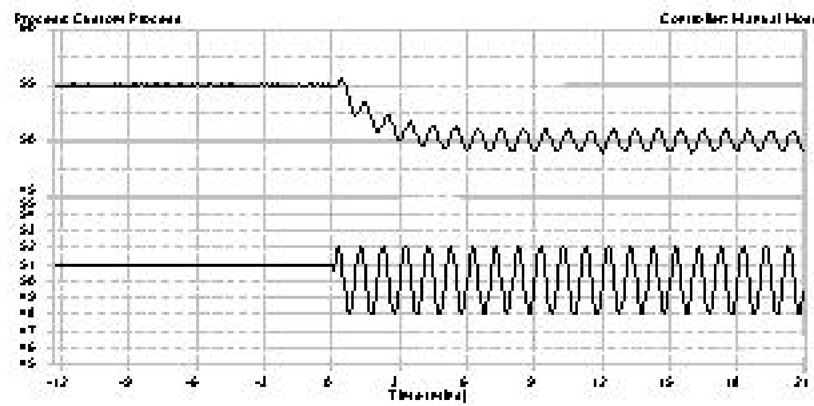
Perturbación de entrada : Amplitud =4 Periodo =100 seg
τo= 1,6 min; ϕ=0,3 min; G=3,2 / 4 = 0.8



Perturbación de entrada : Amplitud =4 Periodo =200 seg
τo= 3,2min; ϕ=0,6 min; G=4 / 4 = 1

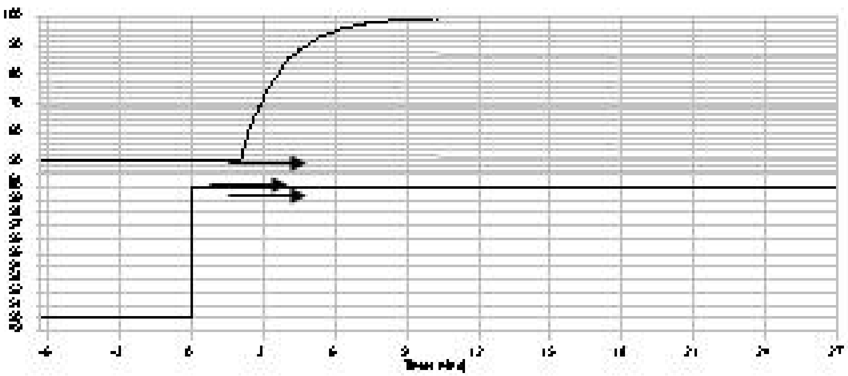


Perturbación de entrada : Amplitud =4 Periodo =60 seg
τo= 1 min; ϕ=0,2min; G=1,1 / 4 = 0.275



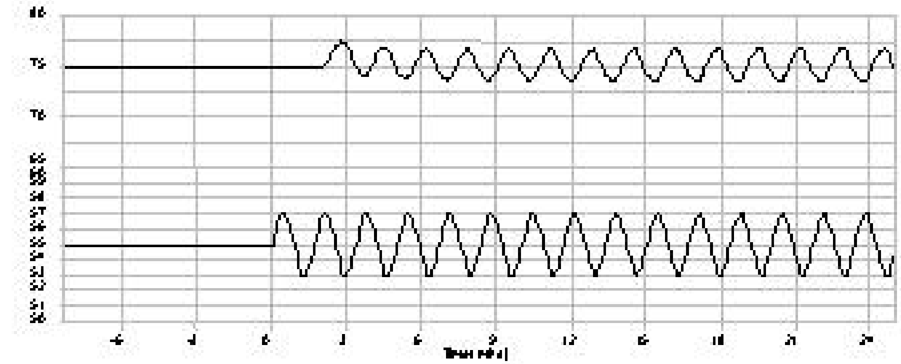
Primer orden + tiempo muerto

$\tau d = 120$ seg $\quad \tau 1 = 100$ seg $\quad$ K=5



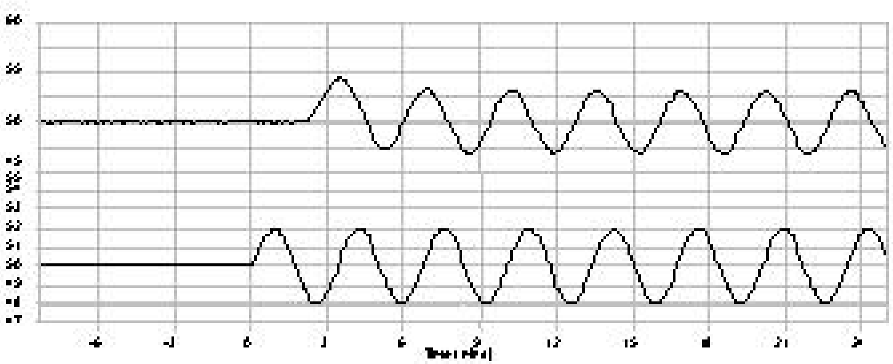
Perturbación de entrada : Amplitud =4 Periodo =100 seg

τo= 1,7 min; ϕ=2,1min G=3,5 / 4 = 0.875



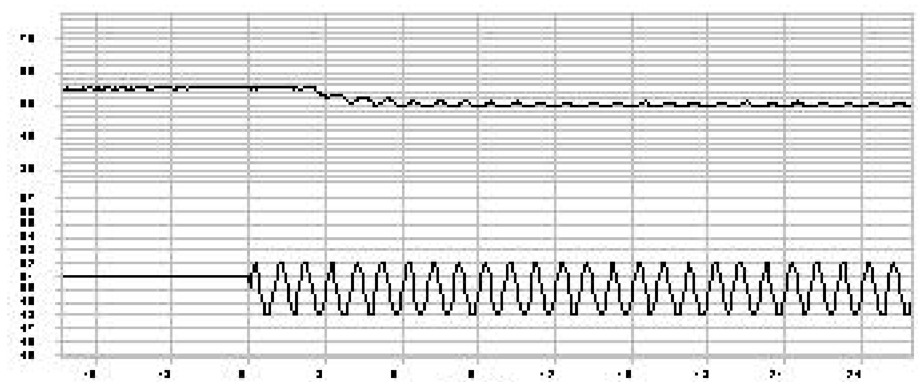
Perturbación de entrada: Amplitud: 4 Período=200 seg

τo= 3,4 min; ϕ=2,2min G=6,1 / 4 = 1,5



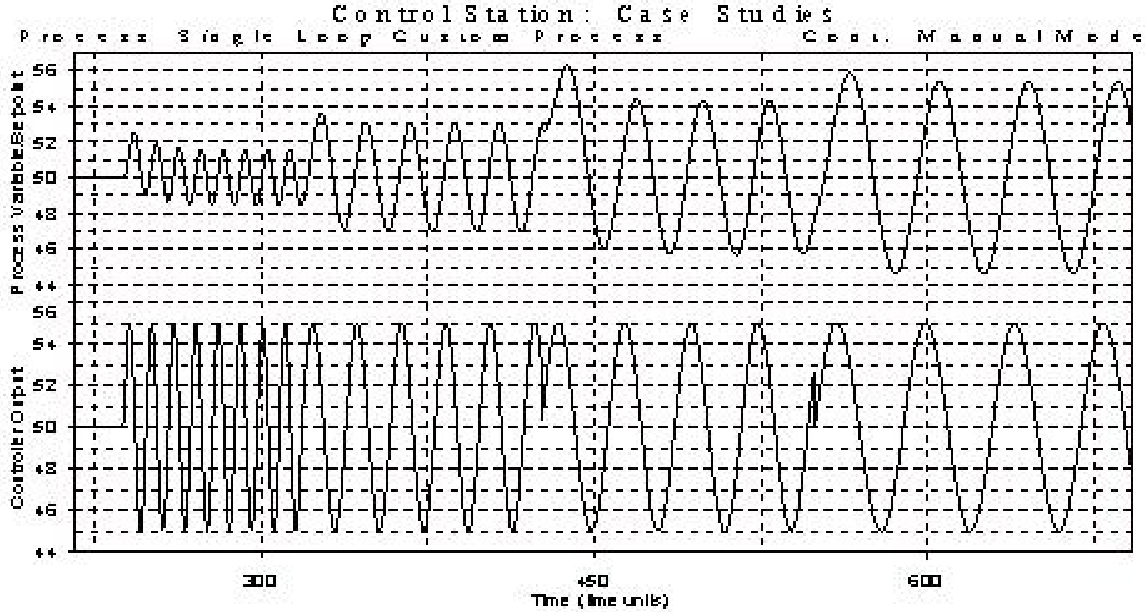
Perturbación de entrada : Amplitud =4 Periodo =60 seg

τo= 1 min; ϕ=2,2min ; G=1,9 / 4 = 0.475



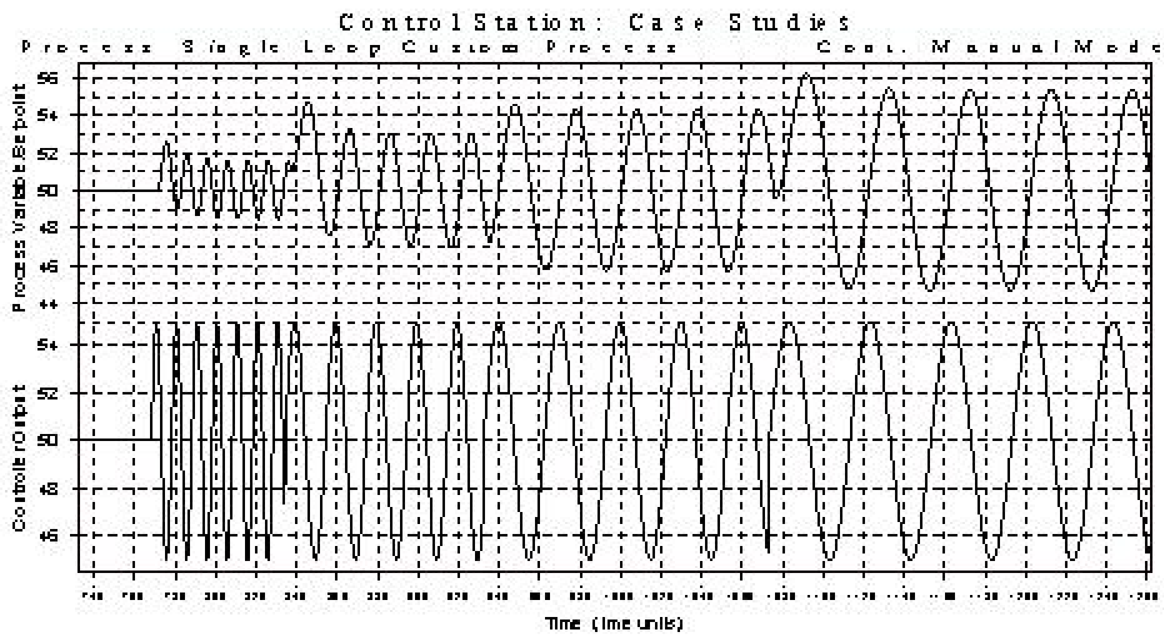
Proceso capacitivo de pimer orden:

$$G(s) = \frac{k}{\tau.s+1}$$



Proceso capacitivo de pimer orden + tiempo muerto:

$$G(s) = \frac{k.e^{-to.s}}{\tau.s+1}$$



Análisis en frecuencia

En capítulos anteriores se consideró la salida del sistema sujetos a una entrada impulso, escalón o rampa. Ahora ampliaremos al caso de una entrada seniodal. Ya que la forma en que ese sistema responde a una función senoidal es una fuente de información muy útil para el análisis y diseño de lazos de control.

El término *respuesta en frecuencia* se define como la respuesta en estado estable de un sistema a una entrada senoidal; la que se monitorea sobre un intervalo de frecuencias. La respuesta en estado estable es la que permanece después de que todos los transitorios han decaído a cero.

Existen varias técnicas para analizar los datos de la respuesta en frecuencia. Estudiaremos dos de ellas: la de Bode y la de Nyquist.

Si a un sistema lineal se aplica una entrada senoidal, la salida es también una senoidal y de la misma frecuencia. La salida puede diferir de la entrada en amplitud y fase. El cociente entre la amplitud de la salida y la amplitud de la entrada se conoce como **ganancia** (también llamada ***magnitud* o *razón de amplitud***). El corrimiento de fase de la senoidal de salida en relación con la fase de la senoidal de entrada se denomina ***fase*.** La variación de la magnitud y la fase con la frecuencia se denomina *respuesta en frecuencia* del sistema.

$$G(s)=\frac{Y(s)}{X(s)}=\frac{K}{\tau.s+1}$$

$$x(t)=X_0.sen\omega t$$

$$X(s)=\frac{X_0\omega}{s^2+\omega^2}$$

$$Y(s)=\frac{K.X_0\omega}{(\tau.s+1).(s^2+\omega^2)} \longrightarrow$$ Aplicando transformada de Laplace

$$\downarrow$$

$$y(t)=\frac{K.X_0\omega.\tau}{(1+\omega^2\tau^2)}.e^{-t/\tau}+\frac{K.X_0}{(1+\omega^2\tau^2)}\left[-\omega\tau.\cos\omega t+sen\omega t\right]$$

Teniendo en cuenta que: A cos at + B sen at = r sen (at + θ) $\longrightarrow$ $r=\sqrt{A^2+B^2}$ $\quad \theta=\tan^{-1}\left(\frac{A}{B}\right)$

$$y(t)=\frac{K.X_0\omega.\tau}{(1+\omega^2\tau^2)}.e^{-t/\tau}+\frac{K.X_0}{(1+\omega^2\tau^2)}.sen(\omega t+\theta) \quad \left\{ \theta=\tan^{-1}(-\omega t) \quad =-\tan^{-1}(\omega t) \right.$$

Pasado el transitorio, para $t \rightarrow \propto$ $$y(t) = \frac{K.X_0}{(1+\omega^2\tau^2)}.sen(\omega t+\theta)$$

La amplitud de la señal de salida es: $$Y_0 = \frac{K.X_0}{\sqrt{1+\omega^2\tau^2}}$$

La razón de amplitud es: $$\frac{Y_0}{X_0} = \frac{K}{\sqrt{1+\omega^2\tau^2}}$$

El ángulo de fase es: $$\theta = -\tan^{-1}(\omega.t)$$

Por lo tanto:

Generalizando:

$$\boxed{G(s)=\frac{Y(s)}{X(s)}} \qquad X(s)=\frac{X_0\omega}{s^2+\omega^2} \longrightarrow Y(s)=G(s).\frac{X_0\omega}{(s^2+\omega^2)}$$

$$Y(s)=\frac{A}{(s+j\omega)}+\frac{B}{(s-j\omega)}$$ + (Términos para los polos de G(s))

$$A=\lim_{s\to -j\omega}\left[\frac{(s+j\omega)X_0\omega G(s)}{(s^2+\omega^2)}\right]=\frac{G(-j\omega)X_0\omega}{-2j\omega}$$

$$B=\lim_{s\to j\omega}\left[\frac{(s-j\omega)X_0\omega G(s)}{(s^2+\omega^2)}\right]=\frac{G(j\omega)X_0\omega}{2j\omega}$$

$r=a+jb$

$a=|r|*\cos\phi$

$b=|r|*sen\phi$

$r=|r|(\cos\phi+jsen\phi)$

$r=|r|*e^{j\phi}$

$$Y(s)=\frac{X(0)\left|G(j\omega)\right|}{2j}\left[\frac{-e^{-j\phi}}{s+j\omega}+\frac{e^{j\phi}}{s-j\omega}\right]$$ + (Términos de G(s))

$$Y(s)=\frac{X(0)\left|G(j\omega)\right|}{2j}\left[\frac{-e^{-j\phi}}{s+j\omega}+\frac{e^{j\phi}}{s-j\omega}\right]$$ + Términos de G(s)

Al antitransformar, podemos encontrar y(t)

$$y(t)=\frac{X(0)\left|G(j\omega)\right|}{2j}\left[-e^{-j\phi}.e^{-j\omega.t}+e^{j\phi}.e^{j\omega.t}\right]$$ + Términos transitorios

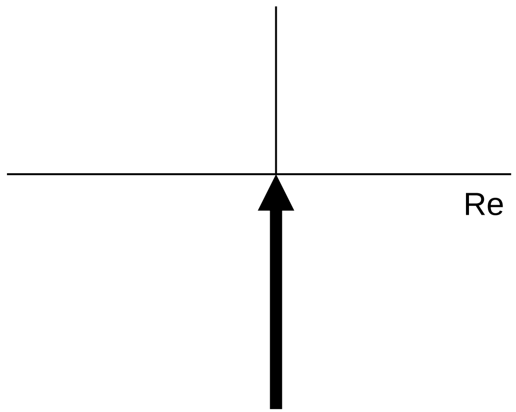
$$y(t)=X(0)\left|G(j\omega)\right|.\left[\frac{e^{j(\omega.t+\phi)}-e^{-j(\omega.t+\phi)}}{2j}\right]$$ + Términos transitorios

$$\boxed{sen(\omega t)=\left[\frac{e^{j\omega t}-e^{-j\omega t}}{2j}\right]}$$

Pasado el transitorio:

$$y(t)=X_0\left|G(j\omega)\right|.\left[sen(\omega.t+\phi)\right]$$

Diagrama de Nyquist- 1ª orden

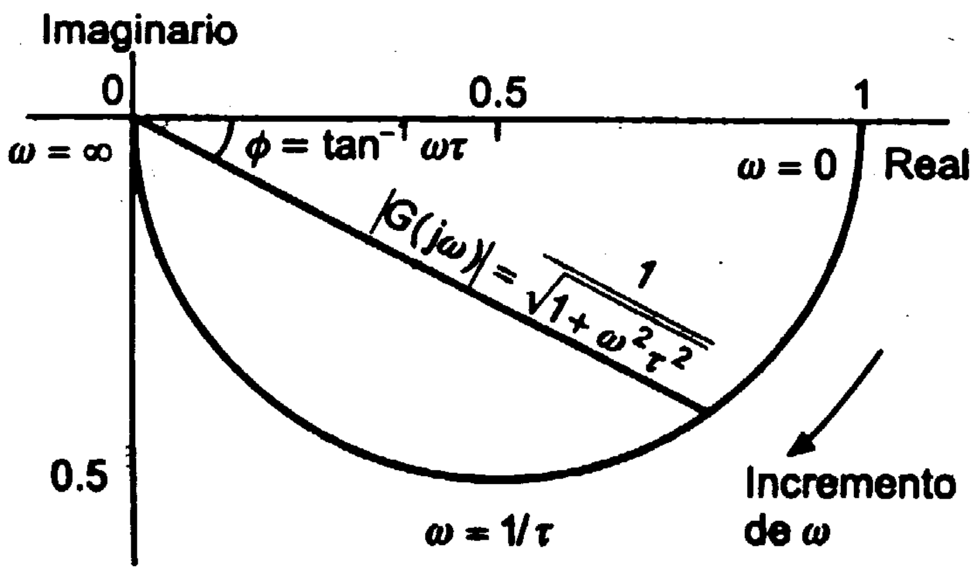


Capacitancia no autorregulada

$$\boxed{G(j\omega) = \frac{1}{j\omega\tau} = \frac{(j\omega\tau)}{\omega^2\tau^2} = -\frac{j}{\omega\tau}}$$

$$|G(j\omega)| = \sqrt{\mathrm{Re}^2 + \mathrm{Im}^2} = \frac{1}{\omega\tau}$$

$$\phi = -\pi/2$$



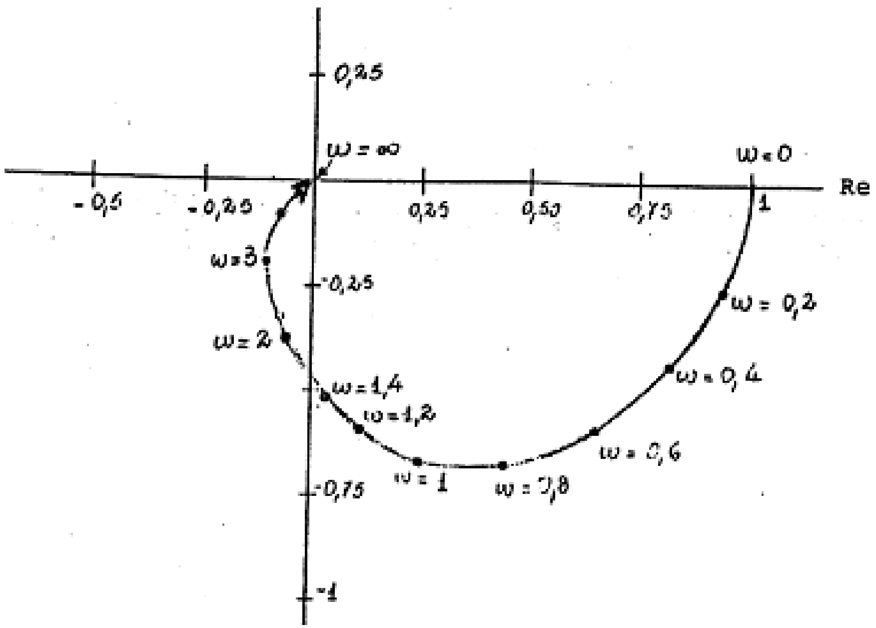
Capacitancia autorregulada

$$\boxed{G(j\omega) = \frac{K}{1 + j\omega\tau} = \frac{K(1 - j\omega\tau)}{1 + \omega^2\tau^2}}$$

$$|G(j\omega)| = \sqrt{\mathrm{Re}^2 + \mathrm{Im}^2} = \frac{K}{\sqrt{1 + (\omega\tau)^2}}$$

$$\phi = -\arctan(\omega\tau)$$

Diagrama de Nyquist- 2ª orden



$$\boxed{G(s) = \frac{K}{(1+\tau_1\ s)\,(1+\tau_2\ s)}}$$

$$G(j\omega) = \frac{K}{(1+\ j\omega\tau_1)\,(1+\ j\omega\tau_2)}$$

$$G(j\omega) = K\,\frac{(1-\ j\omega\tau_1)\,(1-\ j\omega\tau_2)}{\left[1+(\omega\tau_1)^2\right]\left[1+(\omega\tau_2)^2\right]}$$

$$G(j\omega) = K\,\frac{1-\omega^2\tau_1\tau_2 - j\ (\omega\tau_1 + \omega\tau_2)}{\left[1+(\omega\tau_1)^2\right]\left[1+(\omega\tau_2)^2\right]}$$

$$\mathrm{Re}\left[G(j\omega)\right] = K\,\frac{1-\omega^2\tau_1\tau_2}{\left[1+(\omega\tau_1)^2\right]\left[1+(\omega\tau_2)^2\right]}$$

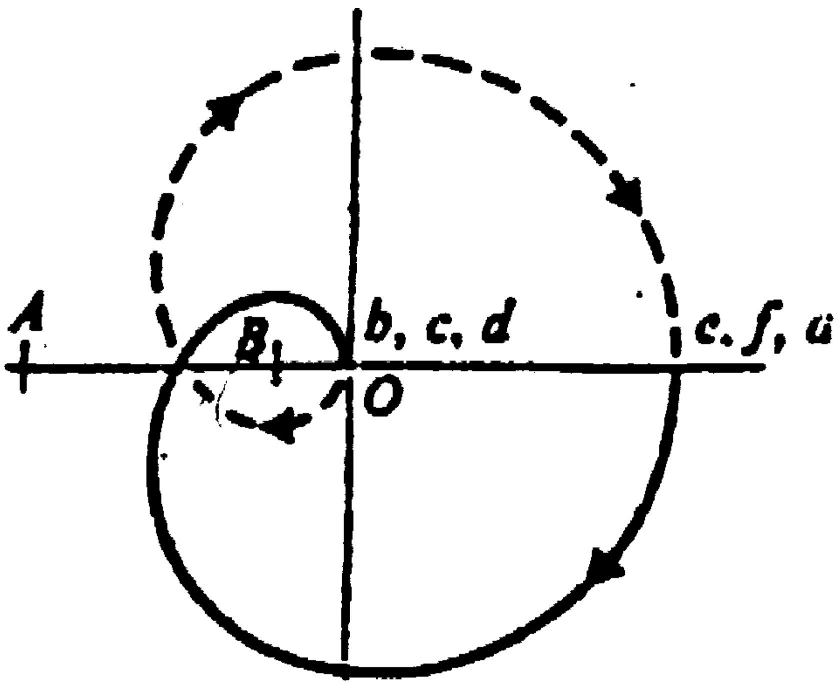
$$\mathrm{Im}\left[G(j\omega)\right] = -K\,\frac{(\omega\tau_1 + \omega\tau_2)}{\left[1+(\omega\tau_1)^2\right]\left[1+(\omega\tau_2)^2\right]}$$

Criterio de estabilidad de Nyquist

$$\boxed{N = Z - P}$$

P= cant de polos a lazo abierto con parte real positiva

Z=cant de polos a lazo cerrado con parte real positiva



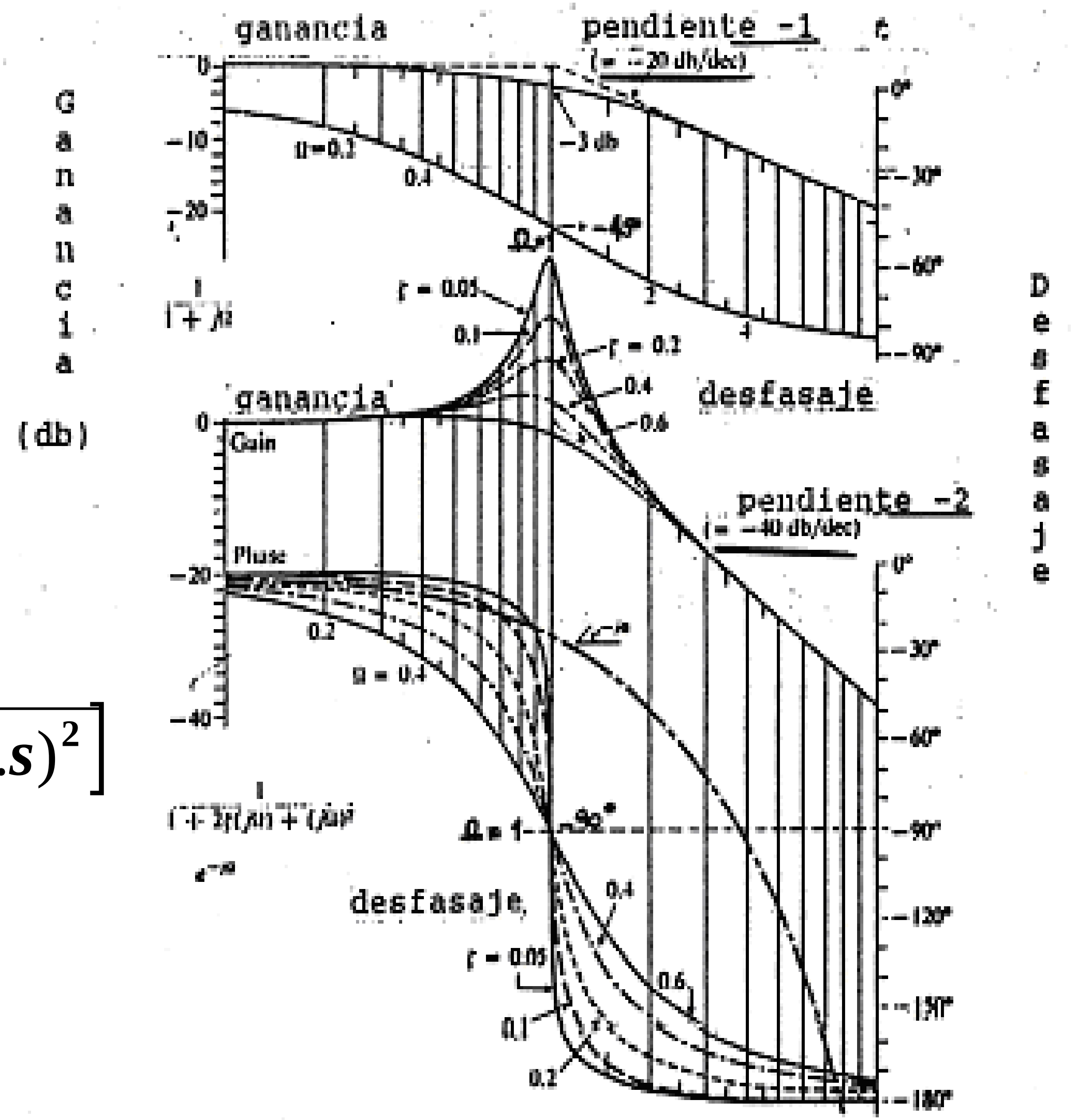
-1 en A $\Longrightarrow$ estable

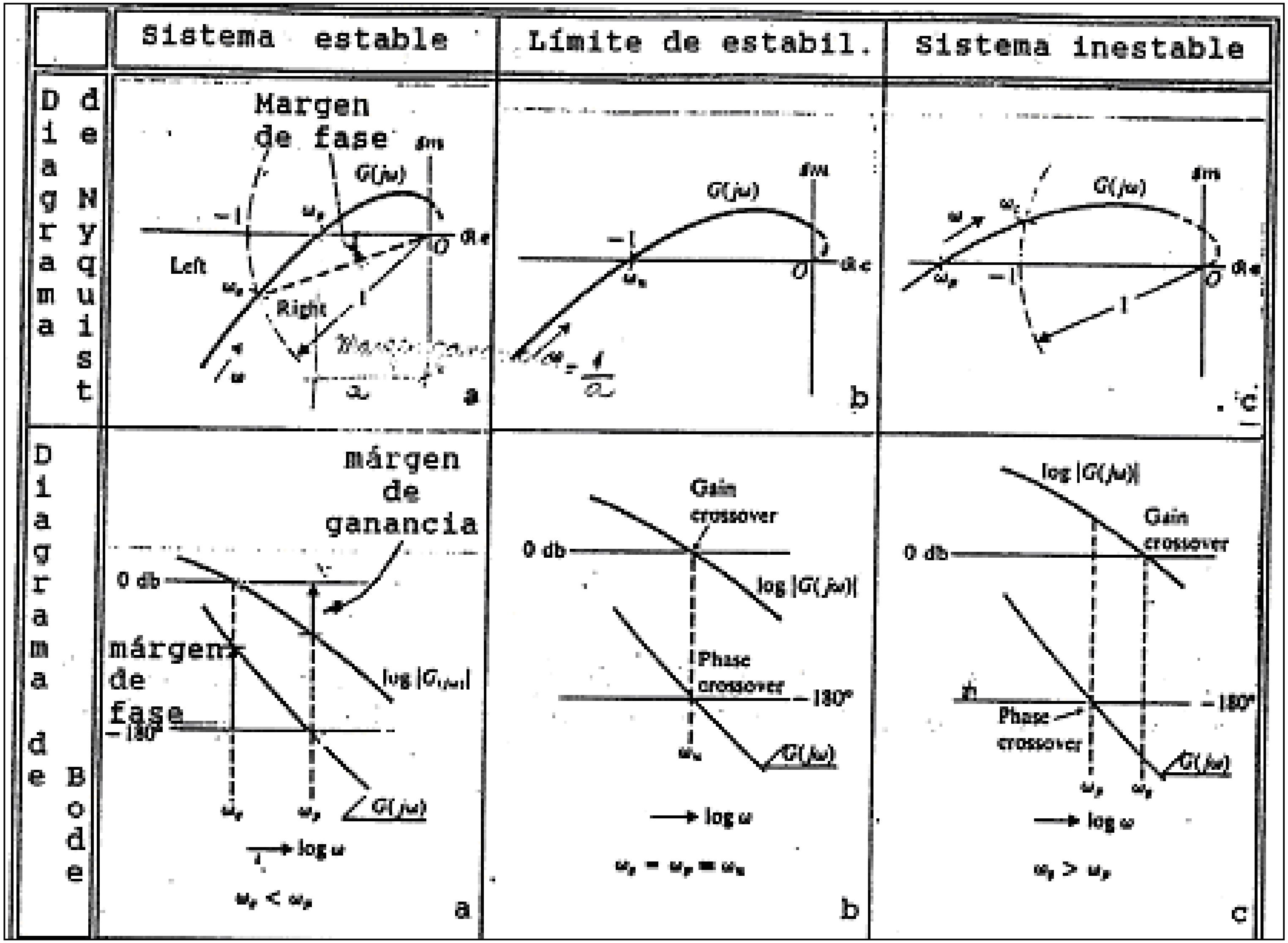
-1 en B $\Longrightarrow$ inestable

<u>**Diagrama de Bode**</u>

$$G(s) = \frac{K}{(\tau.s + 1)}$$

$$G(s) = \frac{K}{\left[1 + 2\zeta\tau.s + (\tau.s)^2\right]}$$



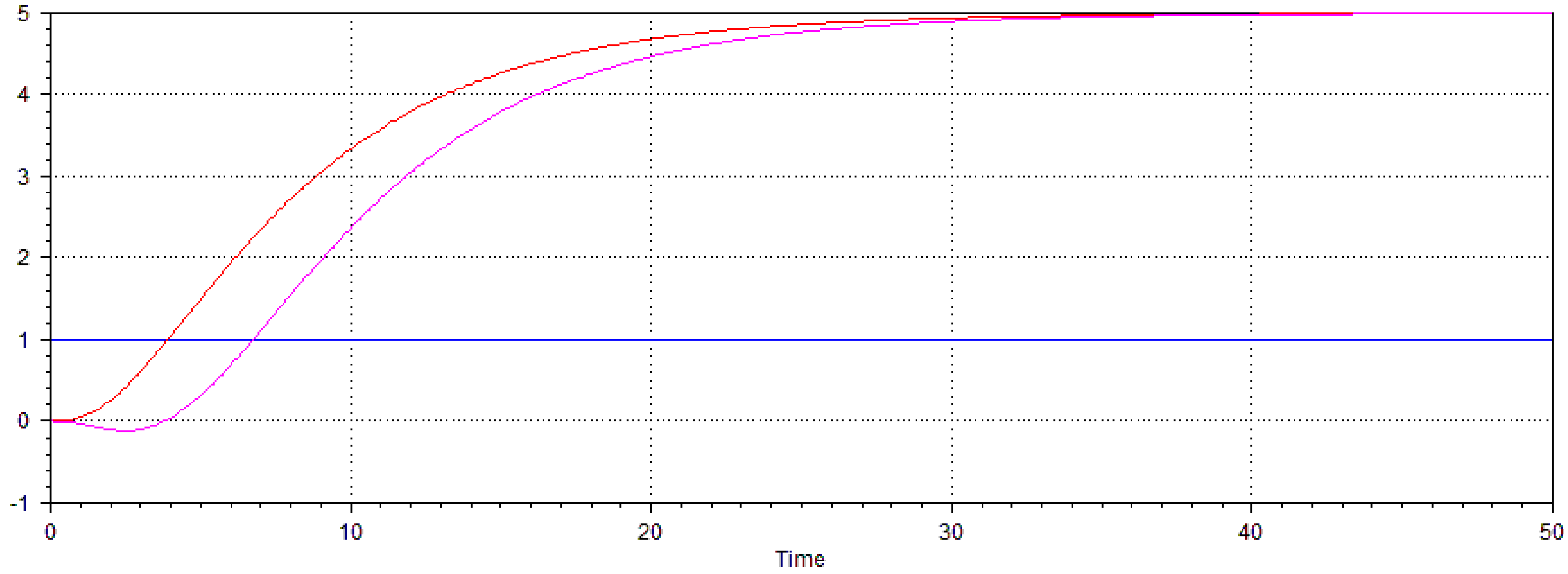


Sistema estable
Límite de estabil.
Sistema inestable
Diagrama de Nyquist
Margen de fase
G(jω)
Im
Re
-1
Left
Right
O
a
b
c
Diagrama de Bode
márgen de ganancia
0 db
log |G(jω)|
márgen de fase
-180°
G(jω)
log ω
Gain crossover
Phase crossover

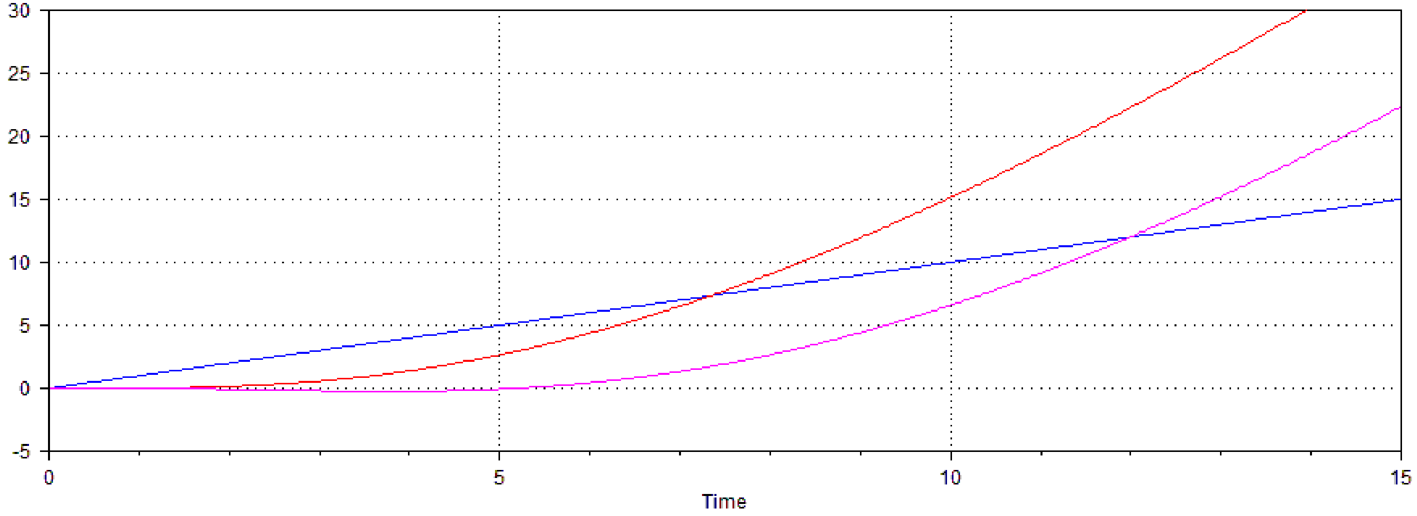
$$G(s) = \frac{5}{((6*s+1)*(2*s+1)*(s+1))}$$

CC>g=5/((6*s+1)*(2*s+1)*(s+1))
CC>g1=pade(3,1)

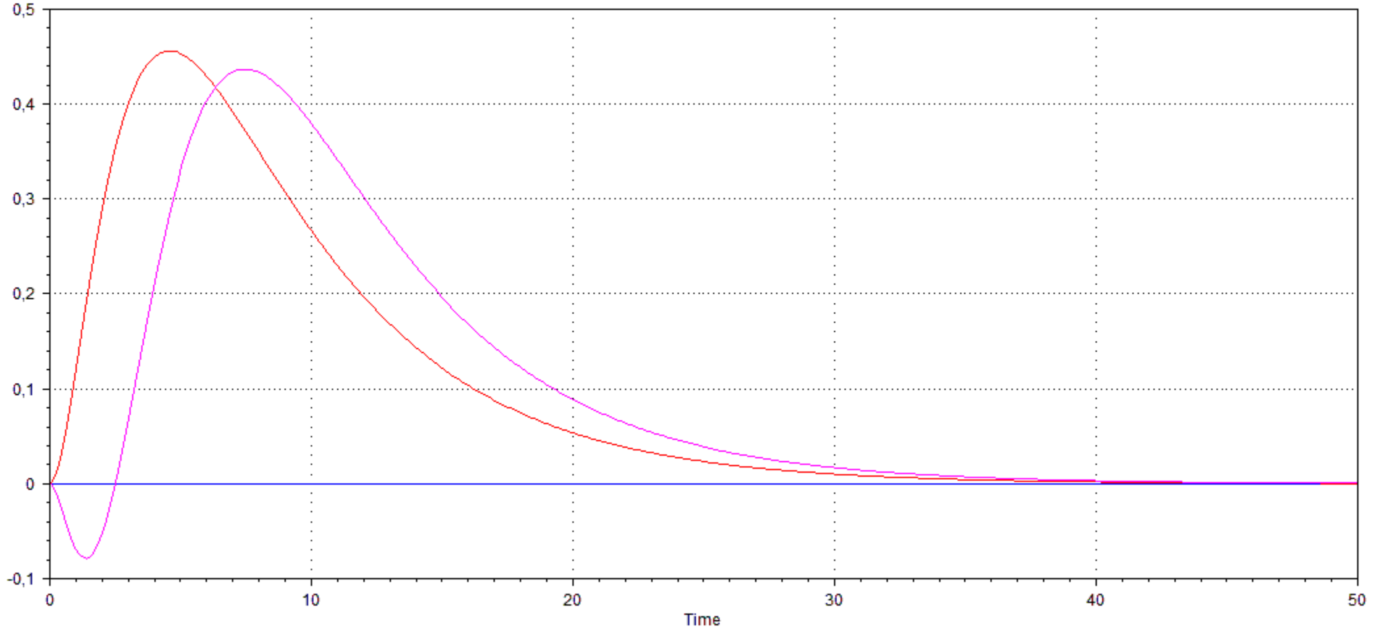
$$g1(s) = \frac{-s+0,6667}{s+0.6667}$$



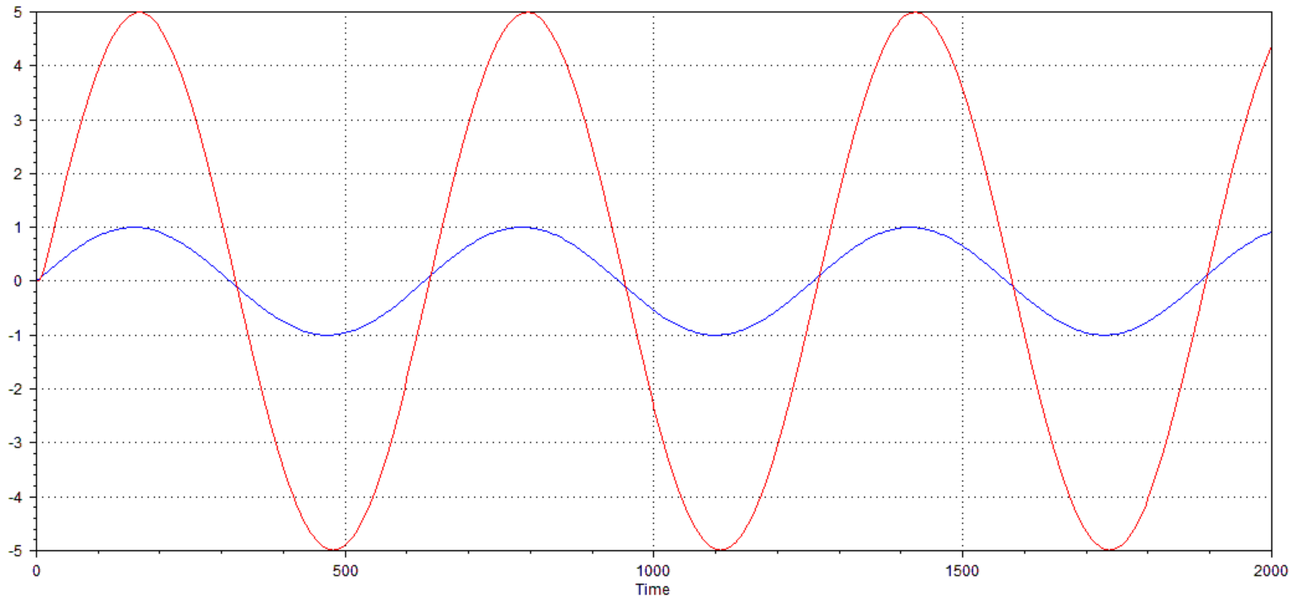
CC>time(1,g,g*g1)- Respuesta al escalón unitario



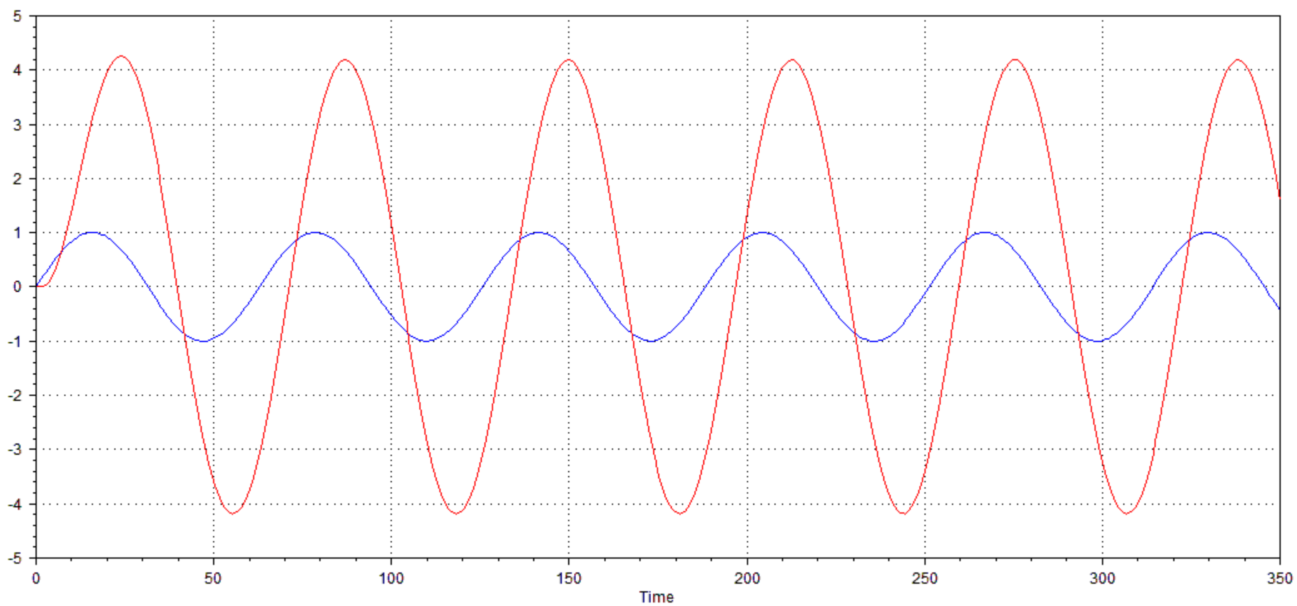
CC>time(1/s,g/s,g*g1/s)- Respuesta a rampa unitaria



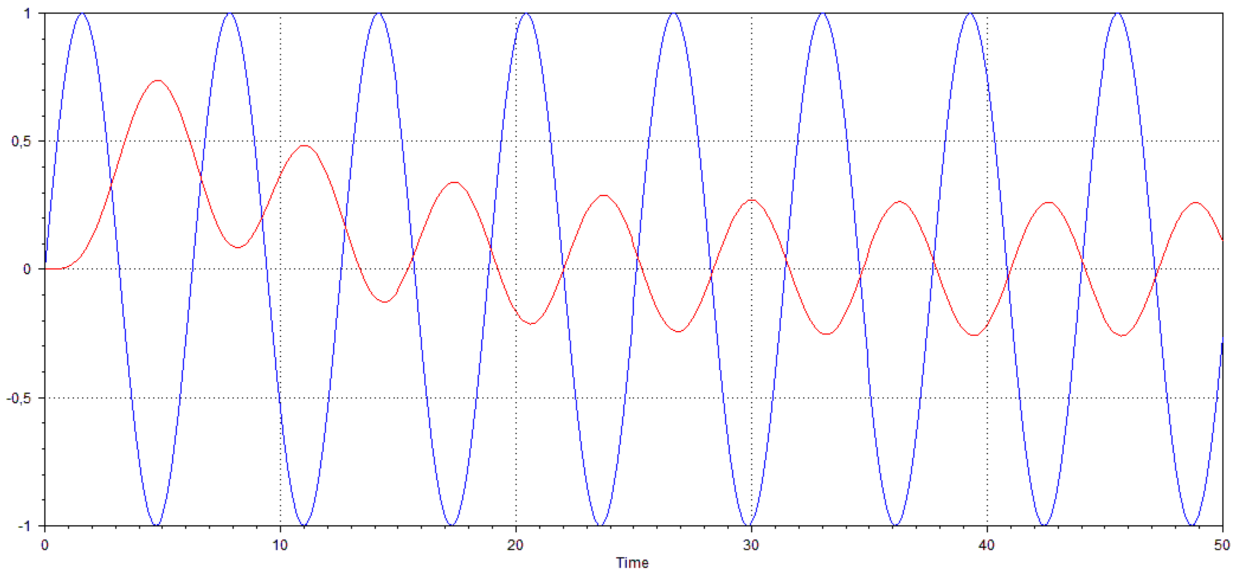
CC>time(1*s,g*s,g*g1*s)- Respuesta a impulso unitario



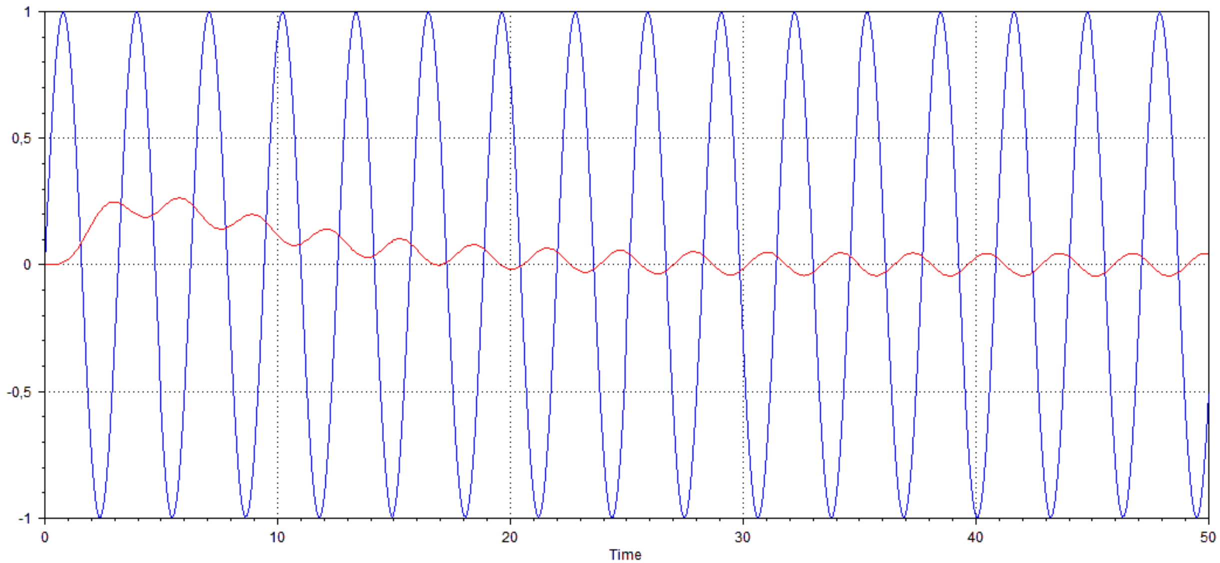
CC>w=0.01
CC>u=w/(s^2+w^2)
CC>time(u*s,g*u*s)



CC>w=0.1
CC>u=w/(s^2+w^2)
CC>time(u*s,g*u*s)

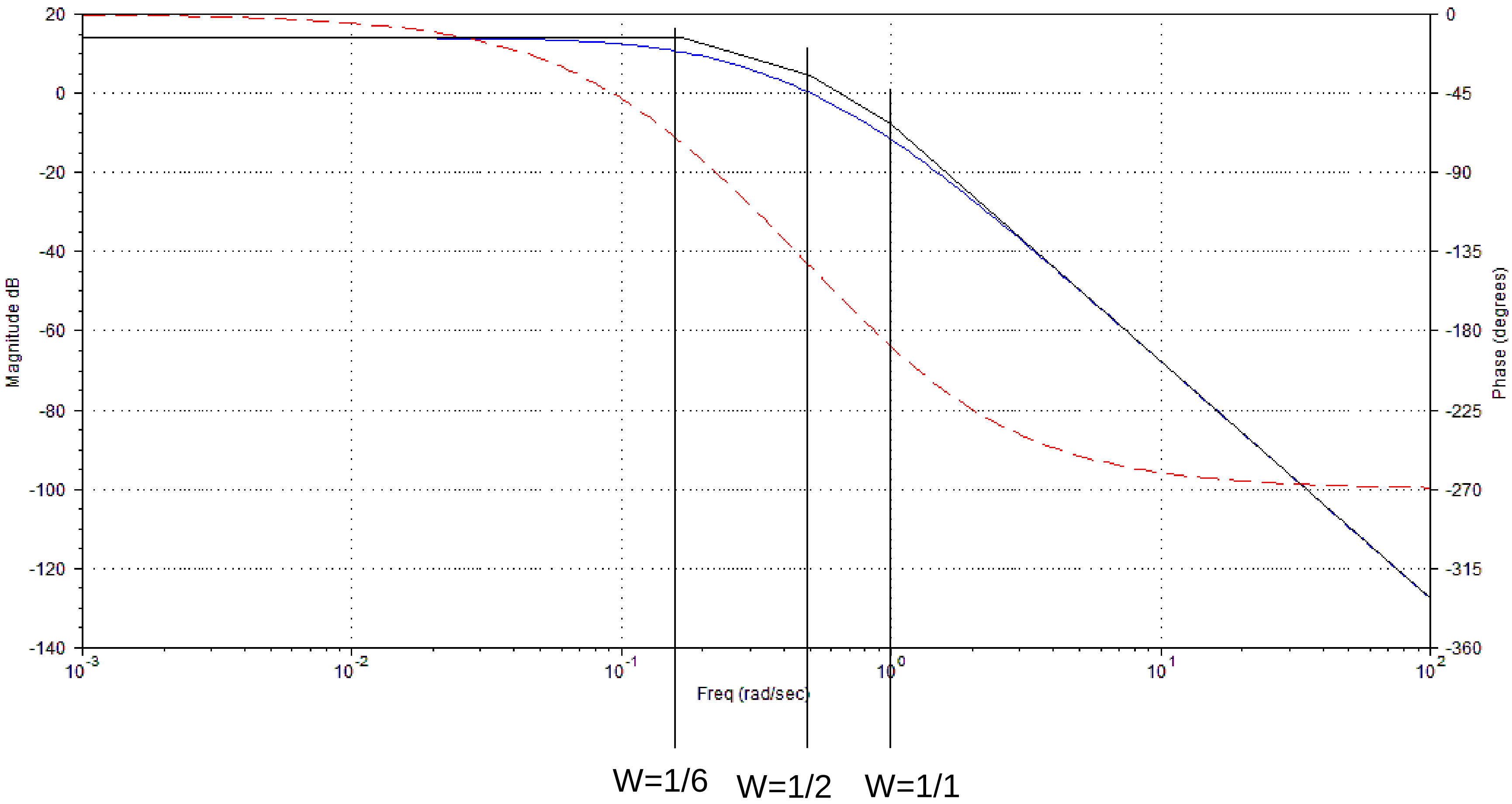


CC>w=1
CC>u=w/(s^2+w^2)
CC>time(u*s,g*u*s)

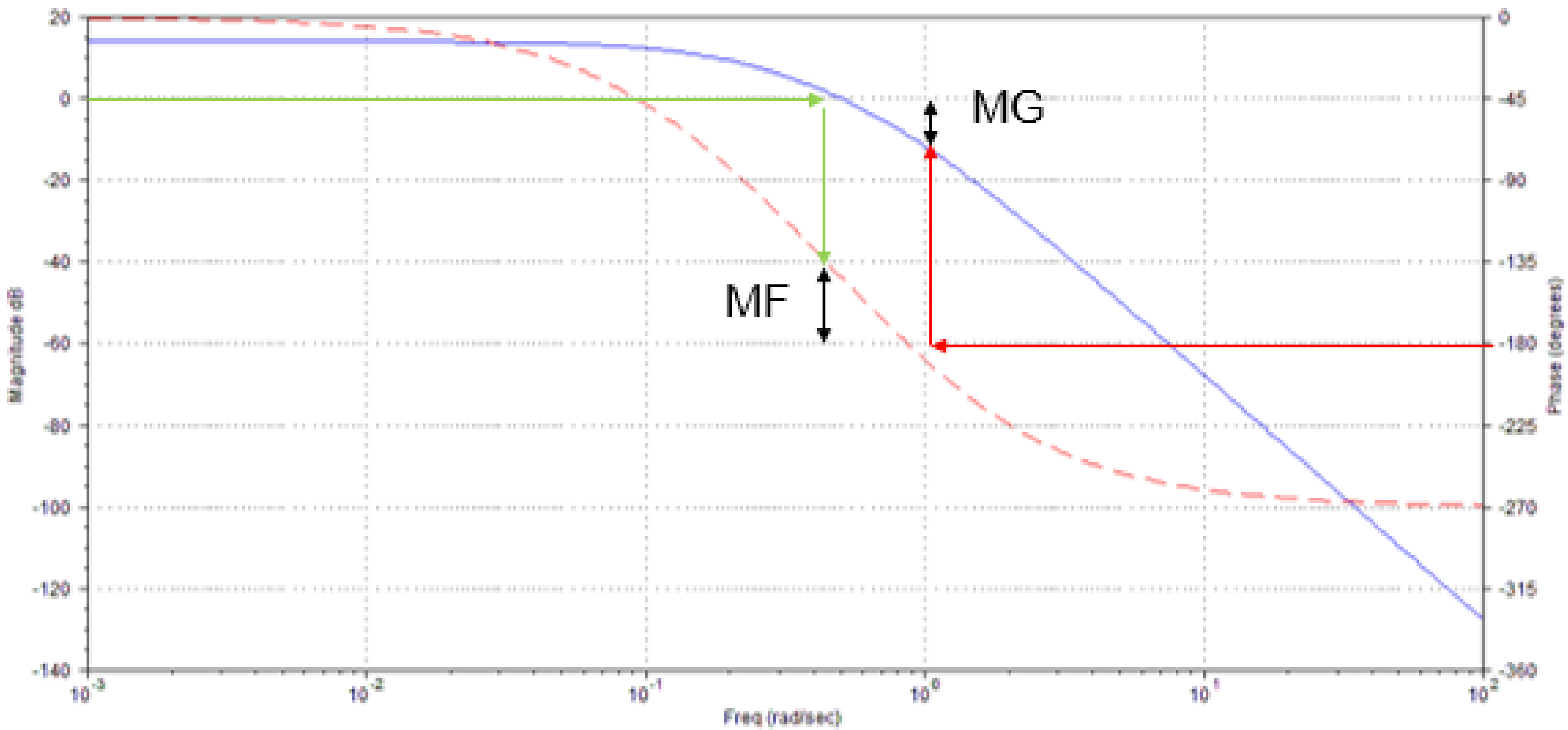


CC>w=2
CC>u=w/(s^2+w^2)
CC>time(u*s,g*u*s)

Diagrama de Bode



CC>g=5/((6*s+1)*(2*s+1)*(s+1))
CC>bode(g)



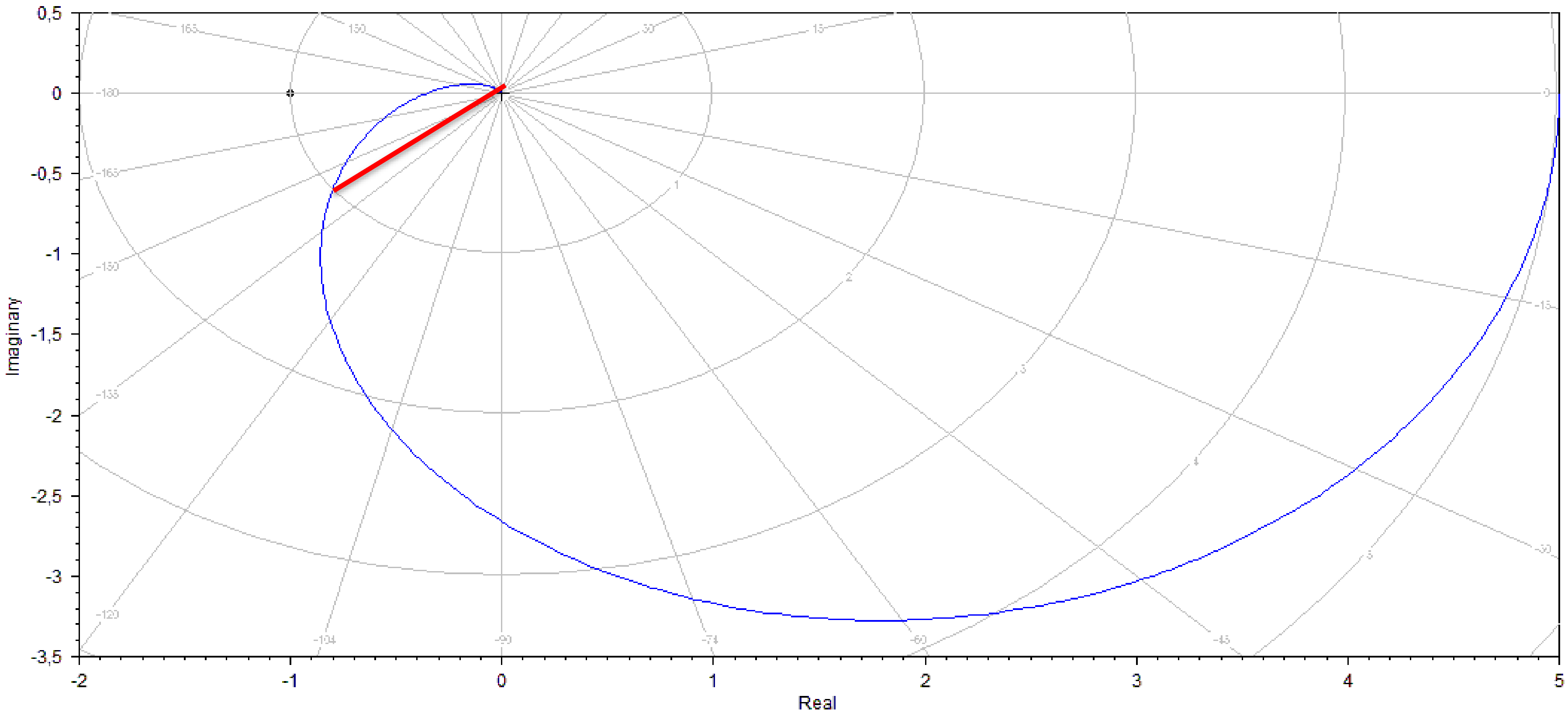
CC>margin(g)

At w= 0,5 r/s, Phase margin= 36,87 deg, Delay margin= 1,29 sec

At w= 0,562 r/s, Mp= 1,7 (4,60 dB)

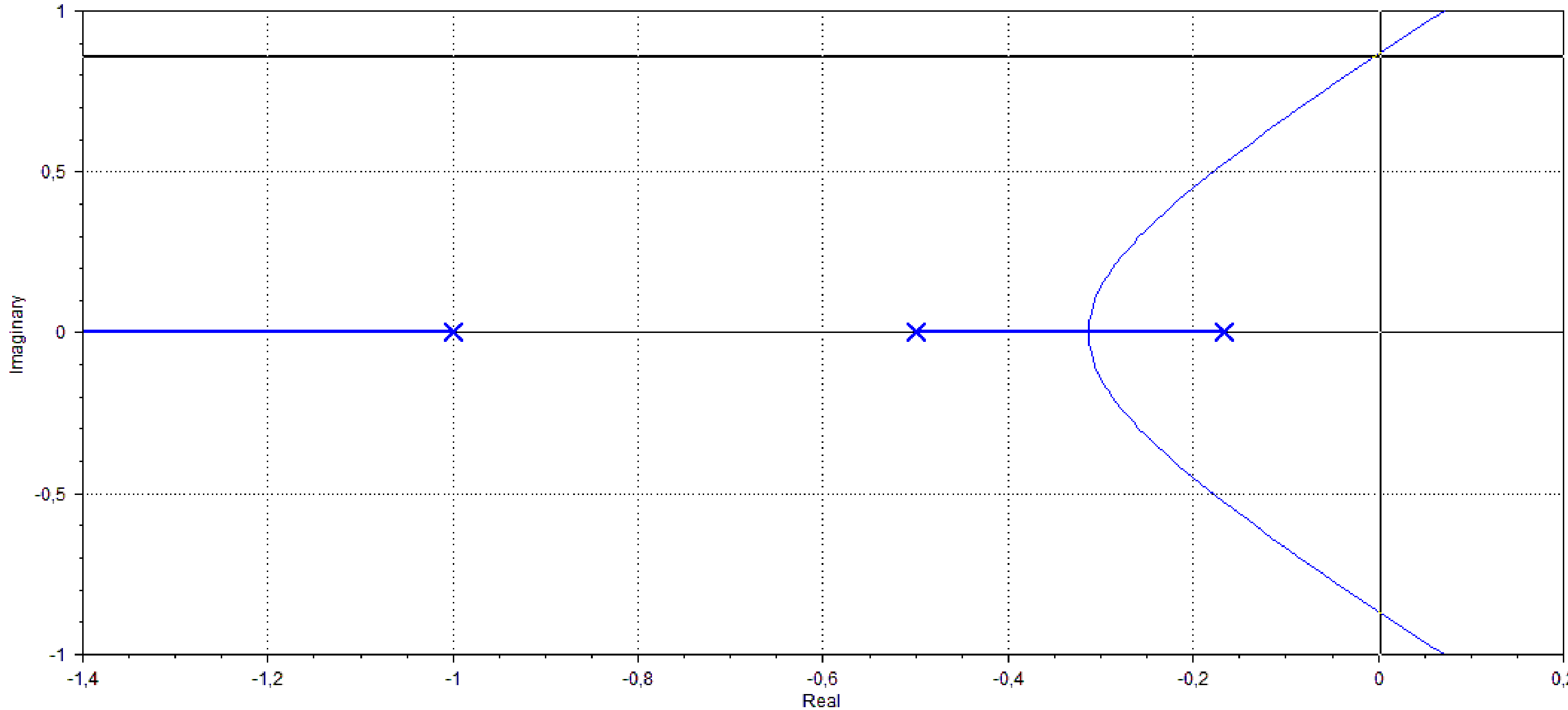
At w= 0,866 r/s, Gain margin= 2,8 (8,94 dB)

Diagrama de Nyquist



CC>g=5/((6*s+1)*(2*s+1)*(s+1))
CC>nyquist(g)

Lugar de Raices

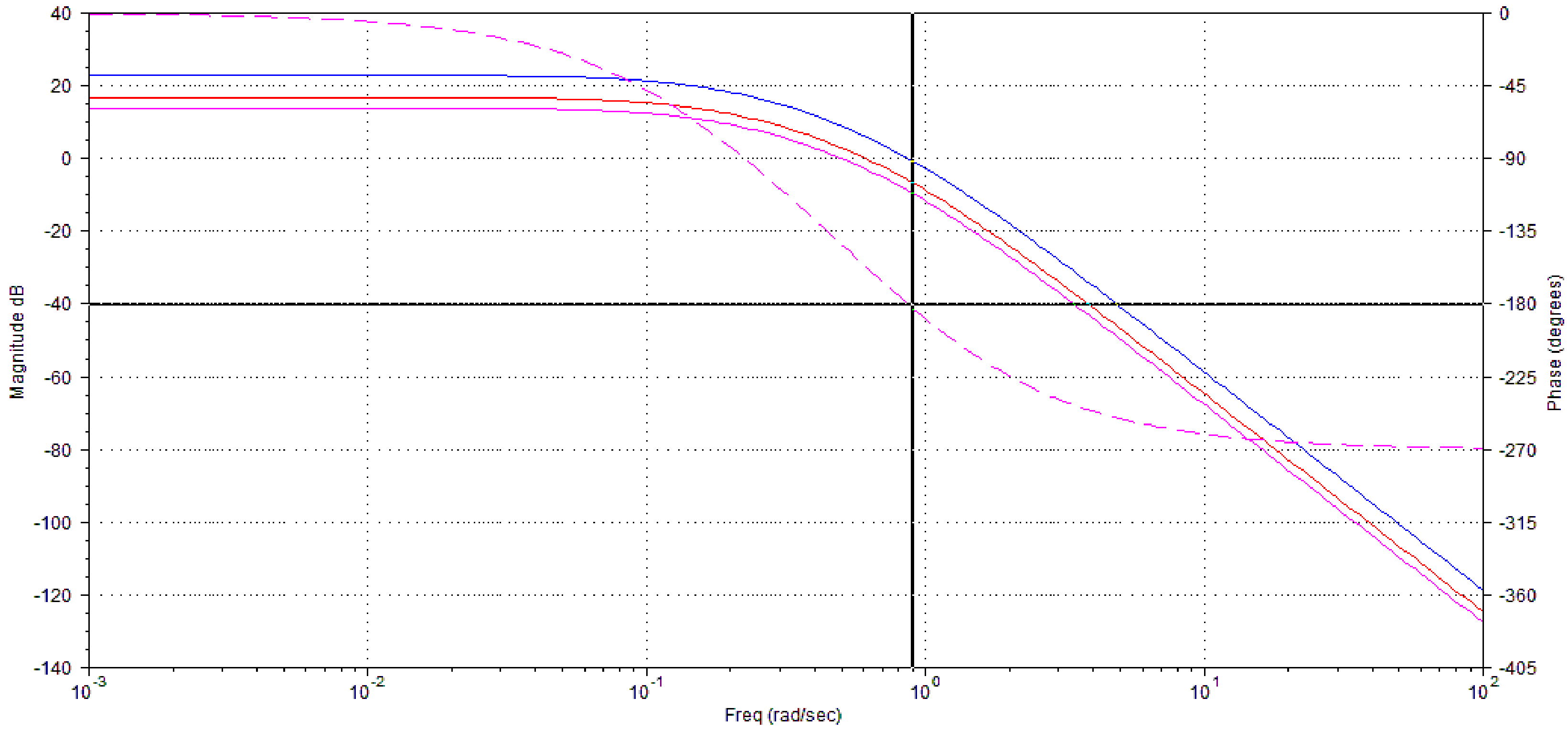


s= 0,002211 +0,8568j (Mag= 0,8568, Zeta= -0,002581)
gain= 2,744 -0,04782j (Mag= 2,745, Phase= -0,9984 deg)

CC>g=5/((6*s+1)*(2*s+1)*(s+1))
CC>rl(g)

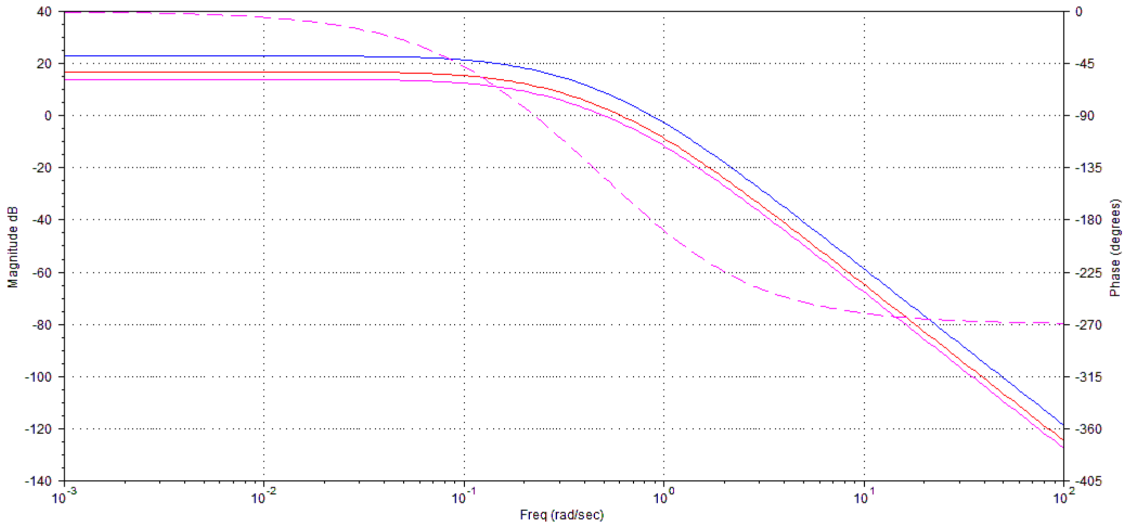
Con controlador solo P

Kc=1,4
Kcu=2,8

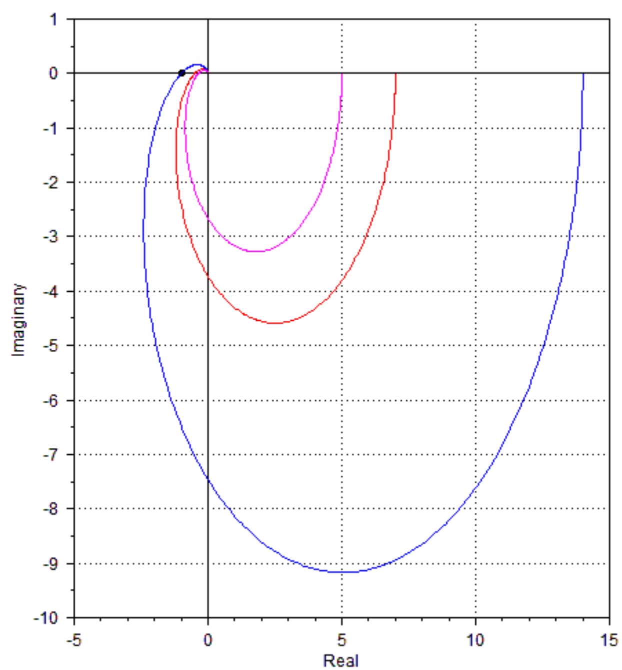


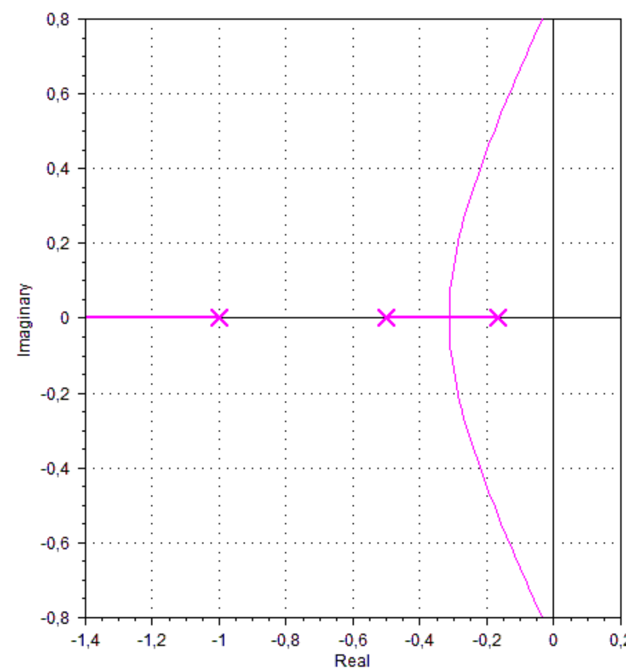
Freq = 0,9027 r/s, Mag = 0,009738 (-40,23 dB), Phase = -180,5 deg

Con controlador solo P



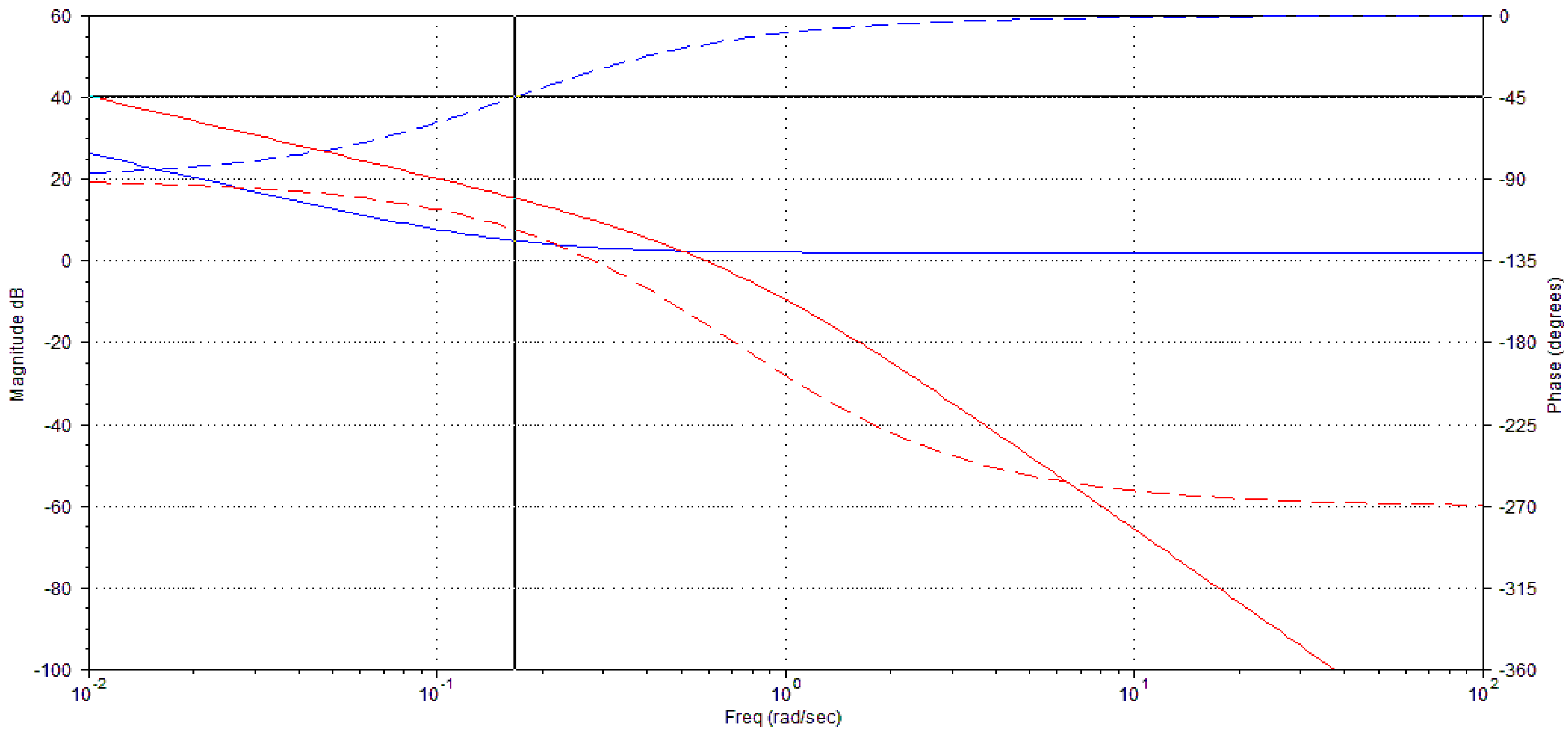
Kc=1,4
Kcu=2,8





Con controlador solo P+I

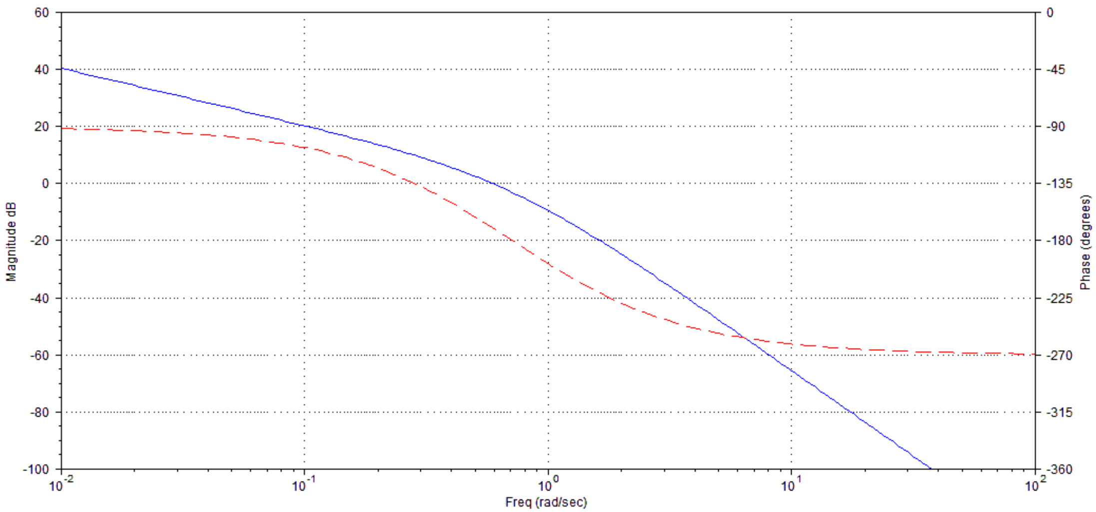
CC>gpi=1.27*(1+(1/(6.09*s)))
CC>bode(gpi,g*gpi)

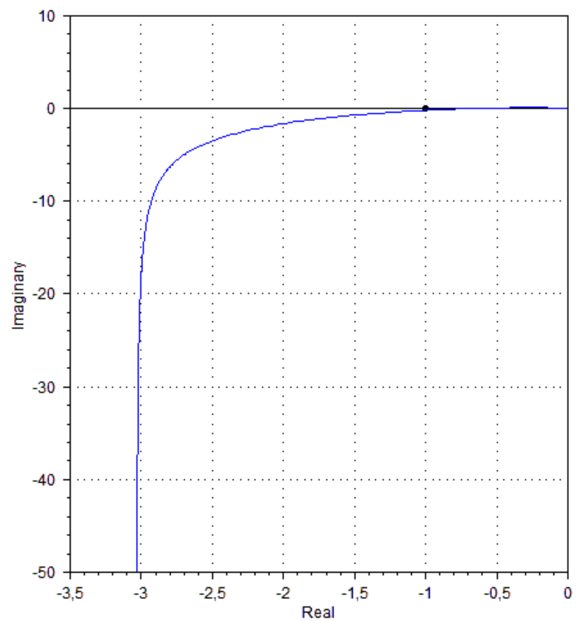


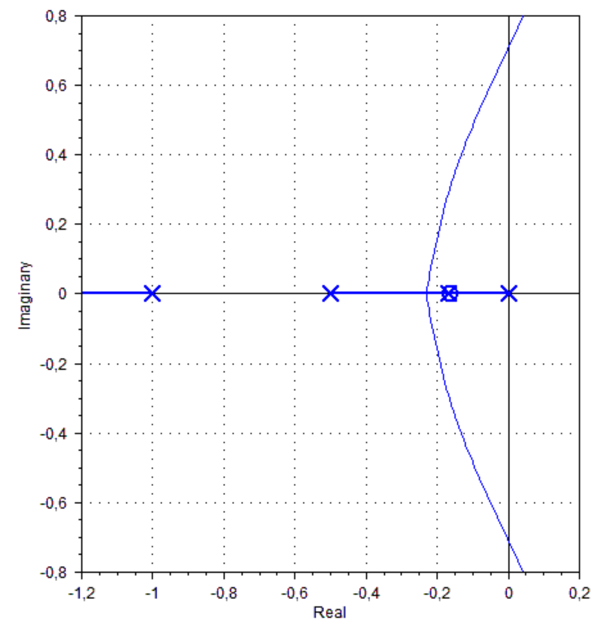
Freq = 0,1673 r/s, Mag = 100,5 (40,05 dB), Phase = -44,9 deg

Con controlador solo P+I

CC>gpi=1.27*(1+(1/(6.09*s)))

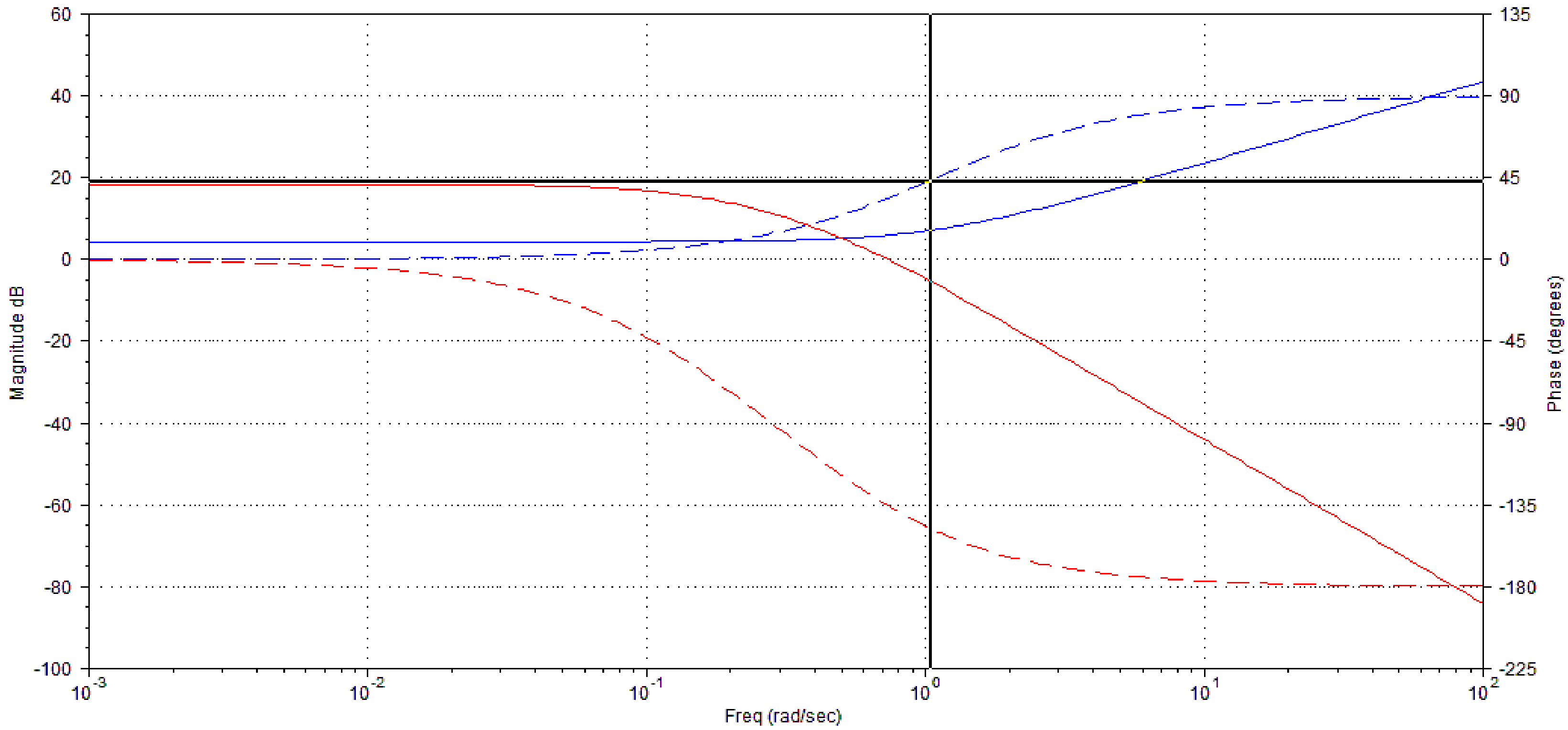






Con controlador solo P+D

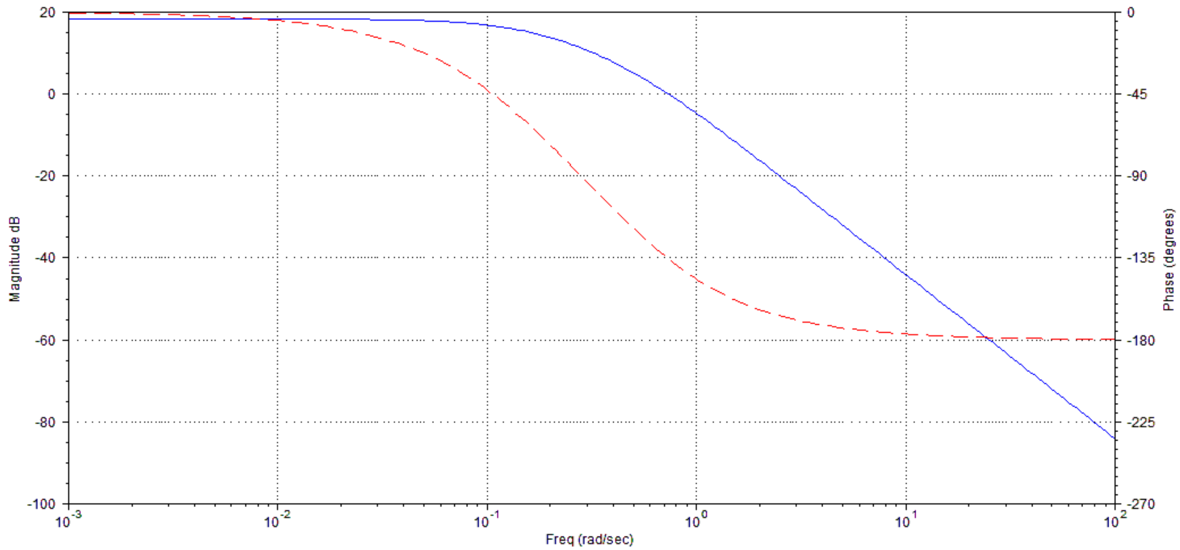
CC>gpd=1.65*(1+(0.91*s))
CC>bode(gpd,g*gpd)

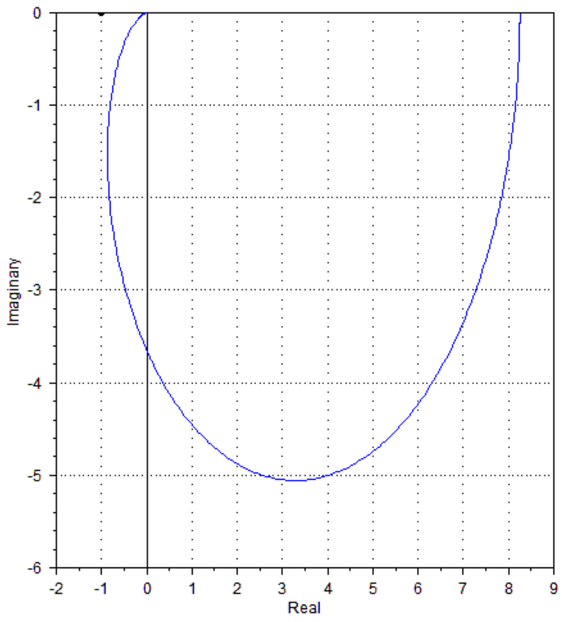


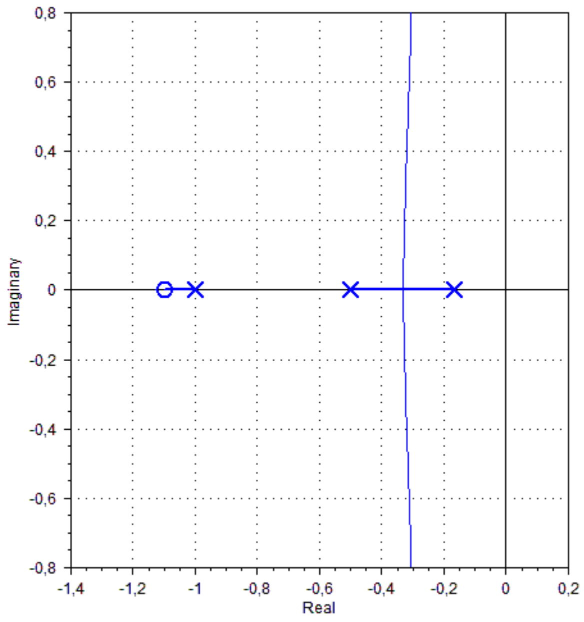
Freq = 1,049 r/s, Mag = 8,896 (18,98 dB), Phase = 42,71 deg

Con controlador solo P+D

CC>gpd=1.65*(1+(0.91*s))

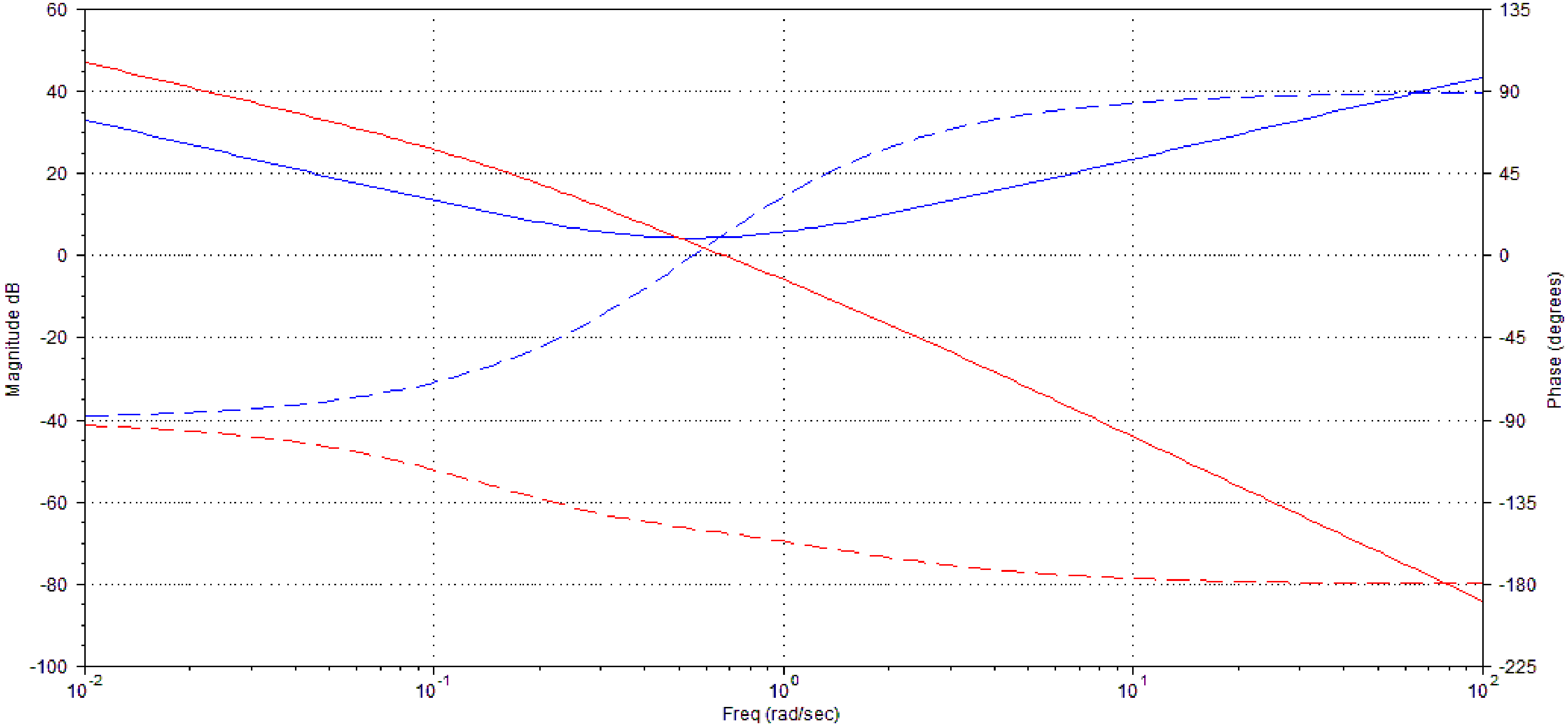






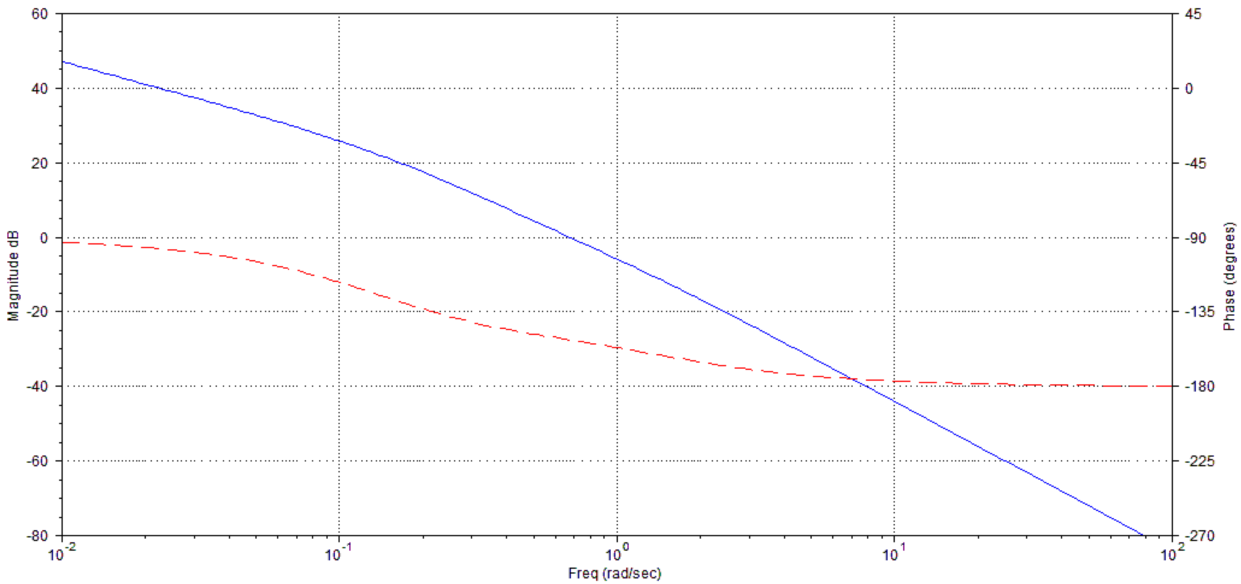
Con controlador solo P+I+D

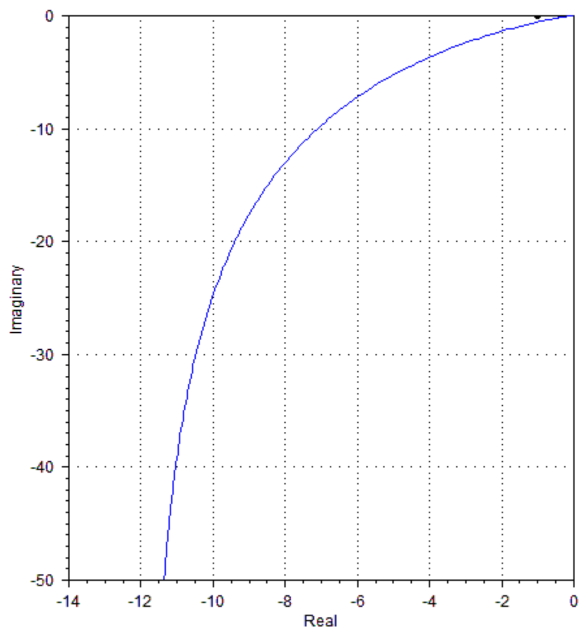
CC>gpid=1.65*(1+(1/(3.65*s))+(0.91*s))
CC>bode(gpid,g*gpid)

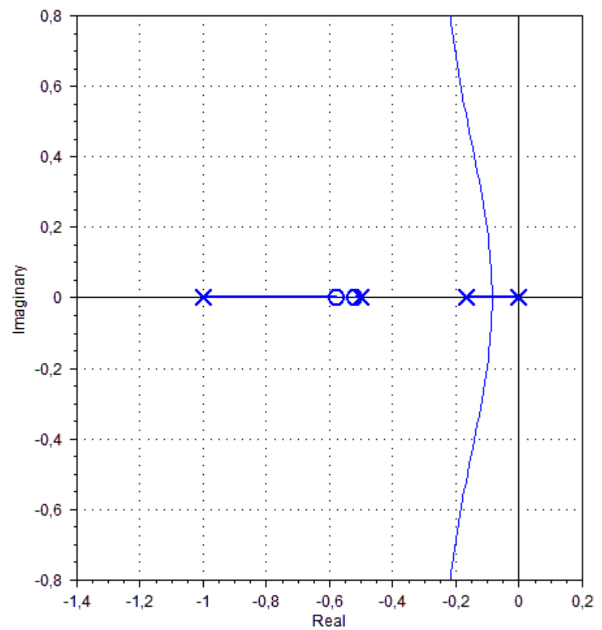


CC>gpid=1.65*(1+(1/(3.65*s))+(0.91*s))

Con controlador solo P+I+D

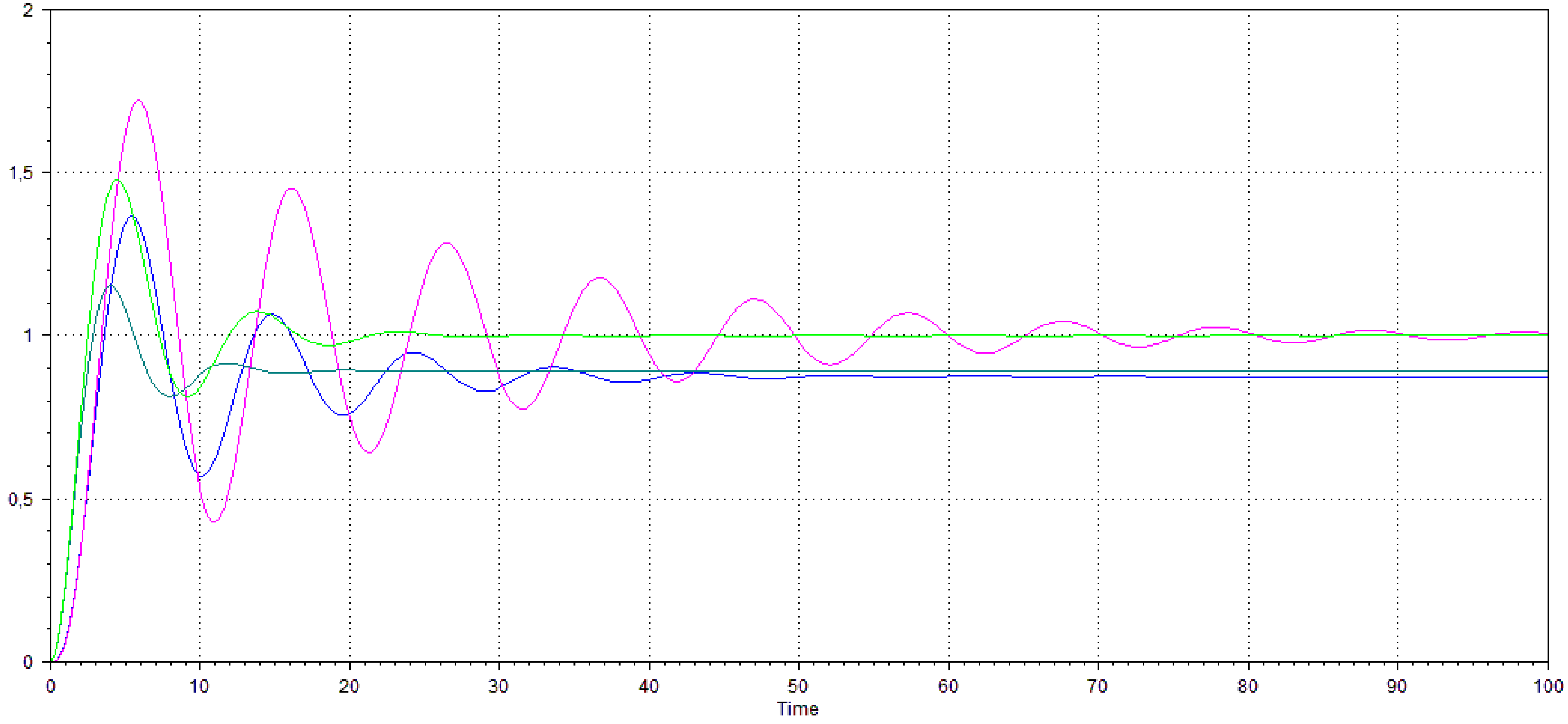


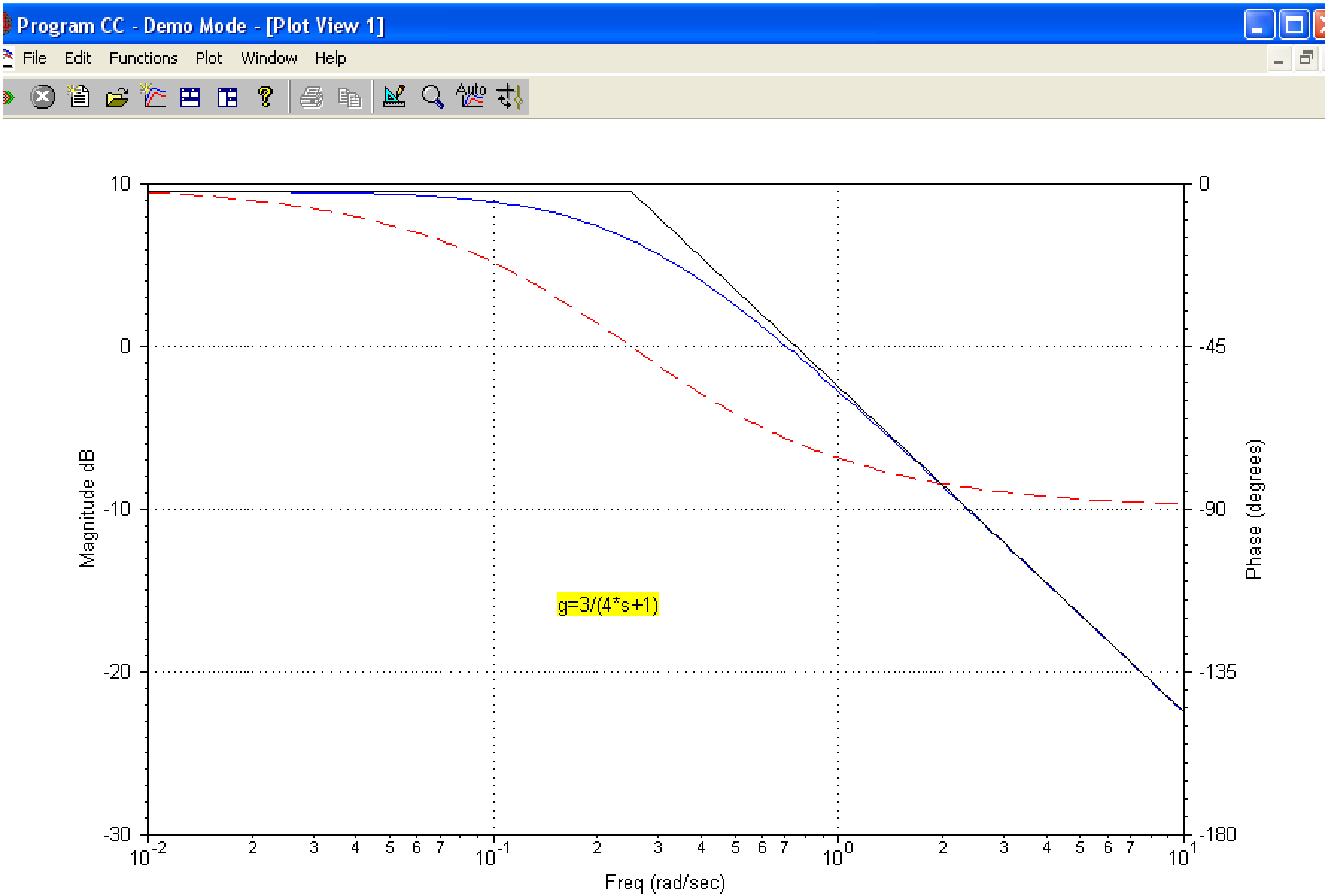


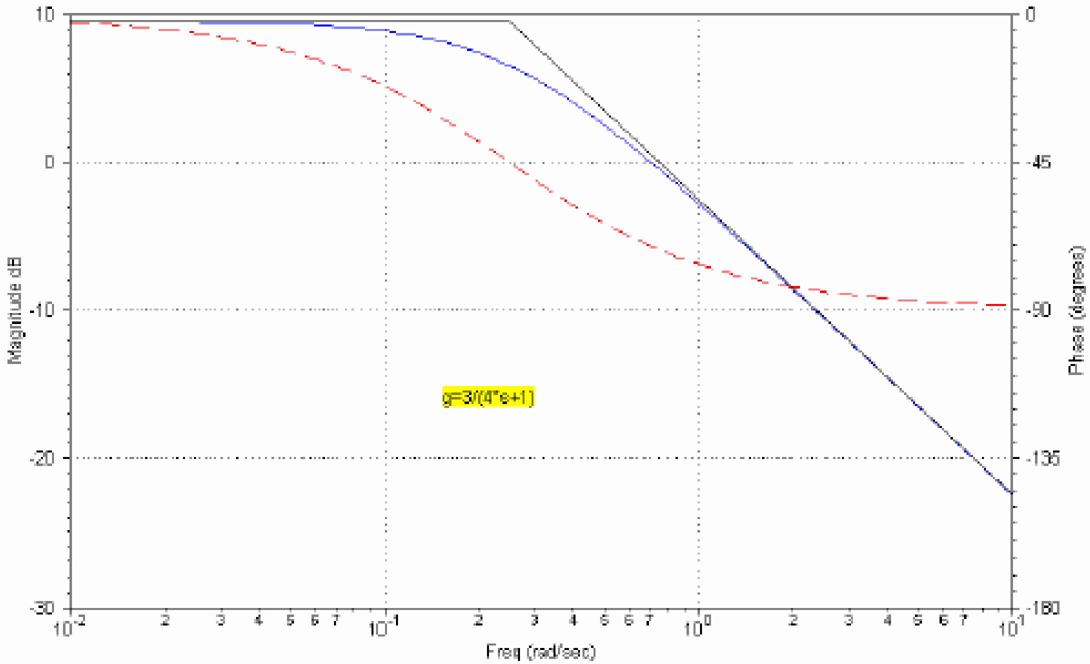


Respuestas temporales con controlador P, PI, PD y PID

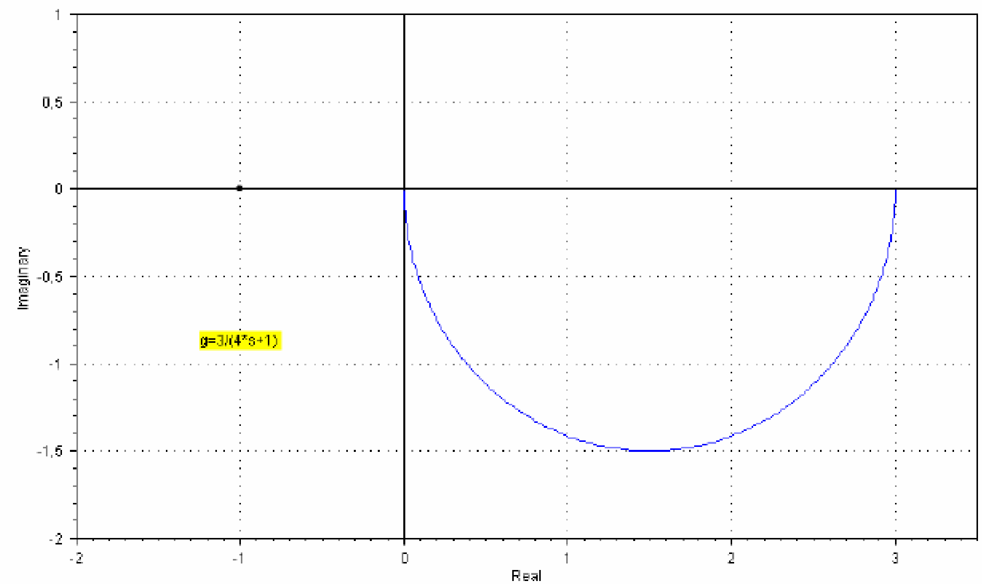
```
CC>h=1
CC>cl=g|h
CC>cl1=(g*1.4)|h
CC>cl2=(g*gpi)|h
CC>cl3=(g*gpd)|h
CC>cl4=(g*gpid)|h
CC>time(cl1,cl2,cl2,cl3,cl4)
```



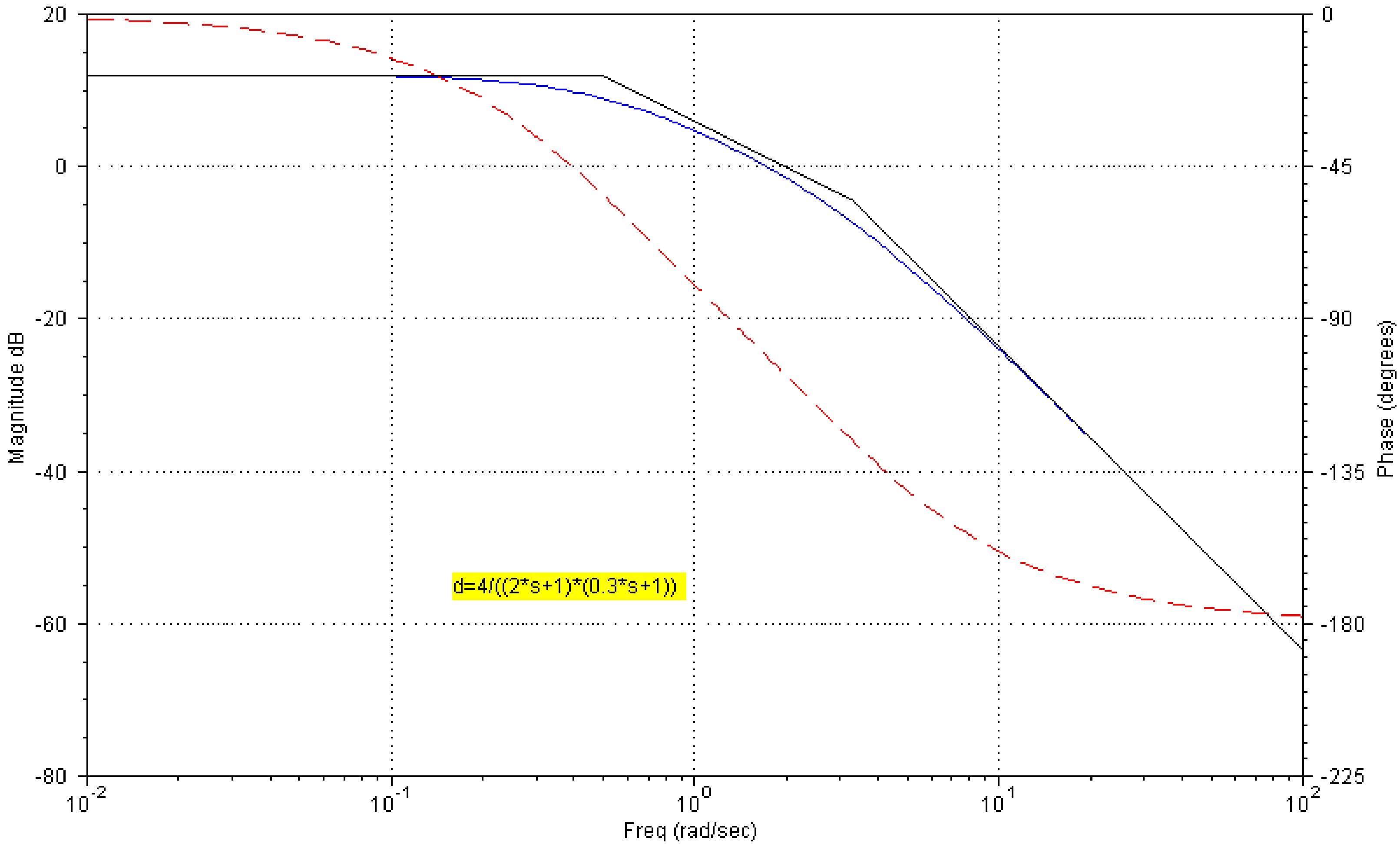

Program CC - Demo Mode - [Plot View 1]
File Edit Functions Plot Window Help
Auto
10
0
-10
-20
-30
0
-45
-90
-135
-180
Magnitude dB
Phase (degrees)
g=3/(4*s+1)
10^{-2} 2 3 4 5 6 7 10^{-1} 2 3 4 5 6 7 10^{0} 2 3 4 5 6 7 10^{1}
Freq (rad/sec)

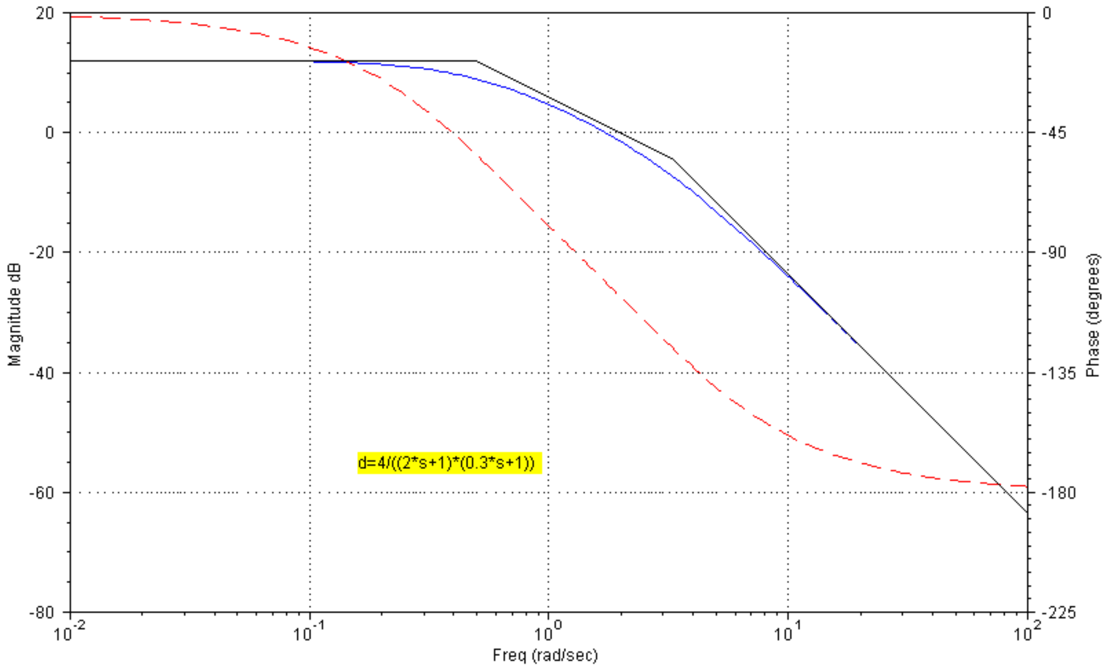


Magnitude dB
Phase (degrees)
Freq (rad/sec)
g=3/(4*s+1)

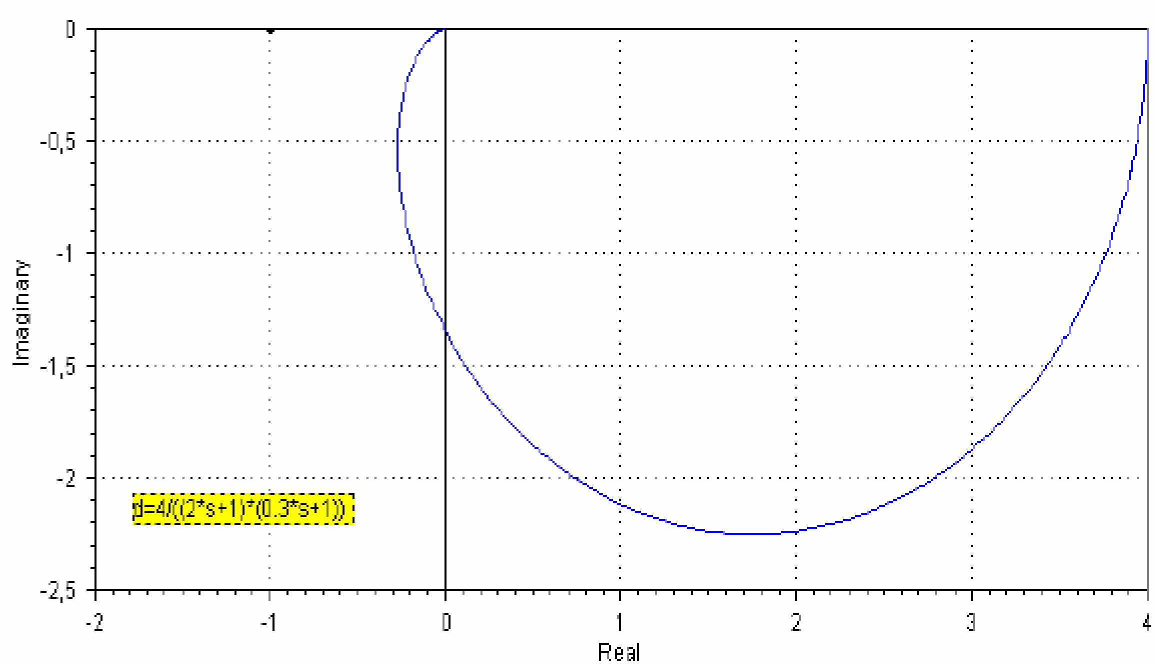


Imaginary
Real
g=3/(4*s+1)

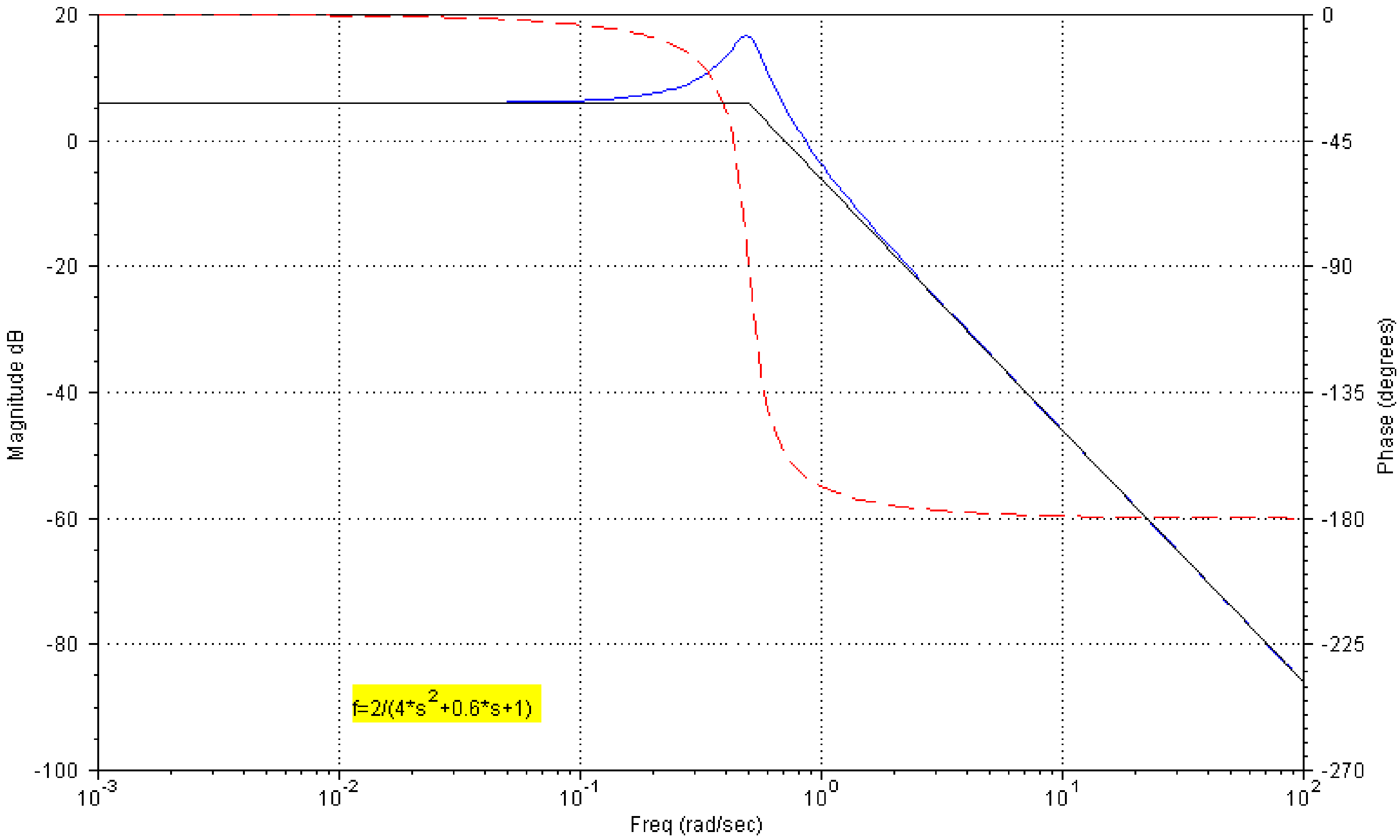

20
0
-20
-40
-60
-80
Magnitude dB
0
-45
-90
-135
-180
-225
Phase (degrees)
10-2
10-1
100
101
102
Freq (rad/sec)
d=4/((2*s+1)*(0.3*s+1))



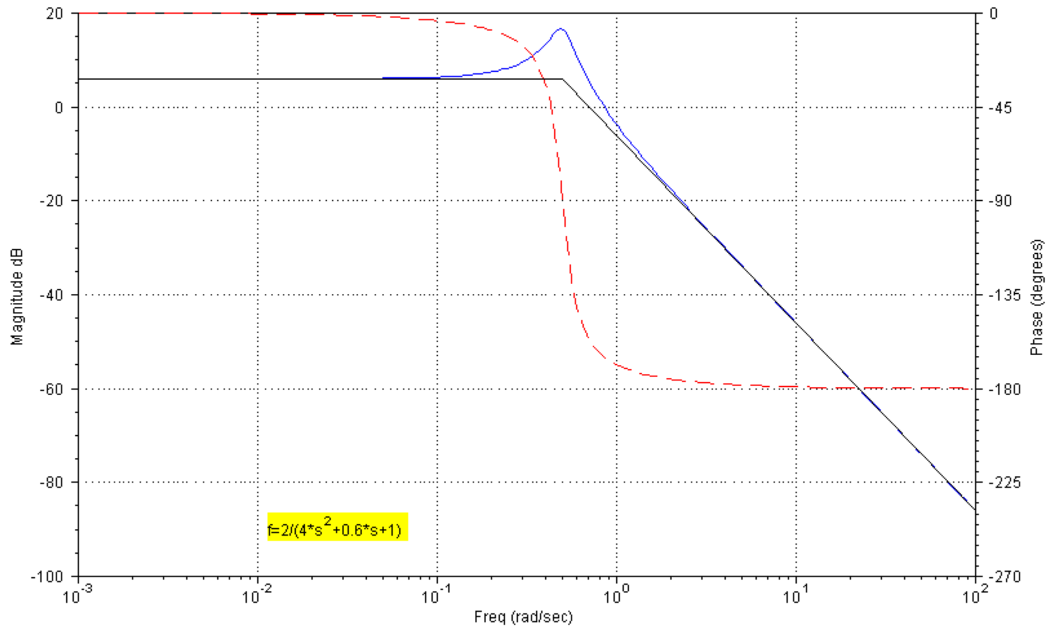
20
0
-20
-40
-60
-80
Magnitude dB
0
-45
-90
-135
-180
-225
Phase (degrees)
d=4/((2*s+1)*(0.3*s+1))
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Freq (rad/sec)



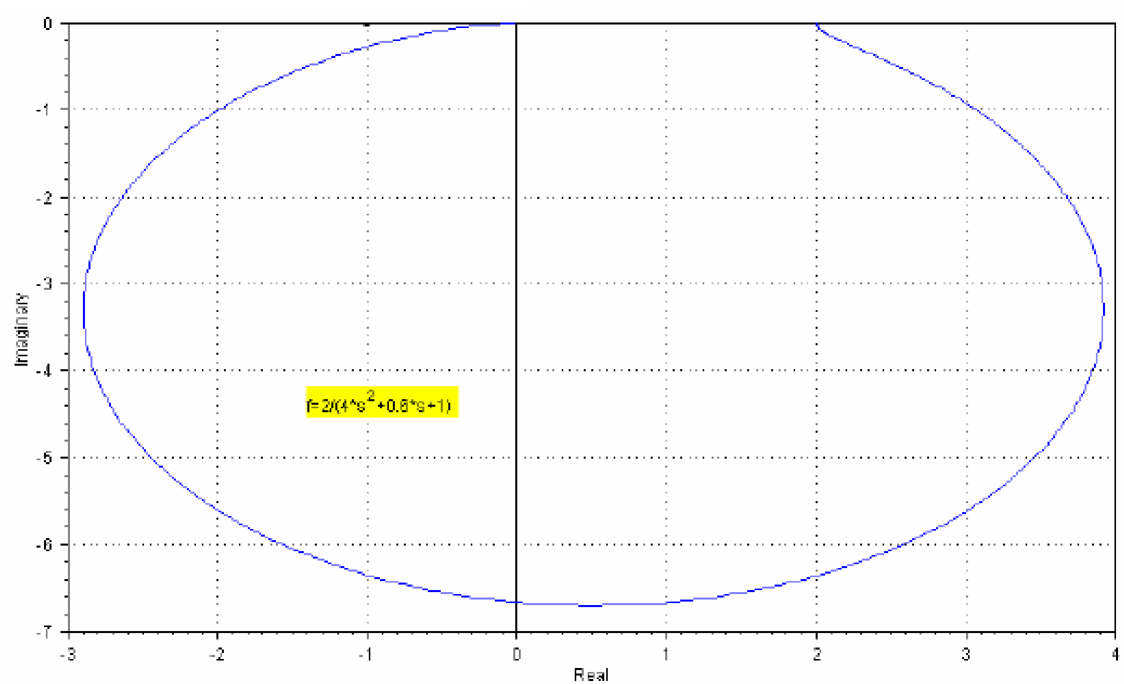
0
-0,5
-1
-1,5
-2
-2,5
Imaginary
d=4/((2*s+1)*(0.3*s+1))
-2
-1
0
1
2
3
4
Real



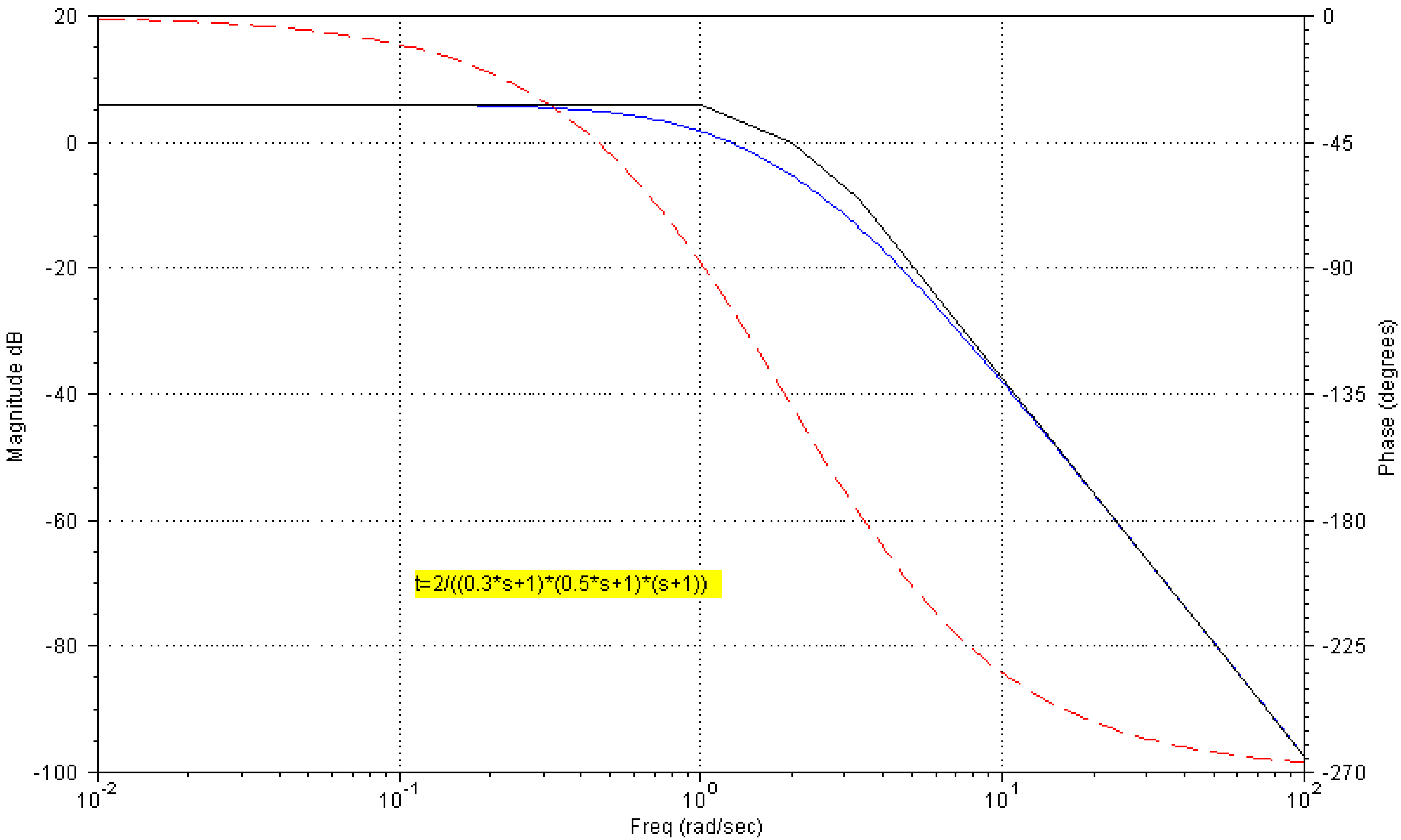
Magnitude dB
Phase (degrees)
Freq (rad/sec)
f=2/(4*s^2+0.6*s+1)
20
0
-20
-40
-60
-80
-100
0
-45
-90
-135
-180
-225
-270
10^-3
10^-2
10^-1
10^0
10^1
10^2



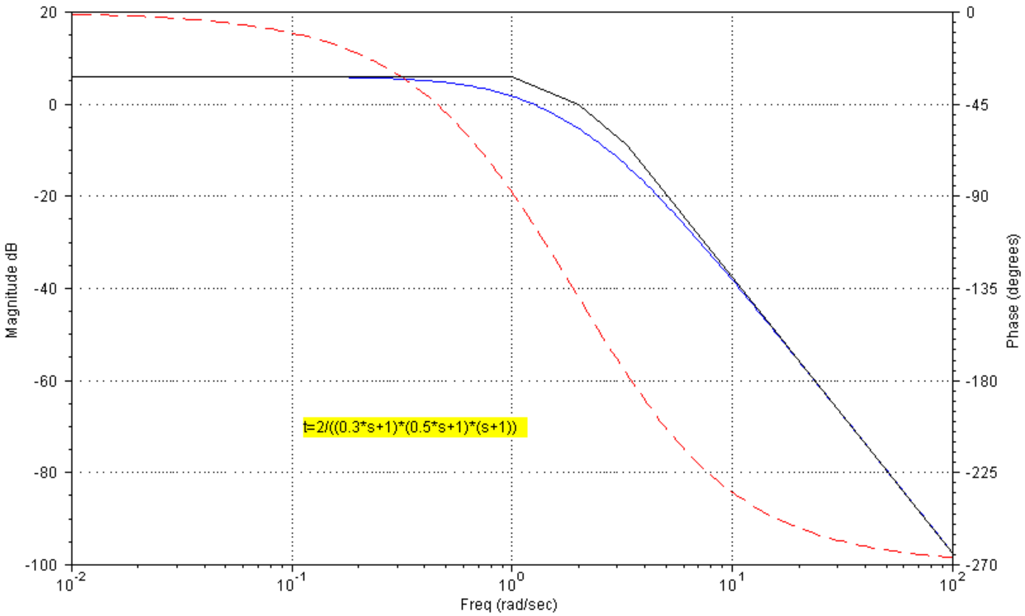
Magnitude dB
Phase (degrees)
Freq (rad/sec)
f=2/(4*s^2+0.6*s+1)



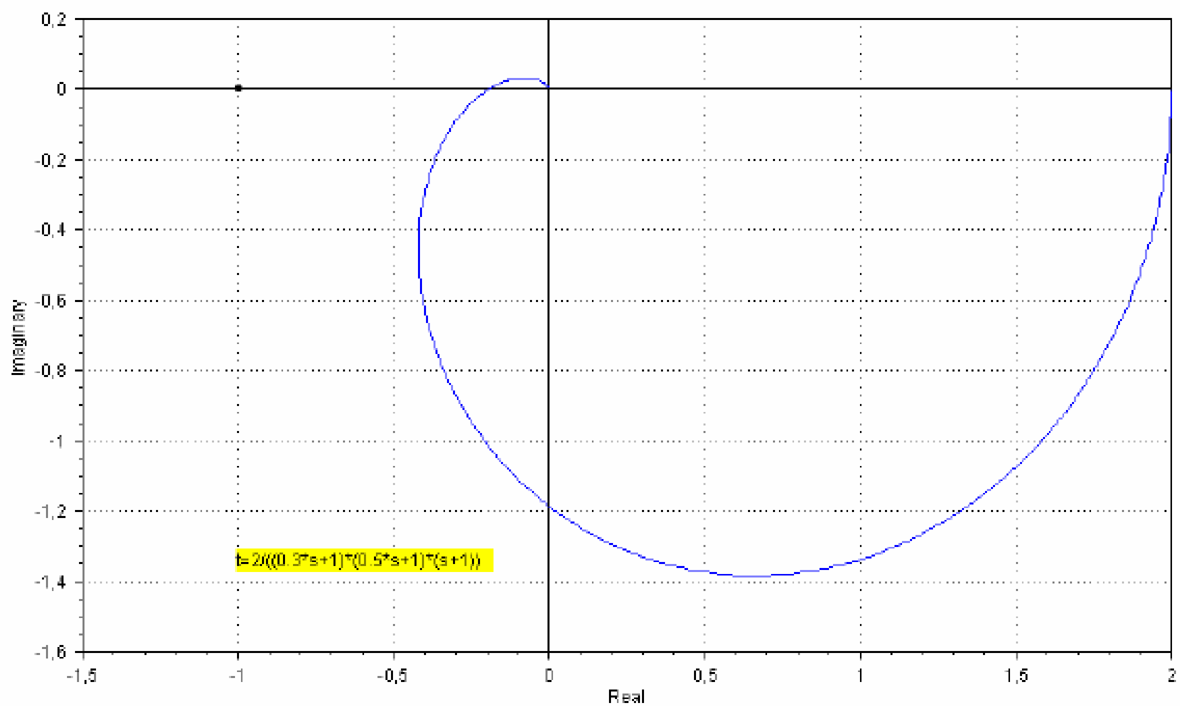
Imaginary
Real
f=2/(4*s^2+0.6*s+1)



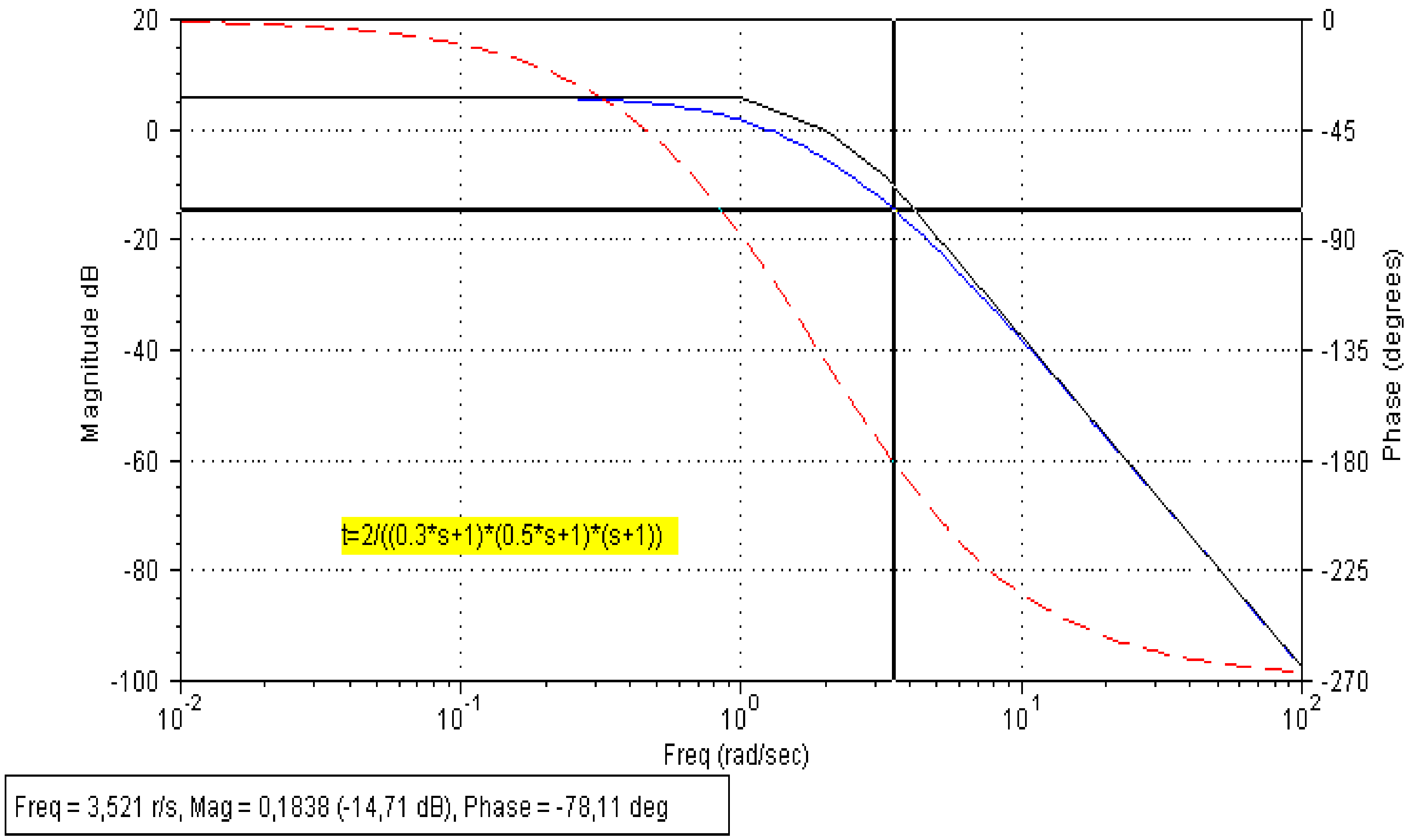
20
0
-20
-40
-60
-80
-100
Magnitude dB
0
-45
-90
-135
-180
-225
-270
Phase (degrees)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Freq (rad/sec)
t=2/((0.3*s+1)*(0.5*s+1)*(s+1))

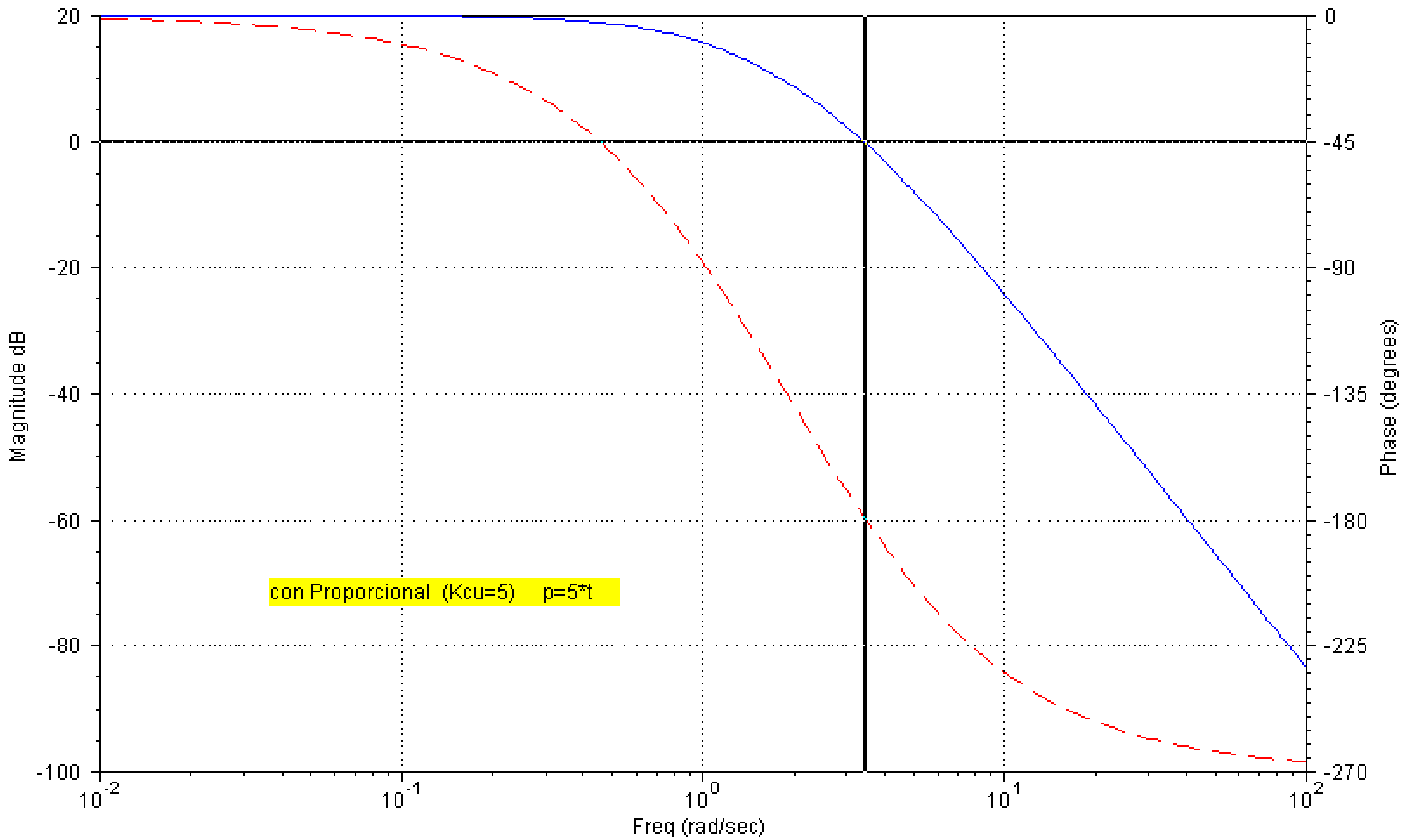


Magnitude dB
Phase (degrees)
Freq (rad/sec)
t=2/((0.3*s+1)*(0.5*s+1)*(s+1))

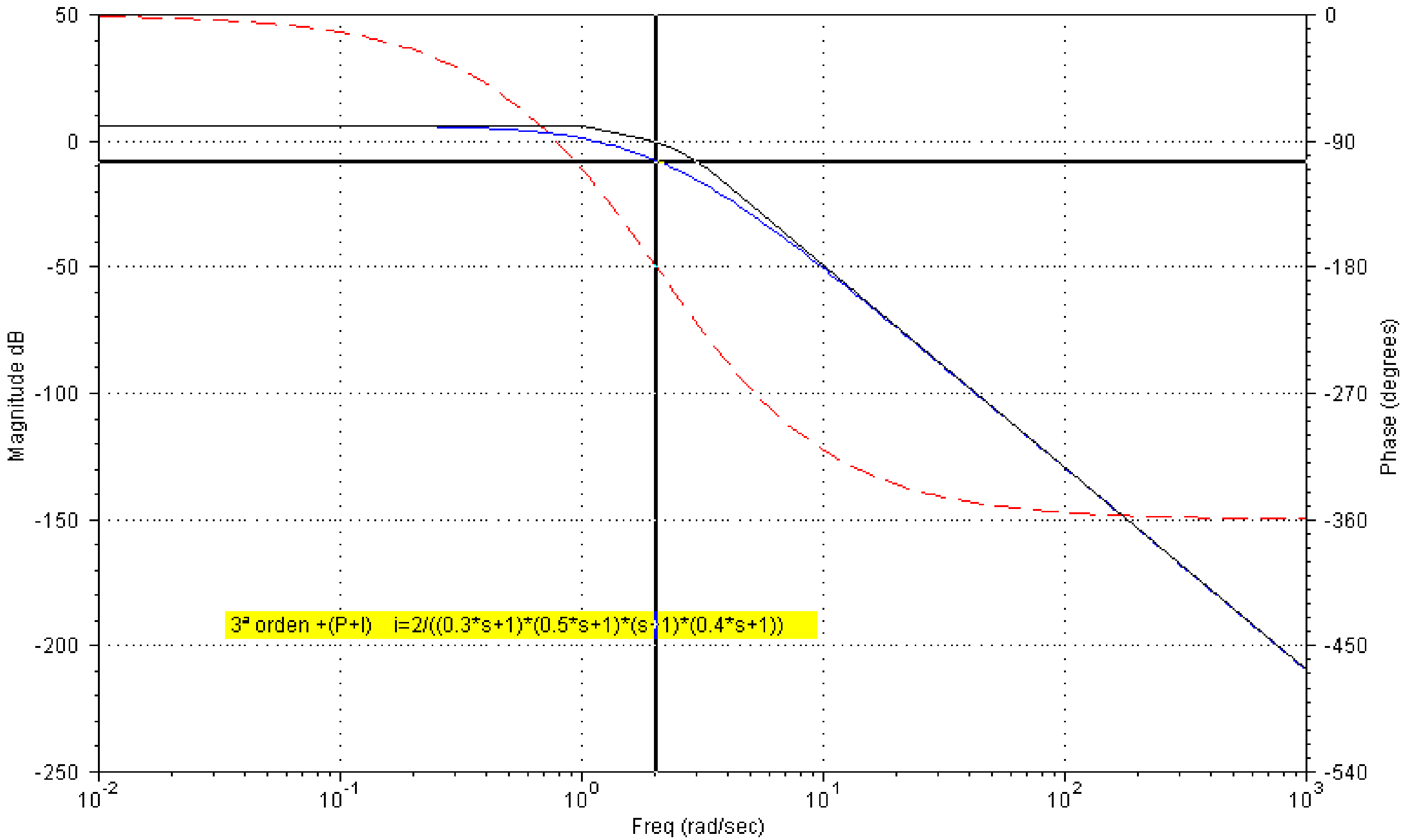


Imaginary
Real
t=2/((0.3*s+1)*(0.5*s+1)*(s+1))

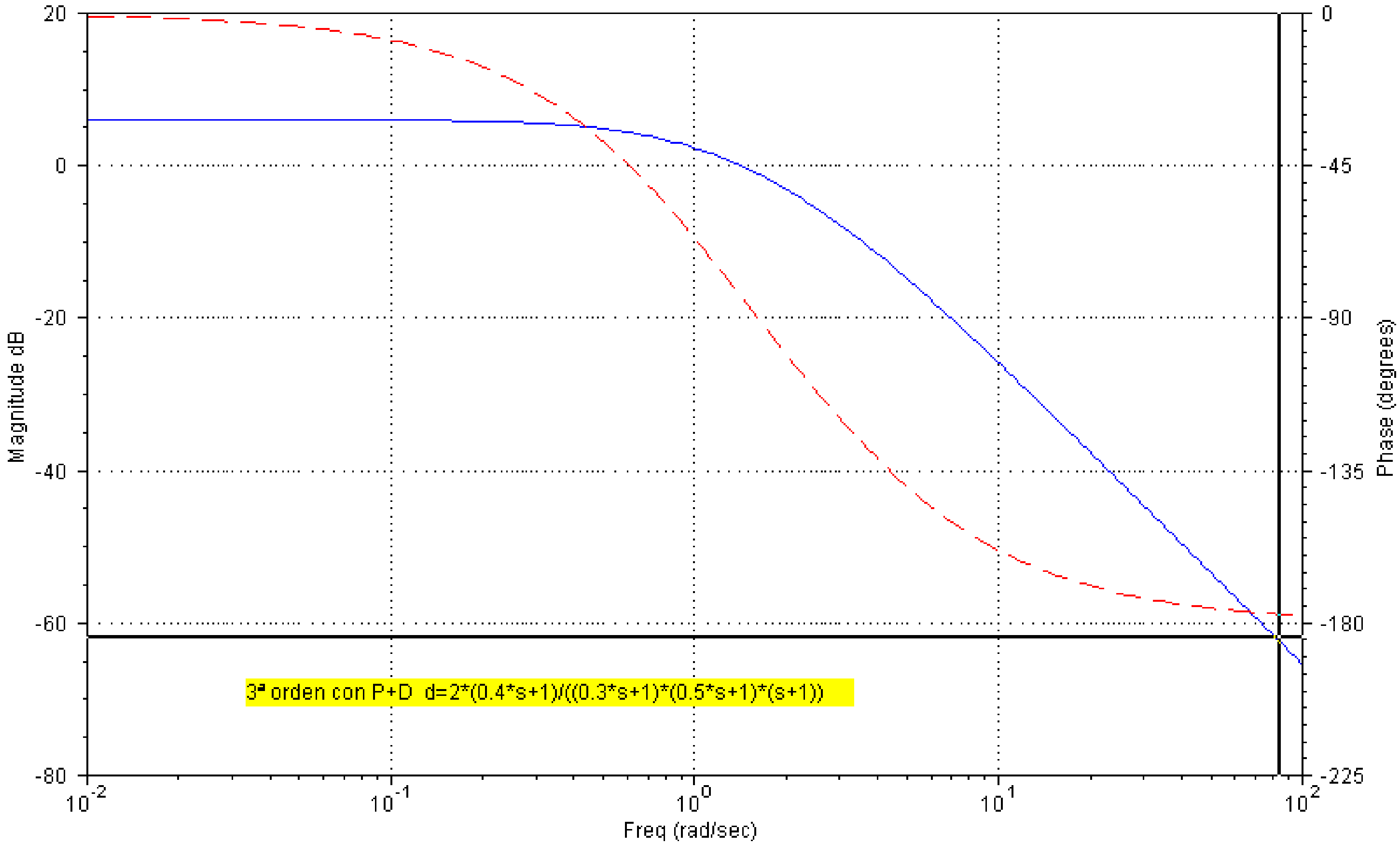

20
0
-20
-40
-60
-80
-100
Magnitude dB
0
-45
-90
-135
-180
-225
-270
Phase (degrees)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Freq (rad/sec)
t=2/((0.3*s+1)*(0.5*s+1)*(s+1))
Freq = 3,521 r/s, Mag = 0,1838 (-14,71 dB), Phase = -78,11 deg



Freq = 3,452 r/s, Mag = 0,9895 (-0,09174 dB), Phase = -45,21 deg



Freq = 2,039 r/s, Mag = 0,3764 (-8,486 dB), Phase = -105,3 deg



Freq = 84,17 r/s, Mag = 0,0008053 (-61,88 dB), Phase = -184,2 deg