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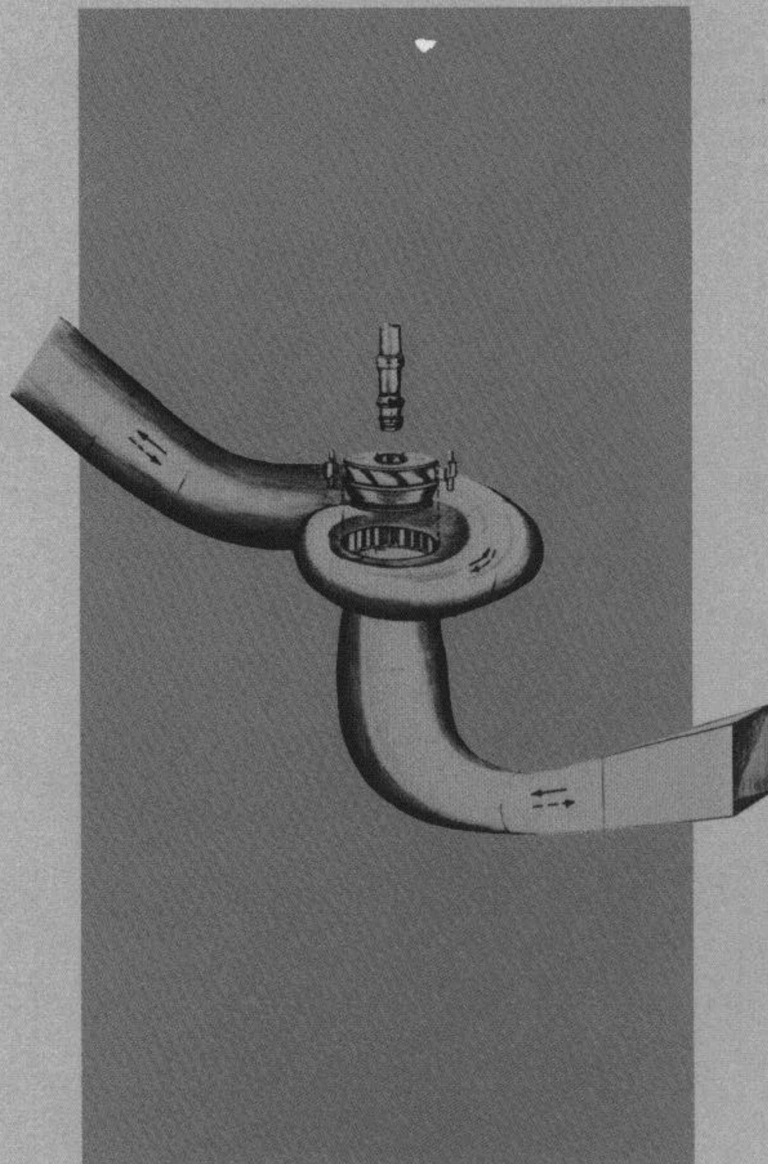

# Estimating Reversible Pump-Turbine Characteristics

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The monograph presents guidelines for designers who plan or maintain pump-turbine installations. Data are presented on recent installations of pump-turbines which include figures showing operating characteristics, hydraulic performance, and sizing of the unit. Specific speed, impeller/runner diameter, and the best efficiency head and discharge were selected as the criteria that characterize the unit performance.



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# Estimating Reversible Pump-Turbine Characteristics

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Bureau of Reclamation

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# Preface

The purpose of this monograph is to devise guidelines for designers who plan or maintain pump-turbine installations. Data are presented on units that the Bureau installed in the past as well as on more recent installations. Figures show operating characteristics and dimensional expressions. International System of Units (SI) as well as U.S. customary units are used.

# Letter Symbols and Quantities

$C_r$=Ratio of fluid radial velocity to spouting velocity
$D_1$=Discharge diameter of impeller or entrance diameter of runner
$D_2$=Minimum opening diameter of impeller or runner
$F_h$=Hydraulic thrust
f=Frequency, Hertz
ft=Foot
g=Gravitation constant (acceleration)
gal=Gallon
H=Head developed (pump) or absorbed (turbine)
$H_{BEP}$=Best efficiency head developed (pump)
$H_{BET}$=Best efficiency head absorbed (turbine)
$H_a$=Atmospheric pressure (head)
$H_f$=Hydraulic losses in pump suction
$H_s$=Static draft head, elevation difference from $D_2$ to minimum tailwater
$H_{so}$=Shutoff head of an impeller or runner
$H_v$=Vapor pressure head of water
$h_d$=Turbine design head
hp=Horsepower
k=Efficiency factor
kg=Kilogram
kVA=Kilovolt-ampere capacity of motor-generator
kW=Kilowatt
lb=Pound
M=Wicket gate height
MW=Megawatt
m=Meter
NPSH=Pump net positive suction head

n=Rotational speed
$n_{sp}$=Pump best (peak) efficiency specific speed
$n_{st}$=Turbine best (peak) efficiency specific speed
$P_{BEP}$=Best efficiency pump power input
$P_d$=Turbine full-gate capacity at $h_d$
$P_p$=Pump power input
$P_t$=Turbine capacity
p=Number of poles in generator-motor
$Q_{BE}$=Pump best efficiency discharge at $H_{BE}$
$Q_t$=Turbine full-gate discharge at $H_d$
r/min=Revolution per minute
s=Second (time)
V=Peripheral velocity of impeller or runner at $D_1$
W=Impeller weight
$W_{pt}$=Total pump-turbine weight
$WR^2$=Product of the weight of revolving parts and the square of the radius of gyration

$\gamma$=Gamma=Specific weight of water
$\eta$=Eta=Hydraulic efficiency
$\eta_g$=$Eta_g$= Generator efficiency
$\eta_p$=$Eta_p$=Pump design efficiency
$\eta_t$=$Eta_t$=Turbine design efficiency
$\pi$=pi=3.14159 . . .
$\sigma$=Sigma=Cavitation coefficient
$\sigma_b$=Sigma begin
$\sigma_{cr}$=Sigma critical
$\phi_1$=$Phi_1$=$\pi n D_1/(60\sqrt{2gH_{BE}})$, velocity ratio at $D_1$

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## FIGURES



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# Introduction

Pumped-storage installations provide the most efficient and practical means for storing large quantities of energy. Large pump-turbines such as the 100-MW units in the Bureau of Reclamation's Mt. Elbert Pumped-Storage Powerplant near Leadville, Colo., can have overall efficiencies of approximately 75 percent. The overall efficiency includes hydraulic losses and electrical losses associated with the generator-motor and transformer.

Although pumped storage does not increase power system energy supply (unless there is natural inflow into the upper reservoir), the large number of pumped-storage systems built worldwide since the 1950's attests to their value in power systems for leveling the peaks and valleys of a typical electrical utility load curve, and for providing emergency power [1].[1] Pumped storage combined with thermal generating capacity allows thermal units to operate at nearly constant output and best (peak) efficiency (BE).

A thermal plant using pump-turbines for providing peaking power can have a higher overall system efficiency than a system that uses gas turbines, another commonly used method for providing peaking power. The pumped-storage efficiency of 75 percent with a baseload plant thermal efficiency of 40 percent gives an overall efficiency of 30 percent as compared to a gas turbine efficiency of 20 percent.

This higher overall efficiency is obtained either with relatively inexpensive nuclear energy or coal fuel as opposed to the increasingly scarce, expensive natural gas or petroleum fuels used in gas turbine operation. Obviously, gas turbines have advantages—design and construction times are considerably less than the 4 to 6 years required to design and build a pumped-storage facility, the initial capital investment is small compared to a pumped-storage installation, and gas turbines are not subject to geographical site limitations as is a pumped-storage facility.

A pumped-storage installation provides flexibility in adapting to power outages. During the pumping cycle the pumping input load is a large interruptable load that can be readily removed from the system. When a pump-turbine is at speed-no-load (spinning reserve) it can assume full output load in 4 to 10 seconds compared to the half hour required for a steam plant to achieve maximum output from spinning reserve. Because of its ability to store energy efficiently, pumped storage could advance energy development from unconventional sources, such as solar and wind.

The Bureau's Flatiron pump-turbine was the first reversible pump-turbine installed in the United States. It began operation in 1954. Presently, the Bureau has 23 reversible units installed and 6 more under construction. None of the units is used exclusively for power generation since all are part of multipurpose water development projects which provide water either for irrigation or municipal and industrial use.

Basically, a pump-turbine is a pump that operates in the reverse to generate power. Figures included compare various characteristics of the three classes of hydraulic machines; i.e., pump, turbine, and pump-turbine.

[1] Numbers in brackets refer to items in the bibliography.

# Unit Selection

## Speed

Many characteristics of the pump-turbine, pump, and turbine are plotted as the ordinate and specific speed as the abscissa. Any hydraulic machine will have a range of specific speeds for a given rotational speed, depending on the operating point; however, the specific speed at the best (peak) efficiency operating point is customarily used as an index number for describing the geometrical configurations of the machine and the shape of the characteristic curves. The specific speed at peak efficiency of a hydraulic machine does not change with changes in rotational speed. All references of pump-turbine specific speed refer to the specific speed at the best efficiency point (BE).

The units of specific speed are noted to remind the reader which parameters express the characteristic speed. For brevity, head and speed are omitted. Comparable formulas are included in the Hydraulic Similarity section.

1. The pump best efficiency specific speed ($n_{sp}$) is defined by the equation:

$$n_{sp}=\frac{n(Q_{BE})^{1/2}}{(H_{BE})^{3/4}} \qquad (1)$$

where:

$n$=rotational speed, r/min,
$n_{sp}$=pump best efficiency specific speed, ft-gal/min (m-m$^3$/s),
$H_{BE}$=pump best efficiency head, ft (m), and
$Q_{BE}$=pump best efficiency discharge, gal/min (m$^3$/s).

2. The turbine design specific speed ($n_{st}$) is defined by the equation:

$$n_{st}=\frac{n(P_d)^{1/2}}{(h_d)^{5/4}} \quad \text{or} \quad \frac{n\left(\frac{P_d}{h_d^{1/2}}\right)^{1/2}}{h_d} \qquad (2)$$

where:

$n$=rotational speed, r/min,
$n_{st}$=turbine design specific speed,
$h_d$=turbine design head, ft (m), and
$P_d$=turbine full-gate capacity (at $h_d$), hp; however, for comparing hydraulic machines the following will establish turbine output of a pump-turbine as that obtained at the gate opening and head ($H_{BE}$) where best (peak) efficiency is attained.

It can be shown that equations (1) and (2) have correlation upon substitution of discharge (Q) times head (H) for power (P) into equation (2).

Stepanoff [2], using the same parameters, Q and H, for both machines, expressed the relationship between pump and turbine specific speeds as:

$$n_{st}=n_{sp}\cdot\eta \qquad (3)$$

where:

$n_{st}$=turbine specific speed,
$n_{sp}$=pump specific speed, and
$\eta$=hydraulic efficiency, approximately equal to the square root of the pump best efficiency.

Because pump criteria dominates the selection of a pump-turbine, the following treatment applies the pump specific speed ($n_{sp}$) formula to all three classes of machines. Of particular interest is the comparison of the calculated specific speeds for a pump-turbine when analyzed at best efficiency turbine operation and best efficiency pump operation. It is apparent from the nearly equal values of pump and turbine specific speeds in table 1 that specific speed does, in fact, define similar hydraulic flow parameters.

Comparative specific speed formulas are shown in the Hydraulic Similarity section.

If an impeller/runner was selected only on the basis of turbine performance and the pump-turbine operated in the turbine direction, expected efficiencies could be equal to or greater than that of a pure turbine [3]. While the turbine best efficiency head is 15 to 30 percent greater than the pump best efficiency head (fig. 10), in actual practice when pumping performance must be considered, there is a sacrifice of approximately 1 percent of turbine efficiency for a pump-turbine as opposed to a pure turbine. The theoretical minimum value of the ratio of turbine best efficiency head ($H_{BET}$) to pump best efficiency head ($H_{BEP}$) for a pump-turbine [4] is:

$$\frac{H_{BET}}{H_{BEP}}=\frac{1}{\eta_p \cdot \eta_t} \qquad (4)$$

where:

$\eta_p$=pump hydraulic efficiency, and
$\eta_t$=turbine hydraulic efficiency.

The value of the ratio of turbine best efficiency head to pump best efficiency head increases as specific speed increases. Using the headwater and tailwater surface elevations as reference, there is a further divergence of the turbine best efficiency head from the pump best efficiency head caused by penstock losses. Adjustment of the turbine best efficiency head to pump best efficiency head ratio is possible by varying the rotational speed (n), such as by using a two-speed generator-motor.

When either pumping or generating, consideration must be given to avoiding cavitation damage and vibration that occur at heads greatly removed from that of the respective

TABLE 1.—*Best efficiency pump and turbine specific speeds for pump-turbines*

| | | Turbine $n_{st}$ | | | | Pump $n_{sp}$ | |
|---|---|---|---|---|---|---|---|
| Pump-turbine | Unit No. | ft-gal/min | m-m³/s | U.S. ft-hp | m-kW | ft-gal/min | m-m³/s |
| San Luis Central Valley Project, California | 1-8 | 1,850 | 35.8 | 28.5 | 108 | 1,950 | 37.7 |
| Flatiron Colorado-Big Thompson Project, Colorado | 3 | 1,960 | 37.9 | 30.2 | 115 | 1,950 | 37.7 |
| Mt. Elbert* Fryingpan-Arkansas Project, Colorado | P/G 1 | 2,160 | 41.8 | 33.2 | 127 | 2,240 | 43.3 |
| Grand Coulee* Columbia Basin Project, Washington | P/G 7 & 8 | 2,320 | 44.9 | 35.7 | 136 | 2,640 | 51.1 |
| Senator Wash Colorado River Front Work and Levee System, California | 1-6 | 4,300 | 83.2 | 66.2 | 252 | 4,650 | 89.9 |

* Model data.

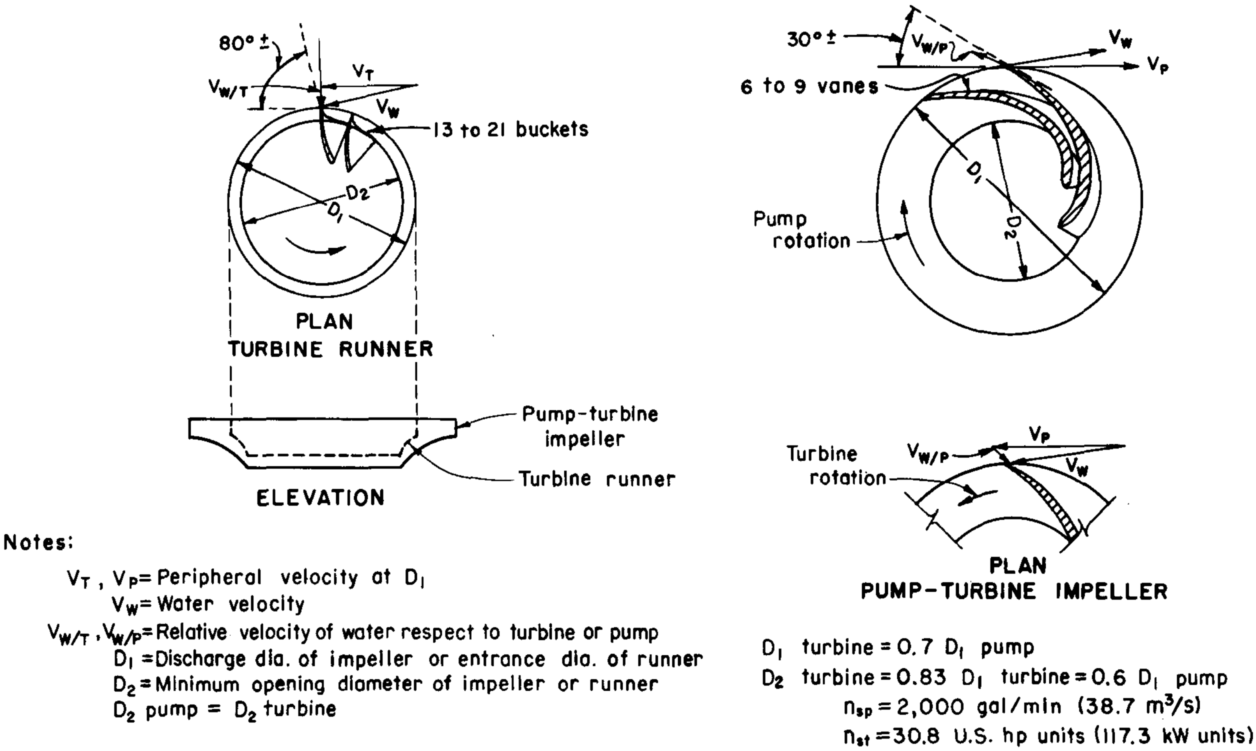


FIGURE 1.—*Comparison of pump-turbine impeller and turbine runner designed for operation at the same head and speed. 106-D-356.*

best efficiency head. A multispeed generator-motor can be used for a pump-turbine operating over a wide head range. In the future, solid-state variable speed control may be practical. The suggested maximum head range relative to pump best efficiency head for a single-speed pump-turbine is shown in table 2.

TABLE 2.—*Permissible operating head range (percent of $H_{BEP}$)*

| Pump specific speed, $n_{sp}$ | | Maximum head | Minimum head |
|---|---|---|---|
| ft-gal/min | m-m³/s | percent | percent |
| less than 1,500 | less than 29 | 110 | 95 |
| 1,500 - 2,000 | 29 - 38.7 | 115 | 90 |
| 2,000 - 3,500 | 38.7 - 67.7 | 125 | 85 |
| 3,500 or greater | 67.7 or greater | 130 | 70 |

The limitations in table 2 are based on data from existing units and should not be considered absolute. Some existing units exceed these limits and a few units with operating ranges that fall within the specified range experience considerable noise, rough operation, and increased maintenance when operating at the extreme. The amount of time that a unit is expected to operate at the extreme high or low heads is a factor in determining the permissible operating head range. Refer to "Submergence of Unit" for operating limitations based on cavitation.

High specific speed pumps have relatively steep head-discharge curves as shown on figure 4; thus, they are more able to operate within a wide head range.

Table 3 shows operating heads of the Bureau's pump-turbine units. The Flatiron pump-turbine operates in the turbine mode at 257 r/min and in the pump mode at 300 r/min. The San Luis pump-turbines can operate either as a pump or a turbine at either 120 or 150 r/min, whichever effects best efficiency for a given head. Flatiron and San Luis, both with two-speed generator-motors, are the Bureau's only reversible pump-turbine installations where a turbine best efficiency head falls within maximum and minimum heads.

TABLE 3.—*Operating heads of Bureau of Reclamation pump-turbines*

| Plant | Flatiron | | San Luis | | Senator Wash | Grand Coulee | Mt. Elbert |
|---|---|---|---|---|---|---|---|
| r/min | 257 | 300 | 120 | 150 | 360 | 200 | 180 |
| Comparable heads for mode of operation - feet (metres) | Turbine Min. 140 (42.7) | ——— | Turbine Min. 109 (33.2) | ——— | Turbine Min. 27 (8.2) | Turbine Min. 263 (80.2) | Turbine Min. 390 (118.9) |
| | | | *Pump Min.* 113 (34.4) | ——— | | | |
| | ——— | *Pump Min.* 170 (51.8) | *Pump BE* 178 (54.2) | ——— | *Pump Min.* 31 (9.4) | *Pump Min.* 272 (82.9) | *Pump Min.* 420 (128.0) |
| | | | *Pump Max.* 200 (61.0) | *Pump Min.* 200 (61.0) | | | |
| | Turbine BE 218 (66.4) | ——— | Turbine BE 215 (65.5) | ——— | *Pump BE* 64 (19.5) | *Pump BE* 287 (87.5) | *Pump BE* 440 (134.1) |
| | ——— | *Pump BE* 250 (76.2) | Turbine Max. 260 (79.2) | Turbine Min. 260 (79.2) | Turbine Max. 71 (21.6) | Turbine Max. 352 (107.3) | Turbine Max. 475 (144.8) |
| | | | ——— | *Pump BE* 272 (82.9) | | | |
| | Turbine Max. 290 (88.4) | ——— | ——— | Turbine Max. 323 (98.5) | *Pump Max.* 74 (22.6) | *Pump Max.* 365 (111.3) | *Pump Max.* 485 (147.8) |
| | | | ——— | *Pump Max.* 332 (101.2) | | | |
| | ——— | *Pump Max.* 300 (91.4) | | | | | |
| | | | ——— | Turbine BE 340 (103.6) | Turbine BE 80 (24.4) | Turbine BE 365± (111.3+) | Turbine BE 525 (160.0) |

* BE = Best efficiency head

The allowable variation of operating head is narrower for pump-turbines than for real turbines. Monograph No. 20 indicates the permissible head range to be between 65 and 125 percent of the design head—". . . the net head at which peak efficiency is desired."—for a Francis turbine [5]. Figure 2 shows pump-turbine characteristics as a function of head and rotational speed for the Bureau's Mt. Elbert Unit P/G 1. This particular pump-turbine is not capable of generating power at heads below 67 percent of the pump best efficiency head.

Because a pump-turbine has a larger diameter ($D_1$) compared to a turbine, the shutoff head is considerably greater for a pump-turbine rotating in the turbine direction than for a comparable turbine operating at the same speed. It is more likely that the turbine shutoff head will be in the operating head range for a pump-turbine than for a turbine. Selection of pump best efficiency head or design head near the lower end of the operating head range favors turbine operation at the expense of pump operation. With the pump design head near the minimum head, turbine operation is possible at minimum head and the turbine best efficiency point is more likely to occur within the operating head range.

For multispeed operation, if rotational speed varies with the square root of the head, the efficiency will remain nearly constant regardless of change in head, and turbine power output will be possible as the head decreases toward zero head. See the four-quadrant diagram on figure 19 for a more detailed description of changes in pump characteristics with changes

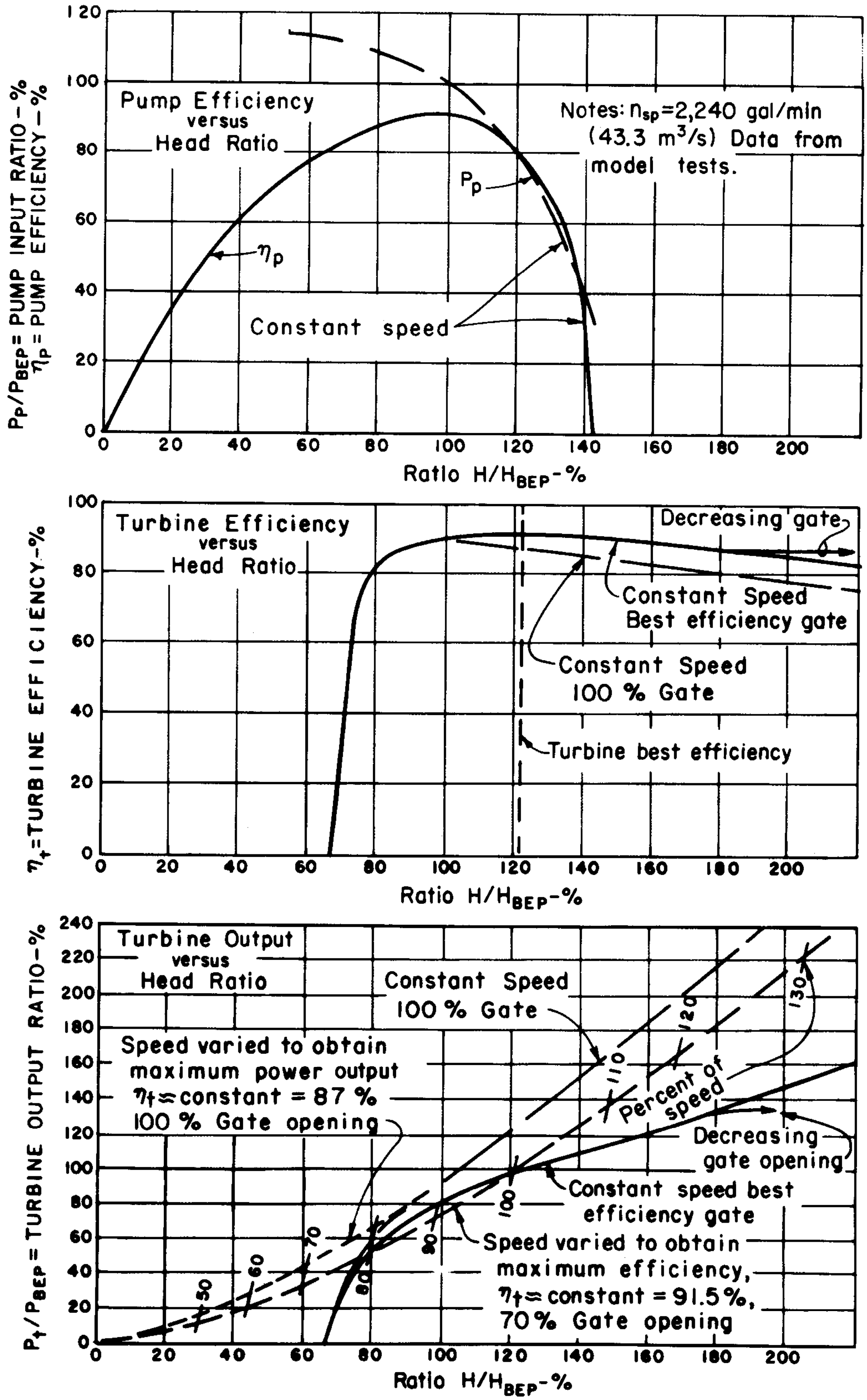


FIGURE 2.—*Variation of pump-turbine efficiency and turbine output with speed and head. 106–D–357.*

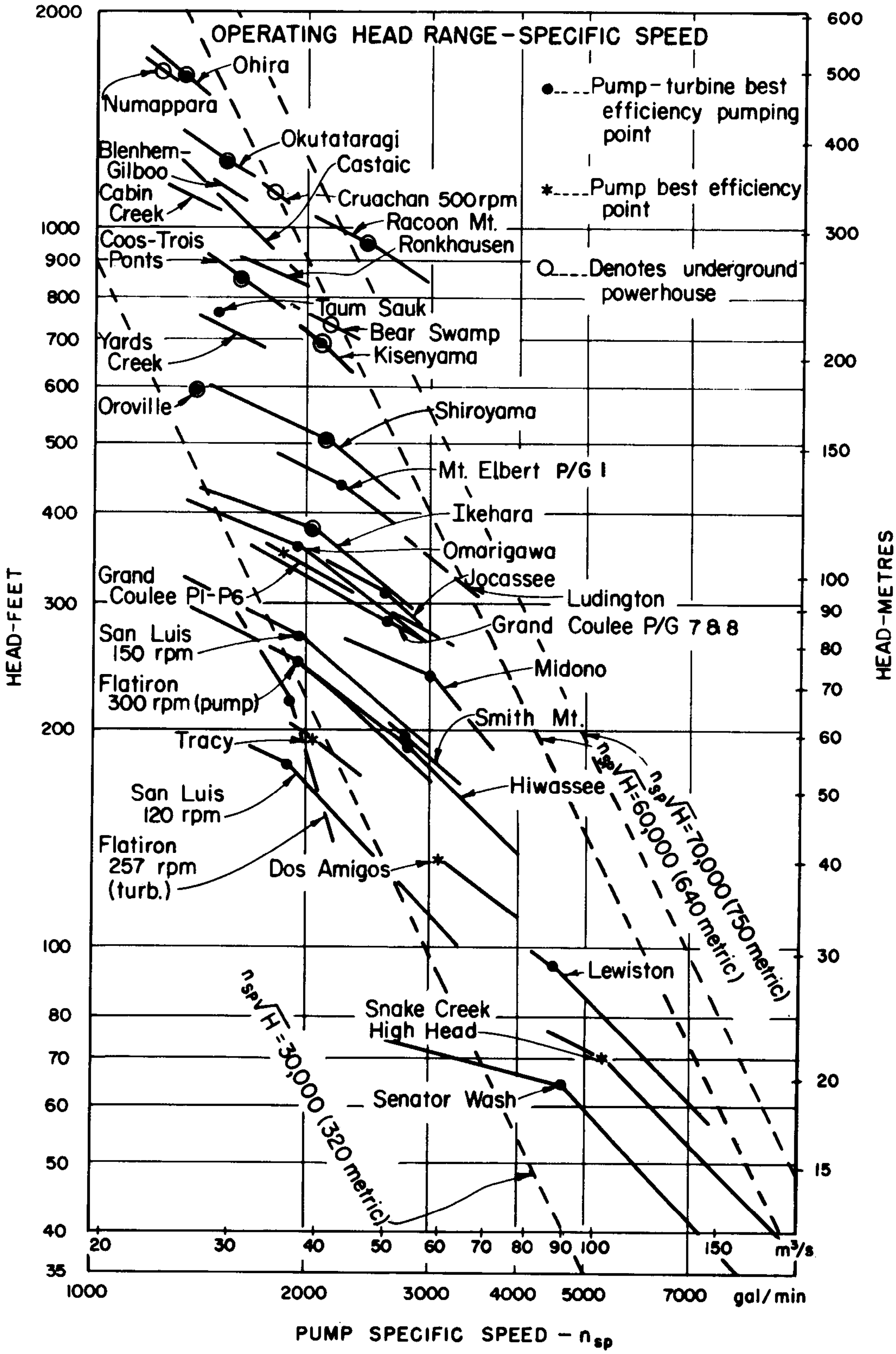


FIGURE 3.—*Operating head range vs. specific speed. 106-D-358.*

in speed or size as established by pump-turbine similarity laws.

Figure 1 shows a turbine runner compared to a pump-turbine impeller having the same specific speed and selected to operate at the same head and rotational speed. The pump-turbine impeller diameter ($D_1$) is 30 percent greater than that of a turbine runner. Because the passages are shorter in a turbine runner, deceleration of water, when the runner is used as a pump, is too abrupt for efficient diffusion within the runner. Conversely, water is accelerated in the turbine direction; however, hydraulic acceleration is inherently more efficient than deceleration. The acceleration process does not cause a significant loss in the efficiency of the turbine runner.

A turbine runner has two to three times more blades than a pump-turbine impeller. The direction of water flow at the outer periphery is more radial for a turbine runner than for a pump-turbine impeller. The throat diameter ($D_2$) will be nearly equal for either a turbine runner or pump-turbine impeller operating at the same head and speed. As indicated on figure 11, the throat diameter ($D_2$) of a pump-turbine is approximately 10 percent greater than that of a comparable centrifugal pump for the same specific speed. The dimensional expressions of diameters shown on figure 1 were derived from figures 10 through 13.

## Effects of Specific Speed on Pump Performance

Figure 3 shows the range for operating head versus pump specific speed of many worldwide

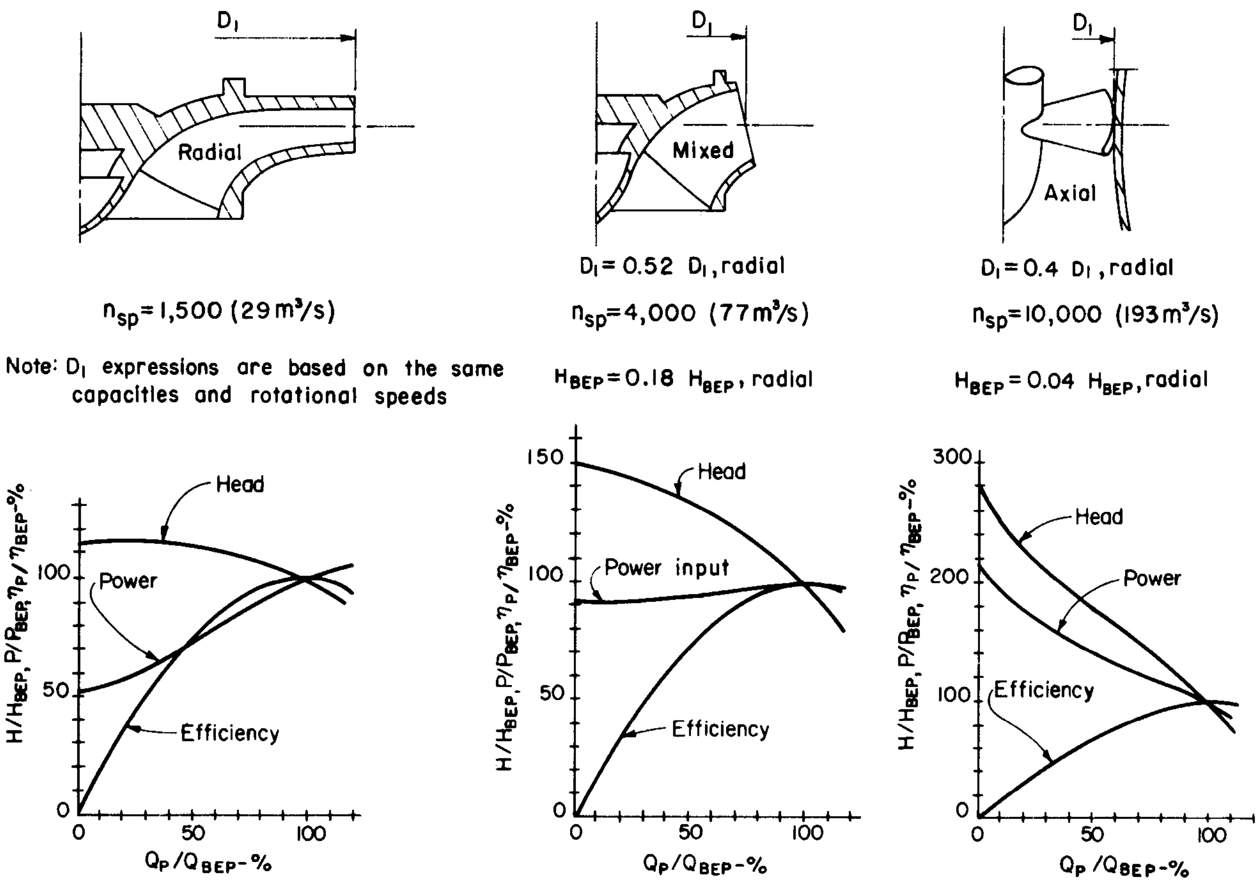


FIGURE 4.—*Comparison of pump impellers and characteristic curves with specific speed. 106-D-359.*

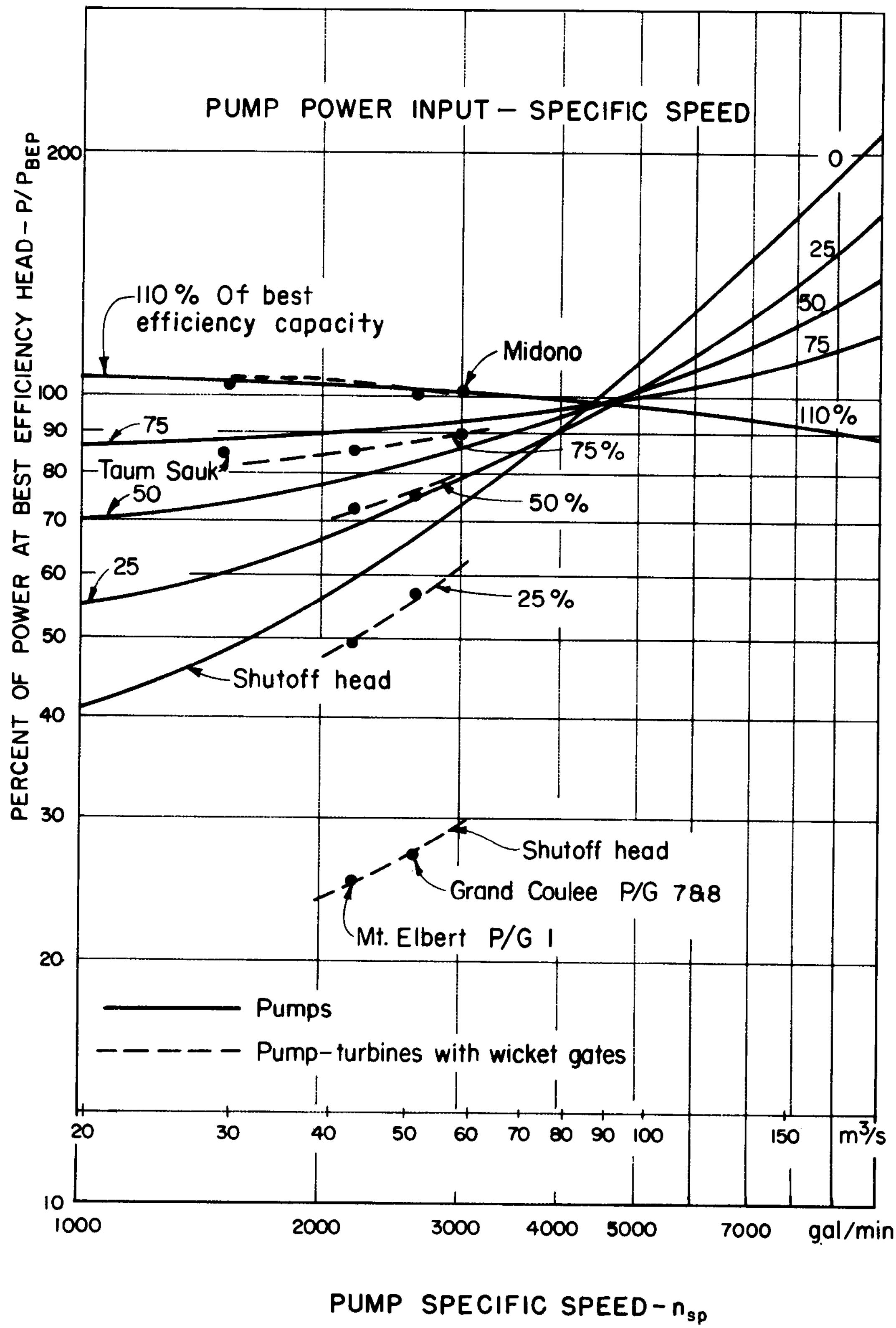


FIGURE 5.—*Percent pump input vs. specific speed. 106-D-360.*

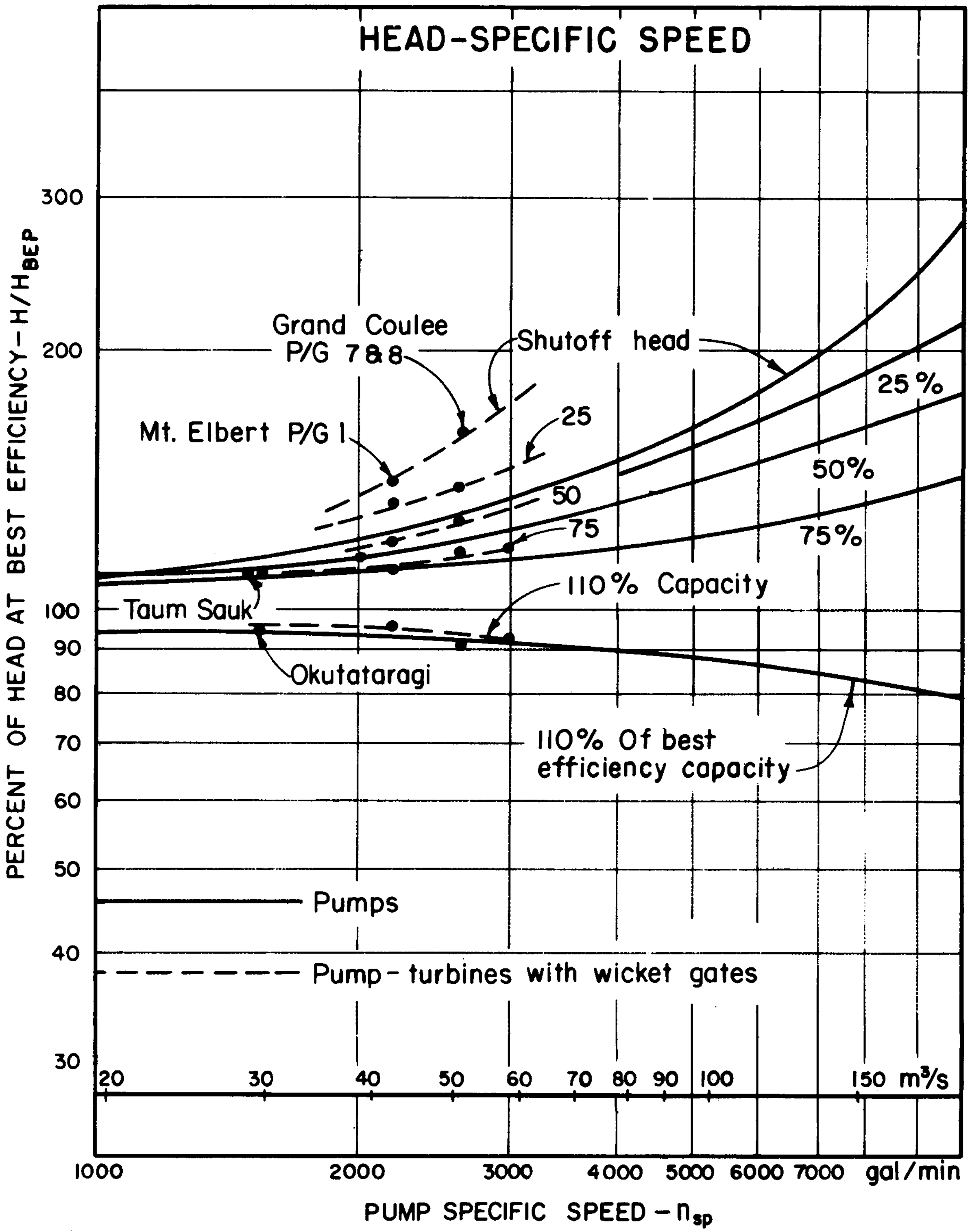


FIGURE 6.—*Percent pump head vs. specific speed. 106-D-361*

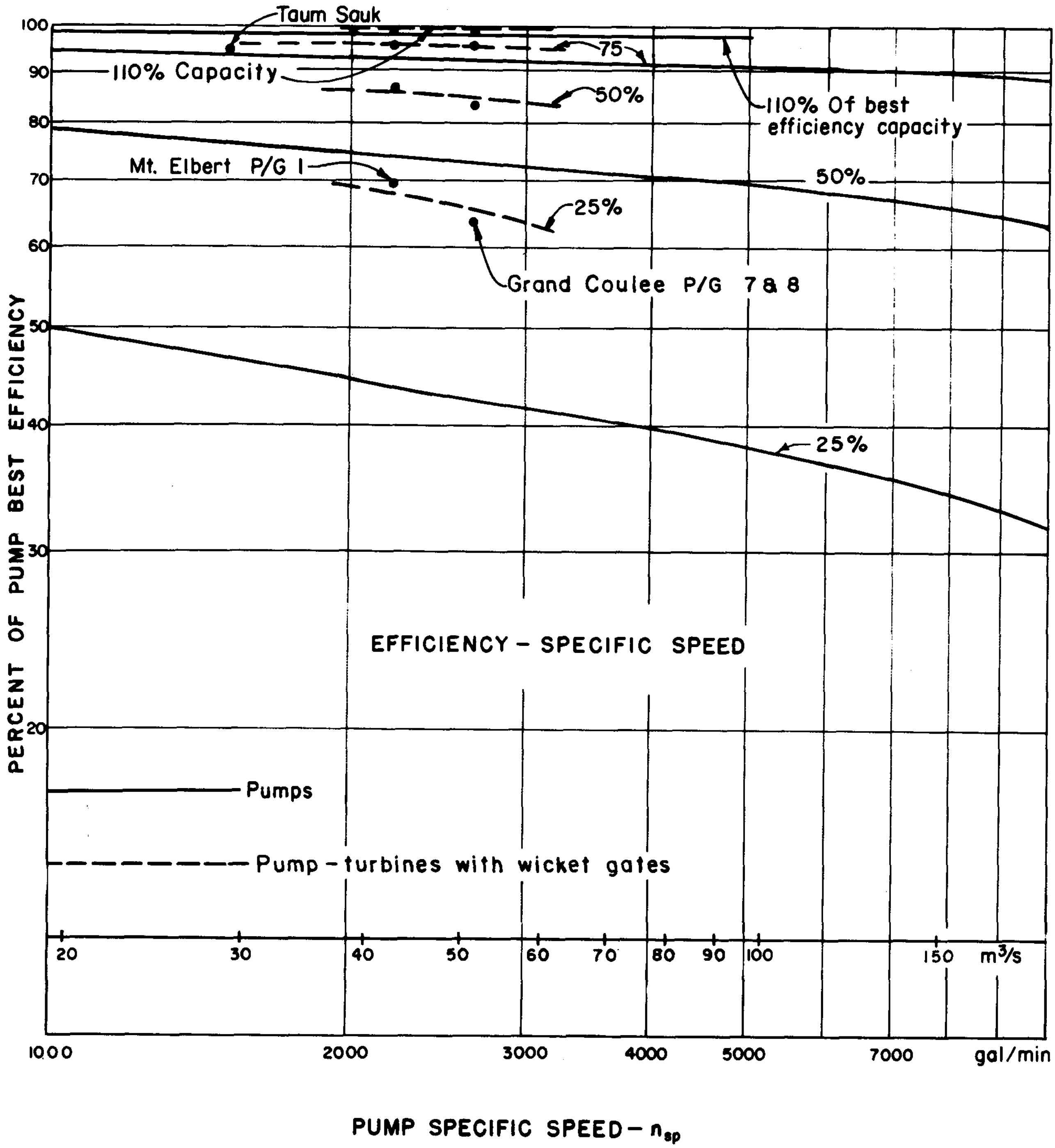


FIGURE 7.—*Percent pump efficiency vs. specific speed. 106-D-362.*

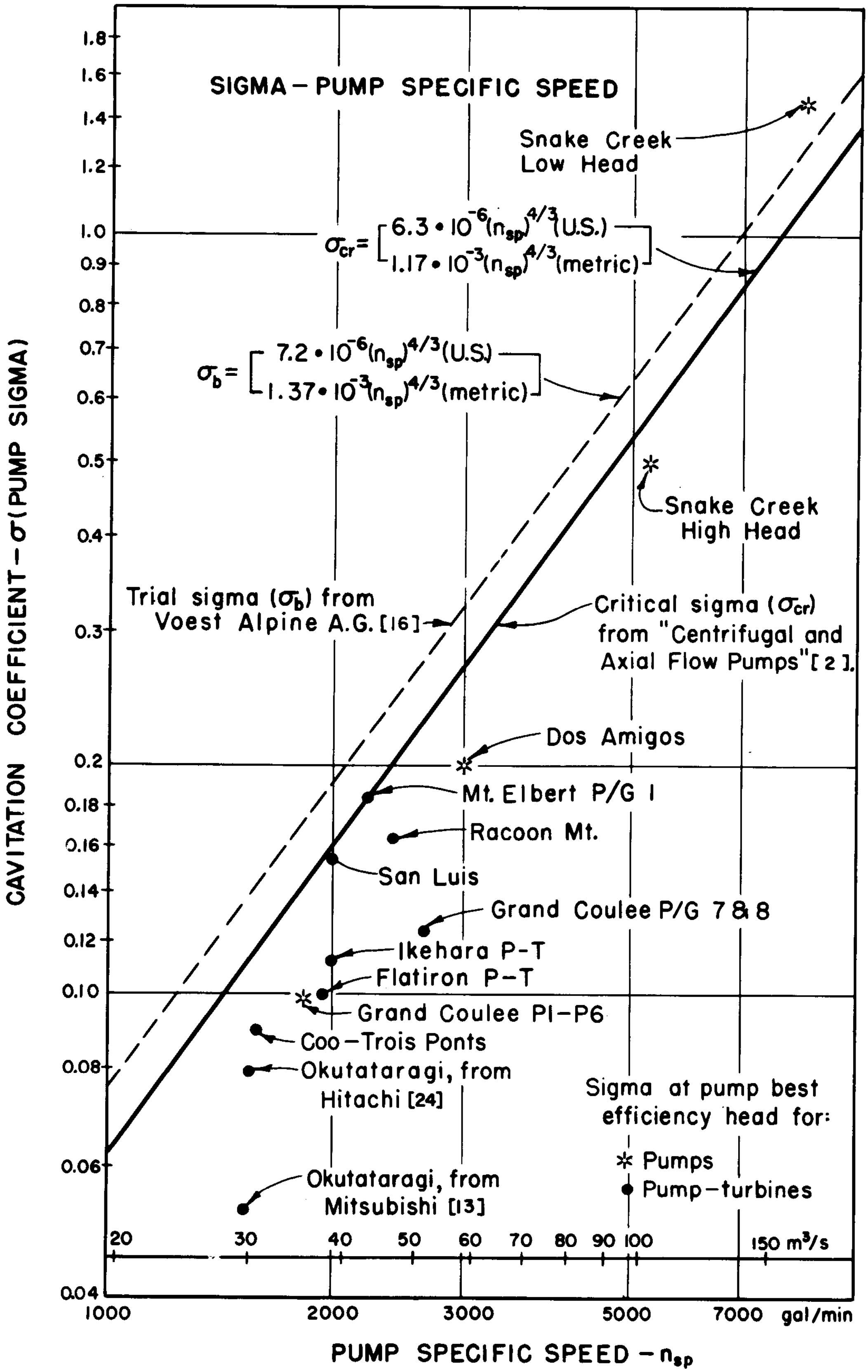


FIGURE 8.—*Cavitation coefficient (pump sigma) vs. specific speed. 106-D-363.*

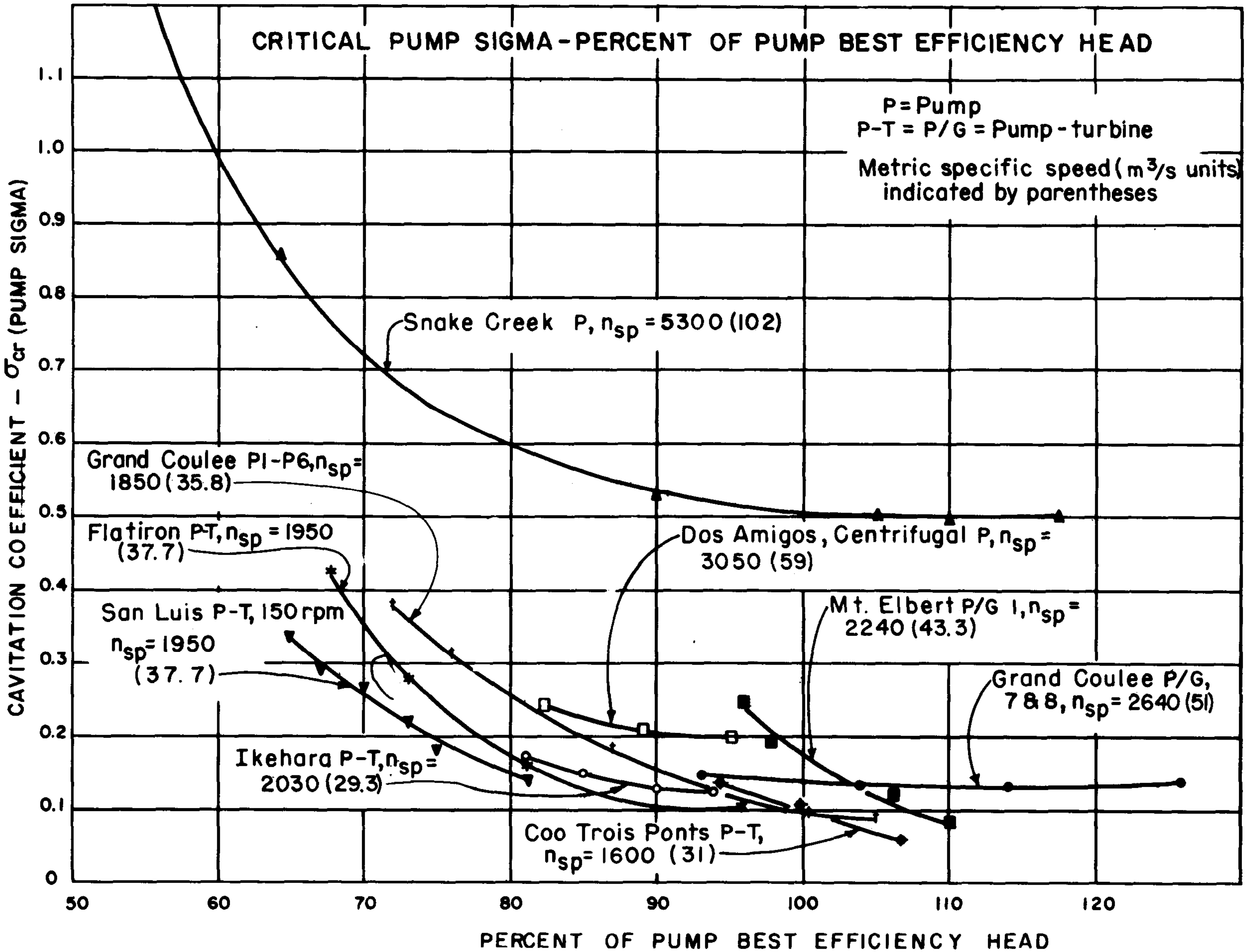


FIGURE 9.—*Critical pump sigma vs. percent of pump best efficiency head. 106-D-364.*

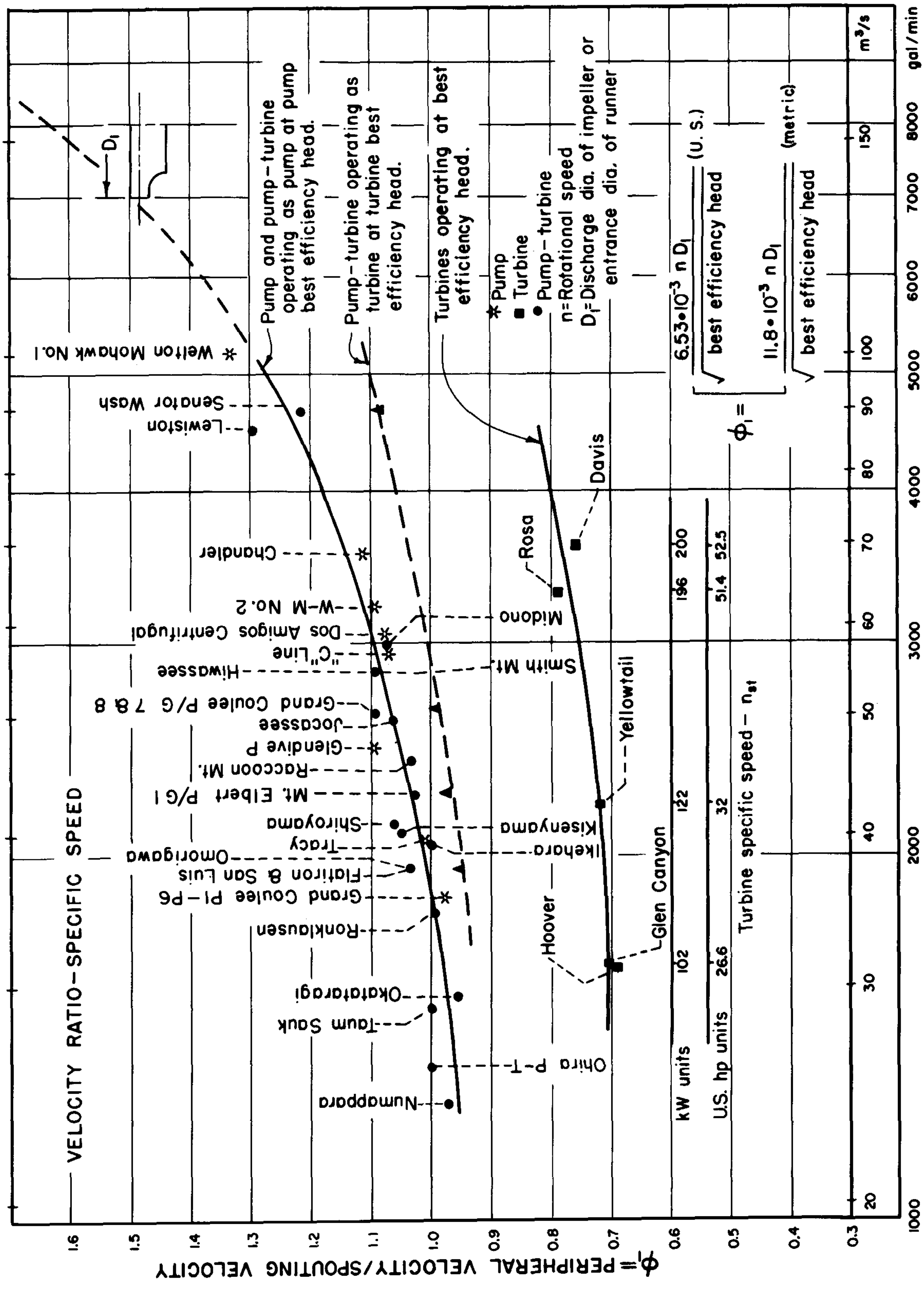


FIGURE 10.—*Velocity ratio* $\phi_1$ *vs. specific speed. 106-D-365.*

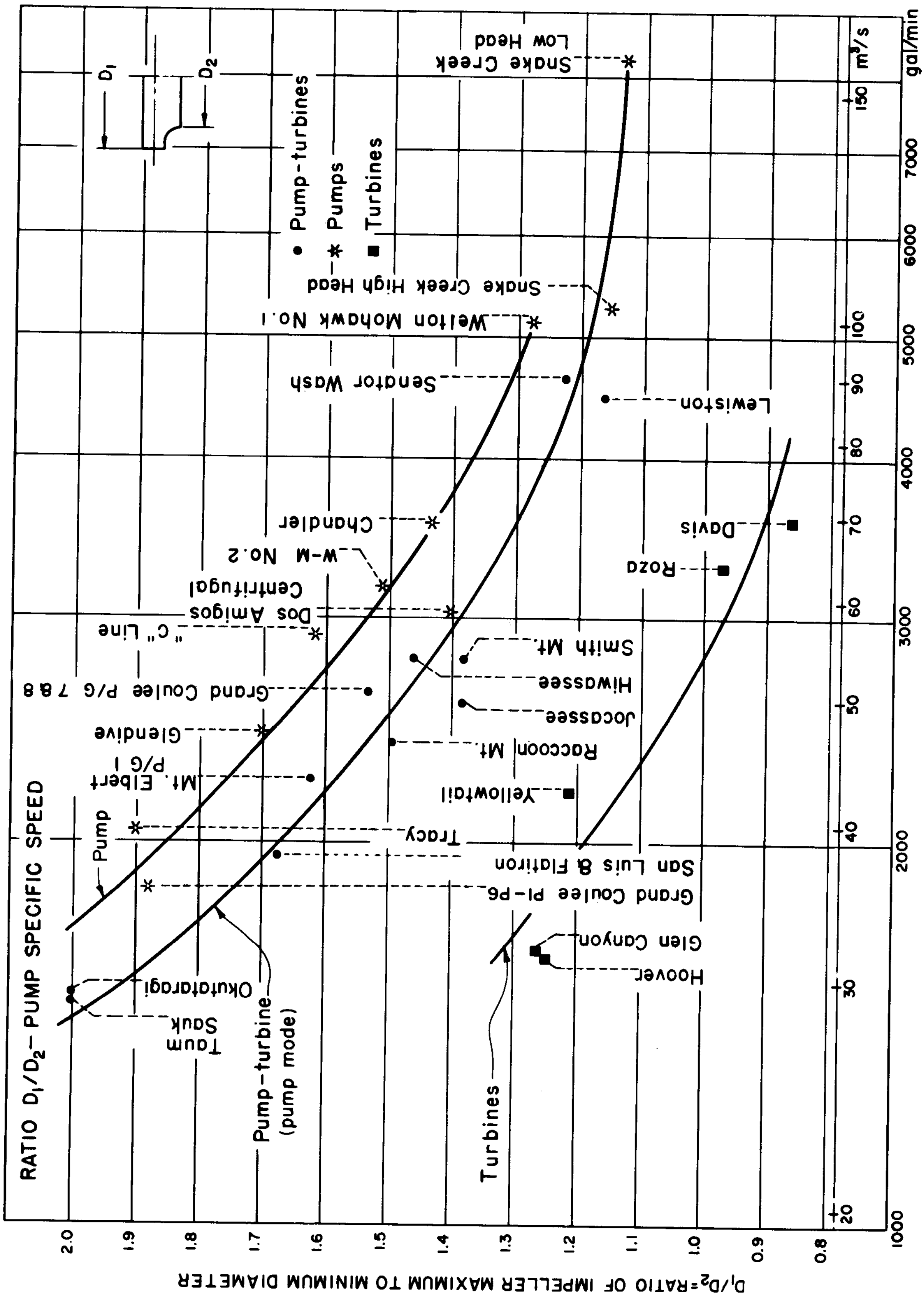


FIGURE 11.—*Ratio of impeller discharge diameter to throat diameter vs. specific speed. 106-D-366.*

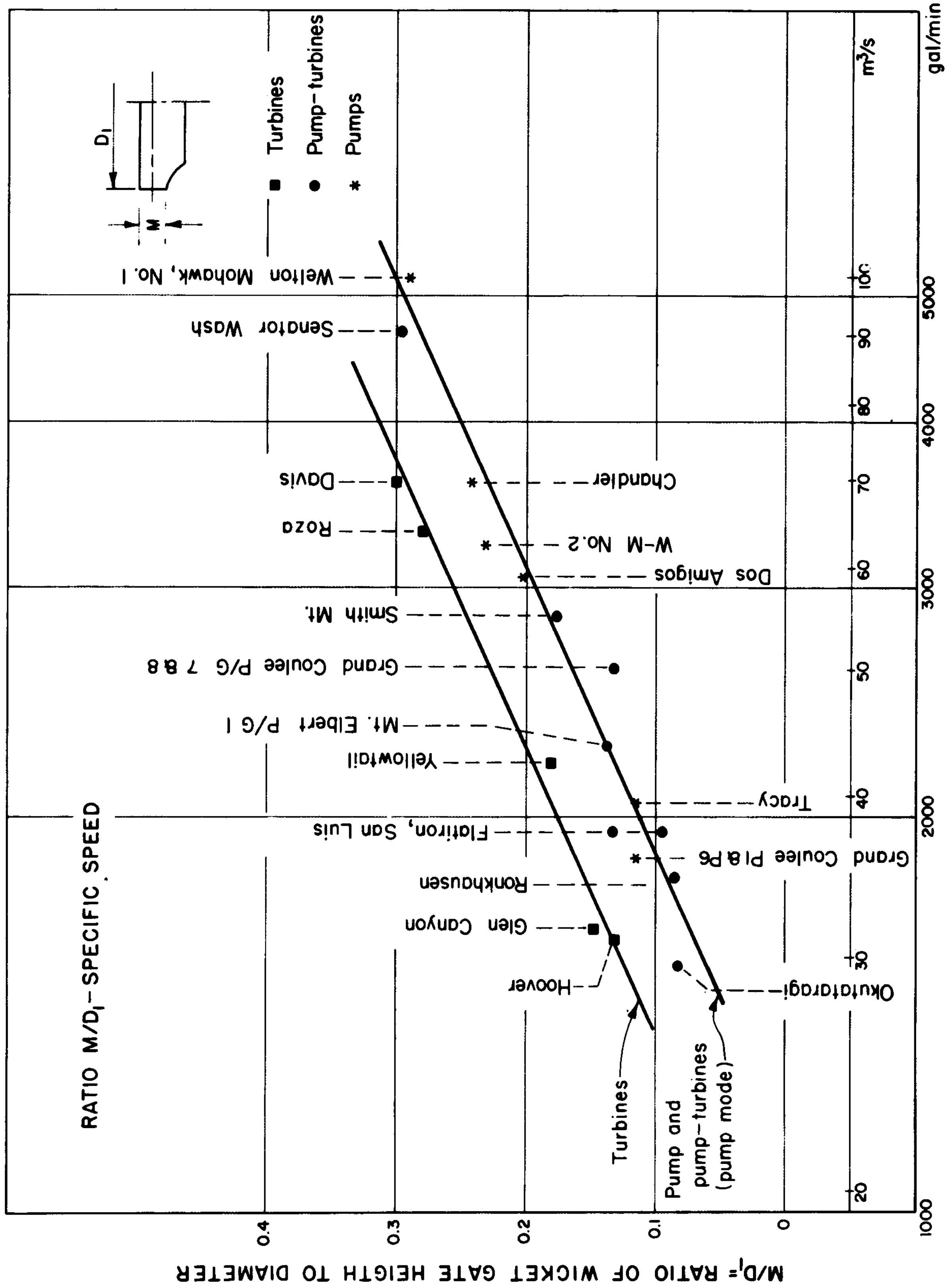


FIGURE 12.—*Ratio of wicket gate height to impeller discharge diameter vs. specific speed. 106-D-367.*

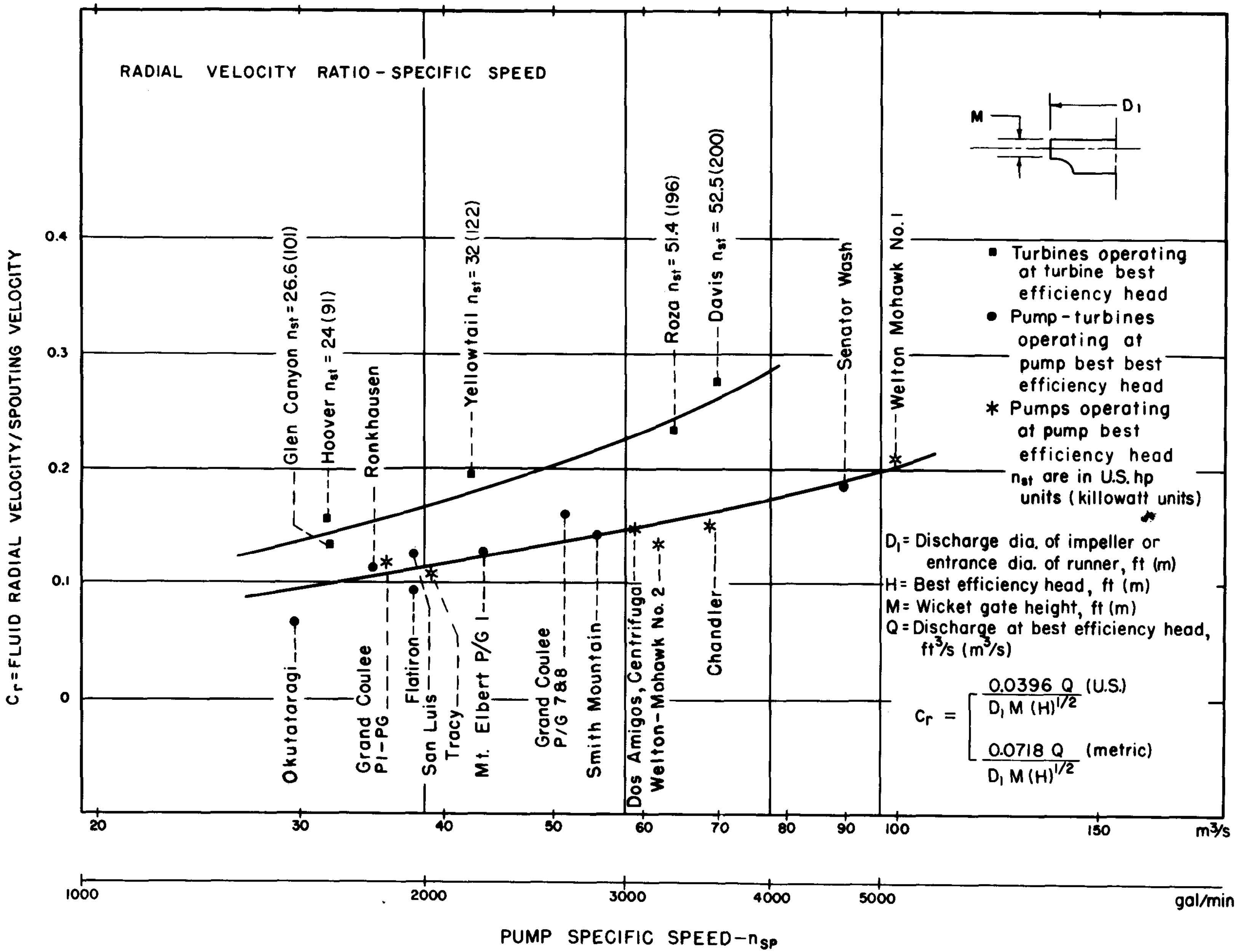


**FIGURE 13.**—*Ratio of fluid radial velocity to spouting velocity vs. specific speed. 106-D-368.*

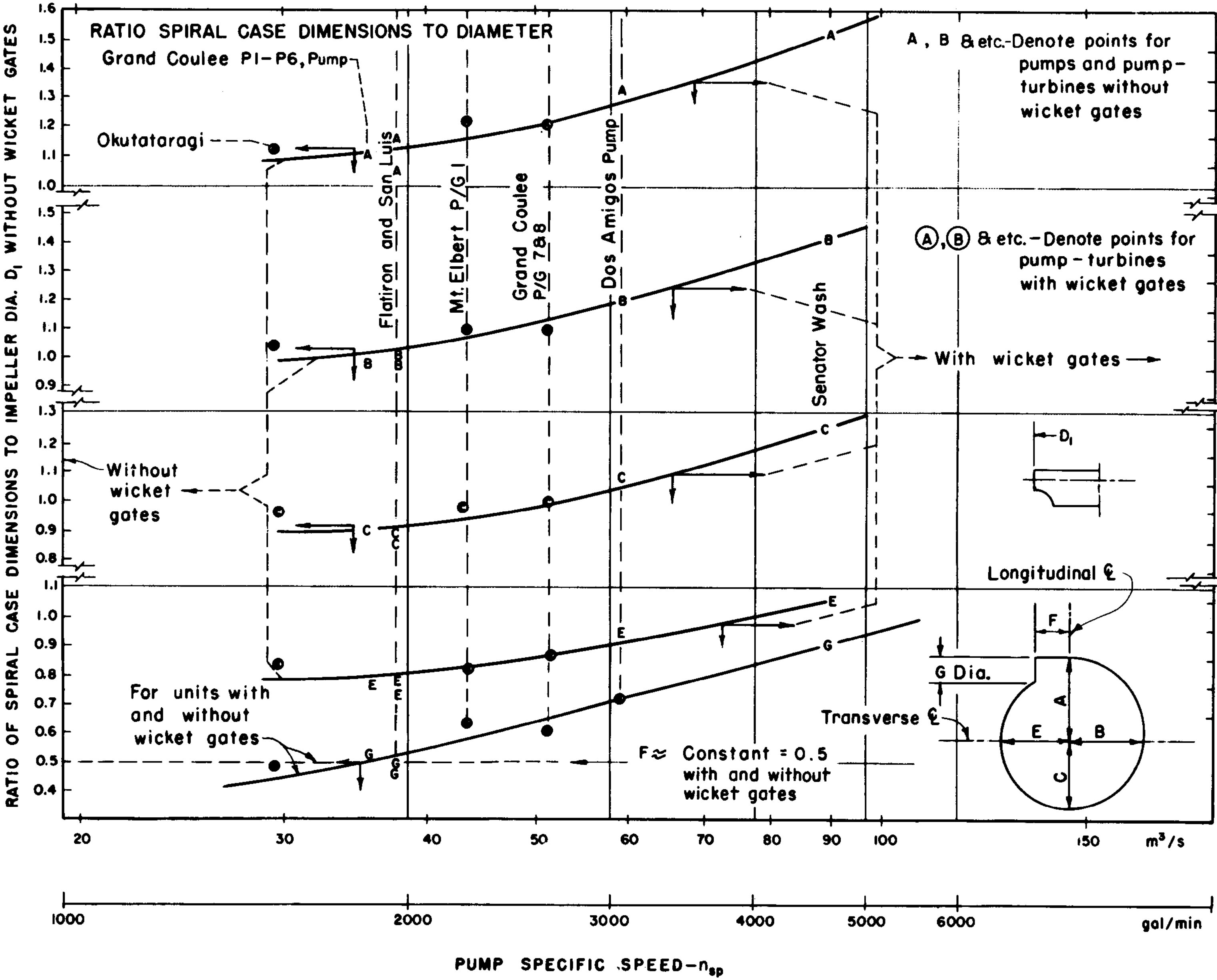


FIGURE 14.—*Ratio of spiral case dimensions to impeller discharge diameter vs. specific speed. 106-D-369.*

pump-turbine units. On figure 3, the term $n_{sp}\sqrt{H}$ plots as a straight line of a constant value. Pump-turbine manufacturers consider this term a basic design parameter, in addition to head and size. There is not a theoretical limit to this parameter. The curves show that the pump-turbine manufacturers' present experience limit is:

$n_{sp}H^{1/2}$=70,000 ft-gal/min (U.S. units) (5)
$n_{sp}H^{1/2}$=750 m-m³/s (metric units)

When deep submergence is prohibitively expensive, the limiting factor will be a lower value of the experience limit, $n_{sp}\sqrt{H}$.

Figure 4 compares impeller geometry and characteristic curves of three different pumps with different specific speeds and having like rotational speeds and capacities. The general category of centrifugal pumps is:

1. Centrifugal pumps (radial-flow) with radial discharge having low values of specific speed—less than 4,000 gal/min (77 $m^3/s$). The hydraulic passages in centrifugal pump impellers are relatively long and the cross sections relatively small. As the specific speed decreases, the passages become longer.
2. Diagonal flow (mixed-flow) pumps having specific speeds in the range of 4,000 to 8,000 gal/min (77 to 155 $m^3/s$). They have axial and radial components of velocity at the impeller discharge.
3. Propeller pumps (axial-flow) with axial discharge having specific speeds greater than 8,000 gal/min (155 $m^3/s$). The hydraulic passages through the impeller are relatively short and larger in cross section.

Upon comparison of low specific speed pumps with high specific speed pumps, figure 4 shows that for a high specific speed pump, the best efficiency point occurs near zero head and maximum discharge. The slope of the head versus discharge curve for a high specific speed pump will rise relatively steep from the best efficiency point to shutoff head, and the power input will increase with increasing head. For a low specific speed pump, the best efficiency point will occur near the shutoff head. The lower the specific speed, the less becomes the slope of the head versus discharge curve from best efficiency head to shutoff head. The maximum power input to the pump usually will occur just below best efficiency head and will decrease with increasing head.

Generally, the higher the specific speed of the pump, the smaller the physical dimensions for a given capacity and head. Pumps that have the highest attained efficiency are those with specific speeds approximating 2,500 gal/min (48 $m^3/s$). To prevent cavitation at a given head, the submergence required for a pump will increase as specific speed increases.

## Effects of Wicket Gates on Pump Performance

Although wicket gates are provided on pump-turbines primarily for controlling turbine load, the plot of pump characteristics for pump-turbines versus wicket gates (represented by dashed lines on figs. 5, 6, and 7), also shows that wicket gates are beneficial for pump operation. During pump operation, the position of the wicket gates is usually set at the point where the best efficiency can be obtained—at the given head—with decreasing gate opening correction for an increasing head. Under certain conditions, such as pumping at low head, the gate position is adjusted to decrease the power input or to reduce capacity for preventing undue cavitation.

When operating at heads other than best efficiency head, the decrease in pump efficiency for pump-turbines with wicket gates is less than the decrease in efficiency of pumps without wicket gates. Also, at shutoff head the conversion of velocity head to pressure head is more efficient for a pump-turbine with wicket gates; thus resulting in a higher shutoff head (gates closed) and a reduced power input than for a comparable pump without wicket gates.

## Submergence of Unit

To prevent excessive cavitation in pump-turbines, submergence requirements are more demanding for pump operation than for turbine operation. The term customarily used for describing the effects of cavitation on unit performance or similarity of cavitation conditions between machines having similar geometry and hydraulic characteristics is defined as the Thoma sigma ($\sigma$):

$$\sigma = \frac{NPSH}{H} \qquad (6)$$

where:

H=head developed (pump) or absorbed (turbine), ft (m),
NPSH=net positive suction head at some location, ft (m), and
$\sigma_{cr}$=critical sigma value at which there is an abrupt decrease in performance of the hydraulic machine.

NPSH (net positive suction head) of the pump is defined as:

$$NPSH = H_s + H_a - H_v - H_f \qquad (7)$$

where:

$H_s$=static draft head with reference to tailwater surface above impeller/runner centerline, ft (m),
$H_a$=atmospheric pressure head for altitude at pump suction supply, ft (m),
$H_v$=vapor pressure head of water for highest expected temperature, ft (m), and
$H_f$=head loss in the pump suction line and impeller approach, ft (m).

Table 4 compares critical sigmas at the same head for both pump and turbine operation for three of the Bureau's reversible units. The values were obtained from manufacturers' model tests.

TABLE 4.—*Comparison of pump and turbine critical sigmas*

| Pump-turbine | Pump BE head | | Turbine $\sigma_{cr}$ | Pump $\sigma_{cr}$ |
|---|---|---|---|---|
| | ft | m | | |
| Flatiron | 250 | 76.2 | 0.05 | 0.10 |
| Grand Coulee | 287 | 87.5 | .07 | .13 |
| Mt. Elbert | 440 | 134.1 | .08 | .17 |

The table reveals that substantially more submergence is required for pump operation than for turbine operation.

Figure 8 shows critical sigma as a function of specific speed at best efficiency pumping head. Although critical sigma increases with increasing specific speed, cavitation can be tolerated at the lower heads which are usually associated with high specific speed pumps. The destructive effects of cavitation increase with the cube of the head [7].

Figure 9 shows how critical sigma varies with head for several large pumps and pump-turbines. The relationship of critical sigma with specific speed shown by the expressions on figure 8 can be considered a general law, but the design of an impeller for a particular specific speed can be varied to some extent by varying the throat diameter and the impeller vane entrance angle to obtain the desired suction performance. This variation in design of suction performance is the reason sigma curves cross each other, as shown on figure 9, for impellers of different specific speeds. The interrelations, nevertheless, can be approximated by the general law of critical sigma versus specific speed. Suction performance can be improved by sacrificing pump efficiency [16].

Localized cavitation usually occurs at sigma values higher than the critical sigma values. Although localized cavitation is too limited to have an appreciable effect on the pump efficiency, it is of concern since sufficient material can erode from an impeller to affect the structural integrity of the impeller. The Bureau defines excessive cavitation as the removal of 0.00004 pounds of metal per operating hour per square foot of impeller/runner throat area (at $D_2$) or:

$$\text{Pounds} = 4.0 \cdot 10^{-5} \text{ (hours) (area, ft}^2\text{)} \qquad (8)$$
$$\text{Kilograms} = 19.5 \cdot 10^{-5} \text{ (hours) (area, m}^2\text{)}$$

The above formula is for aluminum bronze and stainless steel. For carbon steel the allowable rate of metal removal is four times greater.

The conditions at which cavitation is first observed in pump model tests, using a transparent suction tube, are defined as sigma begin ($\sigma_b$). The dashed line on figure 8 represents the sigma begin values obtained by a hydraulic equipment manufacturer [16].

As a compromise between the deep setting required of pumps for eliminating cavitation and the likelihood of attendant excavation and structural costs, units under heads of 400 ft

(122 m) are designed with the intent that they will be exposed to limited cavitation. Avoiding cavitation becomes more critical at heads above 400 ft. Because of the relative compactness of high head hydraulic equipment, it is feasible to excavate underground caverns to obtain the required submergence. Figure 3 indicates that the trend is toward underground plants at high heads.

The available submergence at the plant should be studied over the operating head range to ensure that the plant sigma is greater than critical sigma for the major portion of the operating schedule. Inadequate submergence usually presents problems at the maximum and minimum of the operating head range. A pump-turbine that has adequate submergence for pump operation usually has sufficient submergence for turbine operation. Critical sigma for turbine operation at maximum head and minimum tailwater may be greater than the pump critical sigma for this condition.

Cavitation, noise, and vibration will occur regardless of the available submergence when operating under conditions greatly removed from the best efficiency point because of the difference in orientation between the fluid velocity vector and the impeller vane tip at the point where fluid enters the impeller.

## Sizing the Pump-Turbine

Figures 10, 11, and 12 show several impeller characteristics as a function of specific speed. Figure 10, which shows a plot of the ratio of peripheral velocity at $D_1$ to the water spouting velocity as a function of specific speed, is of particular interest. This velocity ratio is commonly represented by the symbol $\phi_1$. The value of $\phi_1$ defines the relationship between the best efficiency head, rotational speed, and diameter of the impeller in the following expression:

$$\phi_1 = \frac{\pi n D_1}{60(2g H_{BE})^{1/2}} \quad (9)$$

$$= \frac{6.53 \cdot 10^{-3} n D_1}{(H_{BE})^{1/2}} \text{ (U.S.)}$$

$$= \frac{11.8 \cdot 10^{-3} n D_1}{(H_{BE})^{1/2}} \text{ (metric)}$$

where:

$n$=rotational speed, r/min,

$D_1$=discharge diameter of impeller or entrance diameter of runner, ft (m), and

$H_{BE}$=best efficiency head developed (pump) or absorbed (turbine), ft (m)

The rotational speed (n) of a generator-motor is given by the equation:

$$n = \frac{60 f}{p/2} \quad (10)$$

where:

f=frequency, Hz

p=an even integer representing the number of poles in the generator-motor. For large machines p should be a multiple of four.

*Hypothetical Case*

An example for estimating pump-turbine characteristics follows:

A pump-turbine is planned with design conditions of:

| | |
|---|---|
| Output : | 100 MW, |
| Head : | 1,000 ft (305 m), |
| Turbine operating range : | 935 to 1,045 ft (285 to 319 m), and |
| Pump operating range : | 955 to 1,065 ft (291 to 325 m). |

*Assumptions:*

- That 1,000 ft (305 m) is the pump best (peak) efficiency head,
- That turbine efficiency is 89 percent at full gate and design head,
- That pump efficiency is 92 percent at best efficiency head, and
- That a value of 32 ft (9.75 m) is used for $H_a$—$H_v$—$H_f$ (pump mode).

Notes:

The assumptions of efficiency values, based on figures in table 5, are reasonable for this size unit. The turbine efficiency is at the net head where the full-gate output of the turbine produces the generator design output (not the turbine best efficiency point, which usually occurs near maximum head).

Since a pump with a specific speed in the 2,500 gal/min (48.4 m³/s) range is the

TABLE 5.—*Reversible pump-turbine data.*

| Powerplant | Pump best efficiency head unless indicated Head H, Discharge, Q at $H_{BEP}$ | | Rotational speed | ** $n_{sp}$ at $H_{BEP}$ | Pump best (peak) efficiency | Pump input power at head H | $D_1$ and $D_2$ | Operating head range | Turbine maximum efficiency | Turbine full-gate efficiency at $H_{BEP}$ | Max. turbine output to pump input power at pump best efficiency head | Submergence distributor ℄ to minimum tailwater | Manufacturer |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ft (m) | ft³/min (m³/s) | r/min | gal/min (m³/s) | percent | hp (MW) | ft (m) | ft (m) | percent | percent | $P_t / P_{BEP}$ | ft (m) | |
| Ohira | 1635 (498) | 1680 (47.6) | 400 | 1340 (25.9) | | | 15.7 (4.78) | 1540-1790<br>(469.4-545.6) | | | | | T |
| Blenheim Gilboa | 1080Min (329) | 2880 (81.6) | 257 | 1770Max (34.2) | | | 19.6 (6.0) | 1080-1170<br>(329.2-356.6) | | | | 50-90<br>(15.2-27.4) | H |
| Coo-Trois Ponts | 850R (259) | 1620R (45,9) | 300 | 1600 $h_r$ (30.9) | | | 14.75 (4.5)<br>7.25 (2.2) | 772-920<br>(235.3-280.4) | | | | 59-85<br>(18-25.9) | A-C |
| Okutataragi | 1250 (381) | 2600 (73.6) | 300 | 1530 (29.6) | 89 | 410,000 (306) | 18.5 (5.64)<br>9.18 (2.8) | 1187-1362<br>(361.8-415.1) | 89 | 88 | 0.95 | 152-250<br>(46.3-76.2) | M,H |
| Cruachan | 1094Min (333) | 1126 (31.9) | 500 | 1850Max (36) | | | 10.5 (3.2) | 1094-1205<br>(333.4-367.3) | | | | 148MIN-152 Norm<br>(45.1-46.3) | B |
| Cruachan | do | do | 600 | 2200Max (43) | | | 9.0 (2.74) | | | | | | EE |
| Ronkhausen | 830Min (252)<br>910Max (277) | 840 (23.8)<br>690 (19.6) | 500 | 2000 (38.7)<br>1600 (30.9) | | 89,000 (66.4)<br>79,500 (59.2) | 9.05 (2.76) | 830-910<br>(253.0-277.4) | | | 1.10 | | EW |
| Ronkhausen | do | 800 (22.6)<br>680 (19.3) | 500 | | | 84,000 (62.6)<br>79,000 (58.9) | 9.18 (2.8) | " | | | 1.22 | | S |
| Bear Swamp | 685Min (208) | 4430 (125.5) | 225 | 2350Max (45.4) | 91.5 | | 19.17 (5.8) | 685-770<br>(208.8-234.7) | 92.5 | | | 70-100<br>(21.3-30.5) | H |
| Castaic | 870Min (265) | 3220 (91.2) | 257 | 1930Max (37.3) | | | 19.16 (5.8 ) | 870-1250<br>(265.2-381) | | | | | H |
| Cabin Creek | 1238R (377) | 840R (23.8) | 360 | 1100 $h_r$ (21.3) | | | 13.7 (4.17)<br>6.75 (2.05) | 1093-1226<br>(333.1-373.7) | | | | | A-C |
| Raccoon Mt. | 955 (291) | 4400 (125) | 300 | 2440 (47.2) | 93 | 520,000 (388) | 16.2 (4.93)<br>10.8 (3.29) | 645-1040<br>(257.6-317) | 93 | 88 | 1.02 | 135 Normal<br>(41.1) | A-C |
| Kisenyama | 650Min (198)<br>695 (212) | 3900 (110)<br>3500 (99.2) | 225 | 2080 (40.2) | 90 | 310,000 (231) | 18.75 (5.71) | 630-720<br>(191-219.4) | | | | | H |
| Shiroyama | 510 (155) | 1325 (37.5) | 300 | 2120 (41) | 90 | 85,000 (63.4) | 12.3 (3.75) | 420-607<br>(128-185) | 90 | 89 | 1.03 | | H |
| Yards Creek | 691Min (210) | 2175 (61.6) | 240 | 1750Max (33.8) | | 186,000 (139) | 8.6 (2.6) | 656-730<br>200-222.5 | | | | 25 Min<br>(7.62) | BLH |
| Taum Sauk | 760 (232) | 2700 (76.5) | 200 | 1500 (29) | 91.5 | 260,000 (194) | 21.2 (6.46)<br>10.6 (3.23) | | 91 | 87 | 1.13 | | A-C |
| Numappara | 1600 (487) | 1770 (50.2) | 375 | 1300 (25) | | 336,000 (250) | 16.3 (4.95) | 1600-1730<br>(487.7-527.3) | | | | 150-256<br>45.7-78) | H |
| | | | | | | | | | | | | | |
| Shin-Naragawa | 261 (80) | 2840 (80.5) | 144 | 2500 (48.3) | 91 | 93,000 (69.3) | | 155-315<br>(47.2-96) | 91 | | 1.08 | 29.5-87<br>(9.0-26.5) | H |
| Ikehara | 384 (117) | 2120 (60.1) | 180 | 2010 (38.9) | 91 | 100,000 (74.6) | 16.7 (5.10) | 295-435<br>(89.9-132.6) | 94 | 91 | 1.0 | 29.5-93<br>(9.0-28.3) | H |
| Grand Coulee P/G7&8 | 287 (87) | 1900 (53.8) | 200 | 2640 (51) | 91.6 | 67,200 (50.1) | 14.3 (4.36)<br>9.35 (2.84) | 263-365<br>(80.2-111.2) | 91.6 | 88 | 1.08 | 5-87<br>(1.5-26.5) | N |
| Mt.Elbert P/G I | 440 (134) | 3200 (90.7) | 180 | 2240 (43.3) | 92.2 | 175,000 (130) | 18.3 (5.58)<br>11.3 (3.44) | 390-485<br>118.9-147.8) | 93 | 92 | 0.94 | 29-56.5<br>(8.8-17.2) | A-C |
| Midono | 240 (73) | 2700 (76.5) | 150 | 3000 (58) | 92.5 | 80,000 (59.6) | 17.1 (5.2) | 189-266<br>(57.6-81.1) | 91 | 91 | 1.0 | 26-72<br>(7.9-21.9) | H |
| Omorigawa | 360 (110) | 390 (11) | 400 | 1960 (37.9) | 89 | 18,100 (13.5) | 7.54 (2.3) | 276-416<br>(84.1-126.8) | 90 | 87 | 0.99 | 16.4-51.5<br>(5-15.7) | H |
| Ludington | 305Min (93) | 11,100 (314) | 112.5 | 3460Max (66.9) | | | 27.5 (8.375) | 292-363<br>(89-110.6) | | | | 25-29<br>(7.62-8.8) | H |
| Jocassee | 312 (95) | 5740 (163) | 120 | 2580 (49.9) | 94 | 218,000 (162) | 24 (7.31)<br>17.3 (5.28) | 280-344<br>(85.3-104.6) | 93.5 | 89 | 1.18 | 12 Min<br>(3.6) | A-C |
| Flatiron * | 250 (76) | 370 (10.5) | 300 Pump<br>257 Turbine | 1950 (37.7) | 91 | 11,600 (8.6) | 8.42 (2.57)<br>5.05 (1.54) | 140-300<br>(42.7-91.4) | 89 | 88 | 0.93 | 9-25<br>(2.7-7.6) | A-C |
| San Luis 150* | 272 (83) | 17,00 (48.2) | 150 | 1930 (37.3) | 92.5 | 57,000 (42.5) | 17.7 (5.39)<br>10.6 (3.23) | 200-332<br>(61-101.2) | 92 | 90 | 0.88 | 15-23<br>(4.6-7) | H |
| | | | | | | | | | | | | | |
| San Luis 120* | 180 (54.8) | 1300 (36.8) | 120 | 1880 (36.4) | 92.5 | 29,000 (21.6) | 17.7 (5.39)<br>10.6 (3.23) | 109-260<br>(33.2-79.2) | 92 | 90 | 1.0 | 15-23<br>(4.6-7) | H |
| Senator Wash * | 64 (19.5) | 194 (5.5) | 360 | 4650 (89.9) | 87 | 1,650 (1.23) | 4.16 (1.27)<br>3.39 (1.03) | 27-74<br>(8.2-22.5) | 80 | 75 | 0.67 | | F-B |
| Smith Mt. | 196 (59.7) | 4250 (120) | 105.9 | 2780 (53.8) | 93.5 | 101,000 (75.3) | 22.17 (6.76)<br>16 (4.88) | 167.5-205<br>(51.1-62.5) | 92 | 90 | 1.08 | | A-C |
| Hiwassee | 190 (57.9) | 4320 (122) | 105.9 | 2800 (54.2) | 93 | 101,000 (75.3) | 22.17 (6.76)<br>15.17 (4.62) | 135-254<br>(41.1-77.4) | 93 | 91 | 0.89 | 1-9<br>(0.3-2.7) | A-C |
| Lewiston | 94 (28.6) | 3200 (90.7) | 112.5 | 4500 (87) | 93 | 36,600 (26.8) | 17.17 (5.23)<br>14.83 (4.52) | 57-99<br>(17.4-30.2) | 92 | 89 | 1.11 | | A-C |

* Units without wicket gates.

** gal/min and m³/s are units representing U.S. and metric specific speeds.

$h_r$ = Rated head

A-C-Allis Chalmers
BLH-Baldwin Lima Hamilton
B -Boving
EE -English Electric
EW -Escher Wyss
F-B-Fairbanks Morse
H -Hitachi
M -Mitsubishi
N -Nohab
T -Toshiba

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most efficient, this value is assumed for the first trial.

In the example, limitations of size, speed, or submergence are not placed on the unit selection.

The turbine discharge ($Q_t$) at full gate is calculated from:

$$P = MW = \frac{Q_t H \gamma \eta_t}{(1{,}341)\ 550} \text{(U.S.)} \qquad (11)$$

$$MW = \frac{Q_t H \gamma \eta_t}{102\ (1{,}000)} \text{(metric)}$$

$$Q_t = \frac{MW\ (1{,}341)}{0.1134\ h_d\ \eta_t} = \frac{100\ (1{,}341)}{0.1134\ \ (1{,}000)\ \ 0.89} = 1{,}329\ \text{ft}^3/\text{s}$$

$$= \frac{MW}{9.804 \cdot 10^{-3}\ h_d\ \eta_t} = \frac{100}{9.804 \cdot 10^{-3}\ (305)\ 0.89} = 37.6\ \text{m}^3/\text{s}$$

Referring again to table 5, an assumed ratio of unity is used for turbine output to pump input at design head. This power ratio ($P_t/P_{BEP}$) can be adjusted, within limits, by the pump-turbine design or wicket gate position to obtain a balance between motor input and generator output over the operating range.

The pump discharge ($Q_{BE}$) calculation is:

$$P = MW = \frac{Q_{BE}\ H_{BE}\ \gamma}{\eta_p\ (1{,}341)\ 550} \text{(U.S.)} \ (12)$$

$$MW = \frac{Q_{BE}\ H_{BE}\ \gamma}{\eta_p\ (102)\ 1{,}000} \text{(metric)}$$

$$Q_{BE} = \frac{MW\ 1{,}341\ \eta_p}{0.1134\ H_{BE}} = \frac{100\ (1{,}341)\ 0.92}{(0.1134)\ 1{,}000} = 1{,}088\ \text{ft}^3/\text{s}$$

$$= \frac{MW\ \eta_p}{9.804 \cdot 10^{-3}\ H_{BE}} = \frac{100\ (0.92)}{9.804 \cdot 10^{-3}\ (305)} = 30.8\ \text{m}^3/\text{s}$$

The pump rotational speed (n) is calculated from the specific speed formula and, of course, is the same for both customary and metric calculations:

$$n = \frac{n_{sp}(H_{BE})^{3/4}}{(Q_{BE})^{1/2}} = \frac{2{,}500\ (1{,}000)^{3/4}}{(60 \cdot 7.48 \cdot 1{,}088)^{1/2}} = 636\ \text{r/min}$$

$$= \frac{n_{sp}(H_{BE})^{3/4}}{(Q_{BE})^{1/2}} = \frac{48.4\ (305)^{3/4}}{(30.8)^{1/2}} = 636\ \text{r/min}$$

The rotational (synchronous) speeds close to 636 are 600 and 720 r/min. Because $n\sqrt{H}$ value of 80,000 gal/min (852 m³/s) (see fig. 3) is beyond the current experience limit at a specific speed of 2,500 gal/min (48.4 m³/s), a rotational speed of 600 r/min is selected. Also, the number of poles which gives 600 r/min is divisible by four. To maintain best efficiency head and discharge for the design conditions, pump specific speed (the required parameter) should be adjusted by the ratio of rotational speeds:

$$n_{sp} = 2{,}500 \left(\frac{600}{636}\right) = 2{,}358\ \text{gal/min}\ (45.6\ \text{m}^3/\text{s})$$

If a specific speed value of 2,500 gal/min is used at 600 r/min, either pump best efficiency head or discharge and power will change. On figure 10, $\phi_1$ equals 1.04 at a pump specific speed of 2,358 gal/min.

The impeller/runner diameter ($D_1$) calculation is:

$$D_1 = \frac{\phi_1\ (H)^{1/2}}{6.53 \cdot 10^{-3}\ n} = \frac{1.04\ (1{,}000)^{1/2}}{6.53 \cdot 10^{-3}\ (600)} = 8.394\ \text{ft}$$

$$= \frac{\phi_1\ (H)^{1/2}}{11.8 \cdot 10^{-3}\ n} = \frac{1.04\ (305)^{1/2}}{11.8 \cdot 10^{-3}\ (600)} = 2.558\ \text{m}$$

Figures 11 and 12 are used to estimate the impeller throat diameter ($D_2$) and the wicket gate height (M), respectively:

Wicket gate height:

$$\frac{M}{D_1} = 0.145$$

thus:

$$M = 0.145\ (8.394) = 1.217\ \text{ft}\ (0.371\ \text{m})$$

Impeller throat diameter:

$$\frac{D_1}{D_2} = 1.54 \text{ or } D_2 = \frac{8.394}{1.54} = 5.451\ \text{ft}\ (1.661\ \text{m})$$

With reference to figure 14, spiral case dimensions are calculated:

A=1.3 (8.394)=10.912 ft (3.326 m) radius
B=1.2 (8.394)=10.073 ft (3.070 m) radius
C=1.1 (8.394)= 9.233 ft (2.814 m) radius
E=1.0 (8.394)= 8.394 ft (2.558 m) radius
G=0.6 (8.394)= 5.036 ft (1.535 m) dia

hence:

$$A+C=20.145 \text{ ft } (6.140 \text{ m}) \text{ dia}$$
$$B+E=18.467 \text{ ft } (5.628 \text{ m}) \text{ dia}$$

Therefore, longitudinal and transverse dimensions of the spiral case will be 20 by 18.5 ft (6.1 by 5.7 m), respectively.

Figure 8 and equations (6) and (7) are used to calculate the required minimum submergence (distributor centerline to minimum tailwater at best efficiency head):

$$\sigma_b=7.2 \cdot 10^{-6} (n_s)^{4/3}=7.2 \cdot 10^{-6} (2{,}358)^{4/3}=0.23$$

$$NPSH=H\sigma_b=1{,}000 (0.23)=230 \text{ ft } (70.1 \text{ m})$$

Let the sum of $H_a—H_v—H_f=32$ ft (9.75 m)

The required minimum submergence ($H_s$) at best efficiency head is:

$$H_s=230—32=198 \text{ ft } (60.4 \text{ m})$$

By comparing 198 ft to actual submergence values listed in table 5 for similar pump-turbine installations, it is apparent that the sigma begin ($\sigma_b$) curve is quite conservative. Frequently, sigma critical rather than sigma begin is used as a submergence criteria for determining the pump-turbine setting. Ideally, submergence calculations are based on the elevation at $D_2$. Because of the uncertainties of the exact impeller dimensions and sigma curve data, during preliminary studies, submergence is frequently based on distributor centerline elevation for convenience. Consideration must be given to the number of units operating (capacity vs. tailwater).

A number of trial solutions may be necessary to determine the submergence, as a compromise of interrelated parameters of speed, specific speed, unit size, and a submergence that is adequate for the conditions. To illustrate, if the above sample should have a limitation of 100 ft (30.5 m) maximum submergence, selection of a specific speed of approximately 1,400 gal/min (27.1 $m^3/s$) operating at 360 r/min would be required. To satisfy the same turbine power output, the lower specific speed unit would be about 50 percent larger in size.

# Performance

## Estimating Turbine Performance

Figure 15 has turbine characteristics superimposed on pump characteristics for a pump-turbine without wicket gates and a specific speed ($n_{sp}$) of 2,000 gal/min (38.7 m³/s). Figures 16 and 17 show turbine characteristics superimposed on pump characteristics for a pump-turbine with wicket gates and a specific speed of 2,640 gal/min (51.1 m³/s). Using the pump best efficiency head, discharge, and power values calculated in the previous example, pump and turbine performance can be determined from the curves at other heads. Note that turbine power output is given in terms of a ratio; i.e., turbine output to pump input ($P_t/P_p$) at the same head.

The maximum torque imposed on a pump or pump-turbine does not occur at a normal operating point. After a pump power failure, maximum torque occurs upon flow reversal followed by speed reversal. Although this torque is transient and can be reduced for a pump-turbine with a normally functioning governor by closure of the wicket gates, this torque should be considered in the design of the shaft and couplings.

The figures in this section are intended to aid in estimating turbine performance over an operating head range. Characteristics for these units can be applied to units of similar specific speeds. Also, allowance should be made for correction of the turbine output to pump input power ratio at pump best efficiency head, since the ratio can vary within a range of 90 to 120 percent for pump-turbines (see table 5).

## Estimating Runaway Speed

The curves on figure 18 give the ratio of reverse runaway speed to rotational speed versus specific speed for hydraulic machines. A curve is shown for pump power failure while operating under pump best efficiency head [8]. Reverse runaway speed for a pump is the same as runaway speed for a pump-turbine resulting from a turbine load rejection. In accordance with pump similarity laws, calculation of reverse runaway speed ($n_r$) for a unit at a head other than pump best efficiency head can be determined as follows:

$$n_r = n_{BEr} (H/H_{BE})^{1/2} \qquad (13)$$

The overspeed values determined from the curves on figure 18 represent steady-state overspeed with wicket gates at best efficiency position. An analysis that accounts for the water conduit and unit characteristics, as well as the rate of gate closure, is necessary for an accurate overspeed determination under transient conditions. However, the steady-state estimate is suitable for a first order approximation since the transient effects tend to cancel each other.

## Pump-Turbine Four-Quadrant Diagram

Figure 19 is a four-quadrant diagram of a pump-turbine with specific speed of 2,200 gal/min (42.6 m³/s). The diagram describes the relation between discharge, head, speed, and gate opening in four quadrants or modes of pump-turbine operation. The diagram includes all combinations of variables which a pump-turbine is likely to encounter. Head-discharge trends may be more readily interpreted from figures 16 and 17, as both coordinates of the four-quadrant diagram (fig.19) are shown as a ratio of two variables.

The quadrants are numbered in the sequence as direction of flow and sense of rotation encountered by the pump or pump-turbine after a pump power failure. The quadrants do not necessarily have to be followed in sequence to arrive at a particular state, as the head and magnitude and sense of rotational speed define the state of the system. The quadrants and conditions are described as follows:

- I. Normal Pumping
  - A. Flow in pump direction
  - B. Rotation in pump sense
- II. Energy Dissipation
  - A. Flow in turbine direction
  - B. Rotation in pump sense
- III. Turbine
  - A. Flow in turbine direction
  - B. Rotation in turbine sense
- IV. Reverse Pumping
  - A. Flow in pump direction
  - B. Rotation in turbine sense

A pump or pump-turbine will not go into the fourth quadrant after a power failure because the region above the zero torque (runaway) line requires a power input; however, if a vertical gate characteristic curve occurs in the zero torque zero flow regime, the machine is unstable and can operate in either the normal turbine (III) or reverse pumping (IV) quadrants for a given head and speed. A pump-turbine with a vertical gate characteristic curve will be unstable for the turbine speed-no-load gate condition. Since the unit is connected across the line, it can go into reverse pumping while drawing power from the system [10]. The situation probably occurs when the pump shutoff head (at small gate openings), with rotation in the turbine direction, exceeds the static hydraulic head.

Stepanoff [2] expressed the theoretical shutoff head ($H_{so}$) of an impeller as a function of the diameter and rotational speed, and is independent of the specific speed. Shutoff head can be calculated by the following formula:

$$H_{so}=k\left(\frac{V^2}{g}\right) \qquad (14)$$

where:

$g$=gravitational constant (acceleration), ft/s$^2$ (m/s$^2$),
$H_{so}$=shutoff head, ft (m),
$V$=peripheral velocity of the impeller at $D_1$, ft/s (m/s), and
$k$=an efficiency for the conversion of velocity head to pressure head (about 0.58 for pumps without wicket gates).

For pump-turbines with wicket gates, the shutoff heads are higher than values calculated with the above 0.58 efficiency. Presumably, this higher shutoff head occurs because the wicket gates restrict the volume in which the water circulates, resulting in a higher velocity to pressure conversion efficiency. Furthermore, the assumption of constant conversion efficiency regardless of specific speed does not seem strictly to apply to pump-turbines with wicket gates. The short, broad impeller passages of high specific speed pump-turbines probably allows more fluid circulation within the impeller passages at shutoff than do low specific speed pump-turbines.

The lower velocity to pressure conversion efficiency for high specific speed pump-turbines (compared to low specific speed pump-turbines), when rotating in the turbine direction, serves as an advantage for turbine operation because it permits turbine operation at a lower head for a given single speed.

In addition to a turbine efficiency improvement resulting from decreased turbine rotational speed as described in the speed section, figure 19 shows there is an increase in turbine power output in certain head ranges with a lower turbine speed. See figure 2, which is based on the same pump-turbine unit as the four-quadrant diagram (fig. 19).

The k values for the Bureau's two pump-turbine installations with wicket gates are shown in table 6.

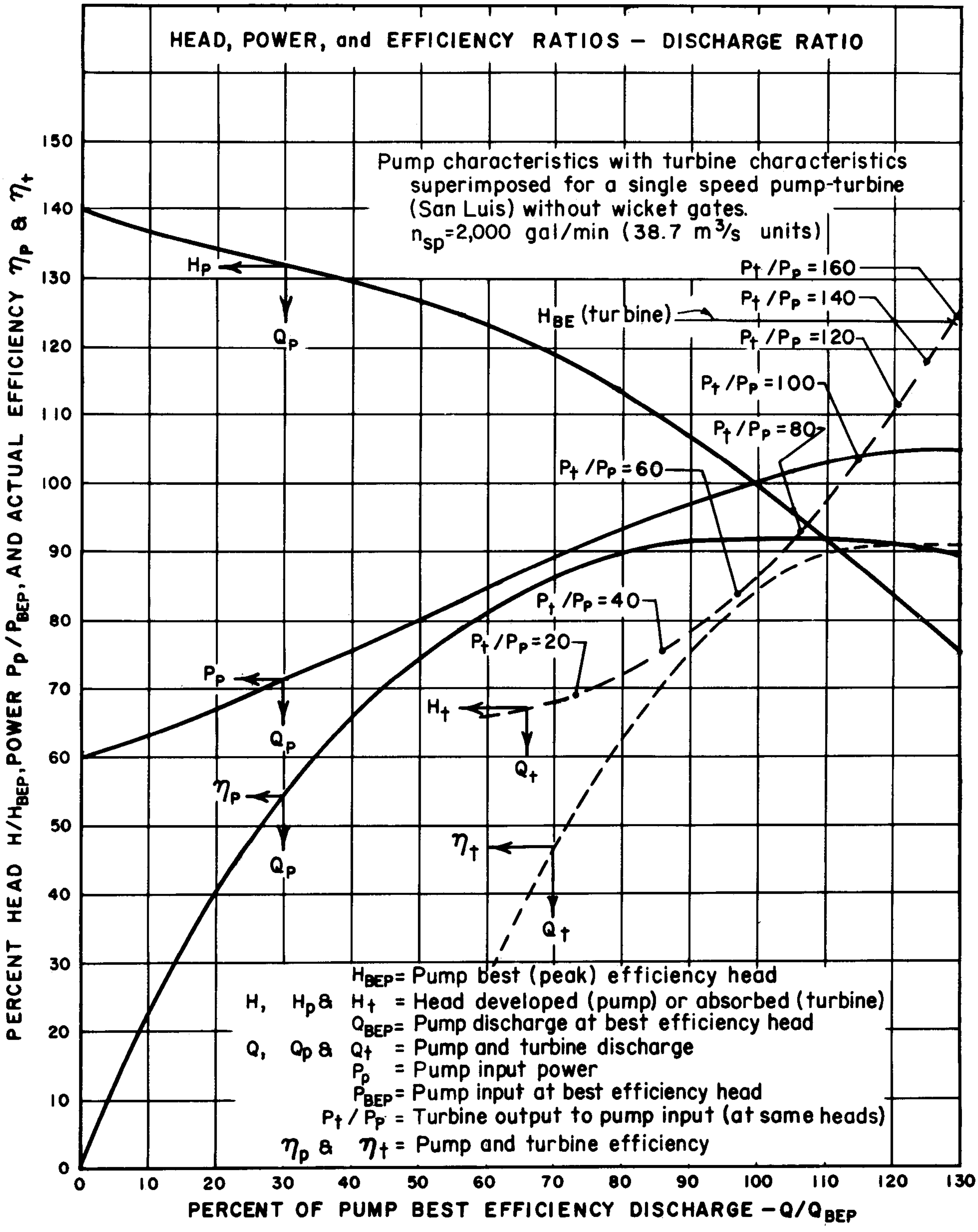


FIGURE 15.—*Turbine characteristics superimposed on pump characteristics - $n_{sp}$ = 2,000. 106-D-370.*

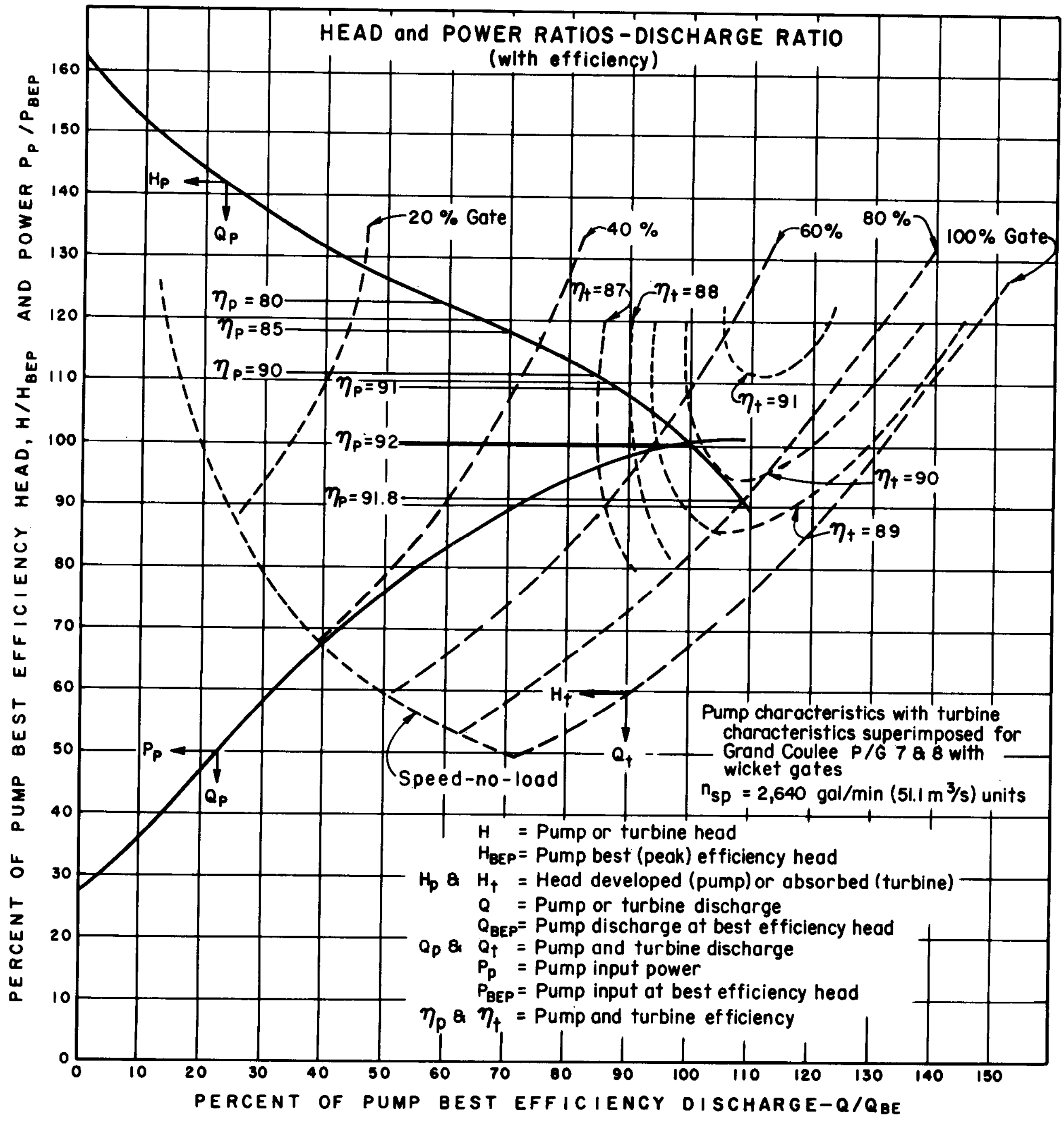


FIGURE 16.—*Turbine head, discharge, and efficiency superimposed on pump characteristics* - $n_{sp} = 2{,}640$. *106-D-371.*

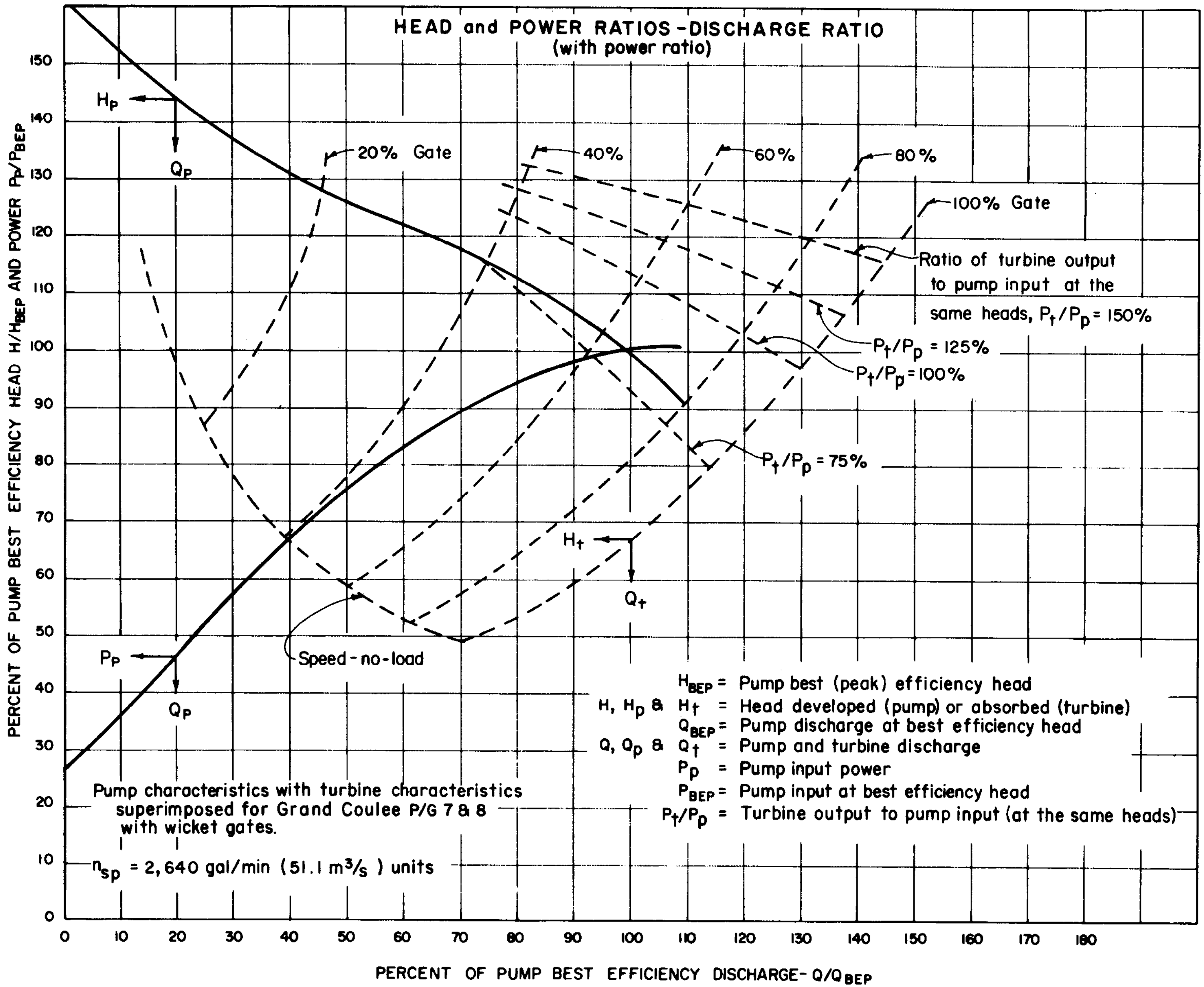


FIGURE 17.—*Turbine head, discharge, and power superimposed on pump characteristics* - $n_{sp} = 2{,}640$. *106-D-372.*

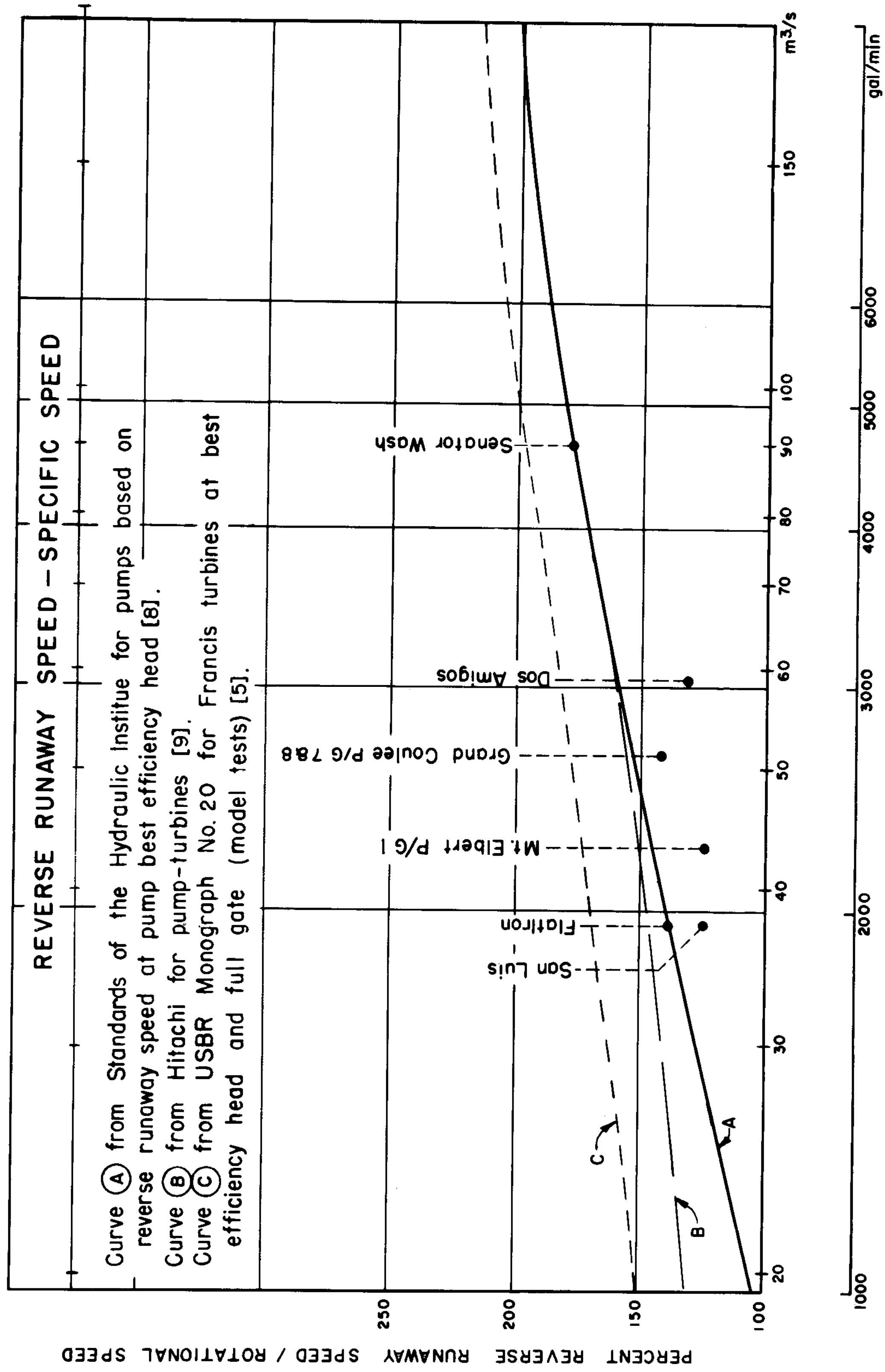


FIGURE 18.—*Percent reverse runaway speed vs. specific speed. 106-D-373.*

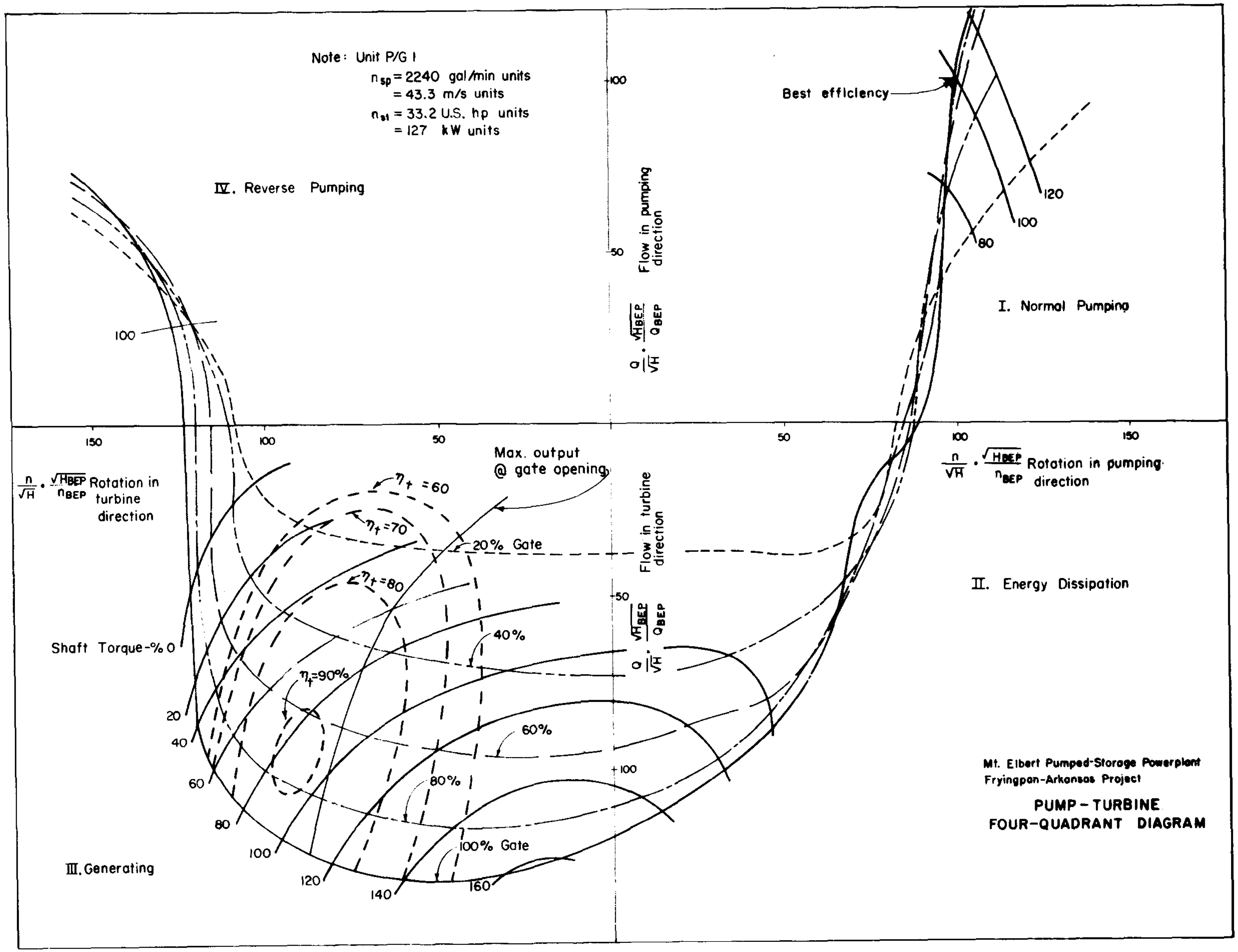


**FIGURE 19.**—*Pump-turbine four-quadrant diagram. 106-D-374.*

TABLE 6.—*Efficiency factor (k) for the conversion of velocity head to pressure head*

| Condition | Mt. Elbert $n_{sp}$=2,240 ft-gal/min (43.3 m-m³/s) | Grand Coulee $n_{sp}$=2,240 ft-gal/min (51.1 m-m³/s) |
|---|---|---|
| Normal pump rotation-zero gate | 0.68 | 0.66 |
| Normal pump rotation-100 percent gate | .62 | .51 |
| Turbine rotation-zero gate | .45 | .37 |
| Turbine rotation-100 percent gate | .32 | .20 |

## Weight, Inertia, and Hydraulic Downthrust

Figures 20 through 23 show impeller weight (W), pump-turbine weight ($W_{pt}$), impeller moment of inertia ($WR^2$), and hydraulic thrust ($F_h$) as a function of impeller diameter ($D_1$). All pumps and pump-turbines plotted on the curves have spiral cases, with the exception of the Snake Creek Unit. The Snake Creek Unit is a vertical column pump having a bowl-type diffuser.

The values obtained from the respective curves are only rough estimates, but it is desirable to have these figures available in the early stages of planning. The generator-motor moment of inertia, in addition to the impeller complement, is used in the calculations to determine transient pressures during pump power failure or turbine load rejection. The impeller and pump-turbine weights are used to obtain cost estimates. The hydraulic thrust affects the generator-motor design with respect to the thrust bearing and bearing support.

The hydraulic thrust shown on figure 23 depends largely on factors other than the basic hydraulics. The clearances at the periphery of the impeller and at the wearing rings, the surface configuration in the annular chambers between the head cover and impeller crown and between the impeller suction band and the suction discharge ring, and the size and arrangement of relief ports for minimizing thrust caused by these annular chambers all affect the magnitude of the hydraulic thrust. The surface configuration of the impeller in the annular chambers affects the impeller pumping action on the fluid in these chambers.

The hydraulic thrust curve shows the maximum net hydraulic downthrust, but for some conditions a net upthrust can occur. This net upthrust can be of the same magnitude as the net downthrust values shown on the curve. Designers attempt to minimize the net upthrust, since the net upthrust can exceed the weight of the rotating parts and lift the rotating parts off the thrust bearing [10].

It may be oversimplification to relate these characteristics to a single function, $D_1$, but this term gives as good a correlation as other more complicated functions which take into account criteria such as head and specific speed.

The scatter of points on the figures is probably because each manufacturer's particular design and fabrication techniques have as great an effect on the variables considered as do the basic design parameters—head, speed, discharge, or specific speed. For instance, the decision whether to cast or fabricate an impeller can make a substantial difference in the final weight.

## Hydraulic Similarity

For geometrically and hydraulically similar machines (i.e., equal specific speeds), certain performance data obtained for a unit of a given size and speed can be used to determine the performance for other size units and different rotational speeds. The following laws give the relationship of a point on a characteristic curve for a given rotational speed and diameter with respect to an equivalent point on another curve with a different rotational speed and/or diameter.

1. The affinity laws used for pump-turbine scaling are:

$$\frac{V_2}{V_1} = \frac{n_2}{n_1} \cdot \frac{D_2}{D_1}$$

$$\frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$$

$$\frac{Q_2}{Q_1} = \frac{n_2}{n_1} \left(\frac{D_2}{D_1}\right)^3$$

$$\frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5$$

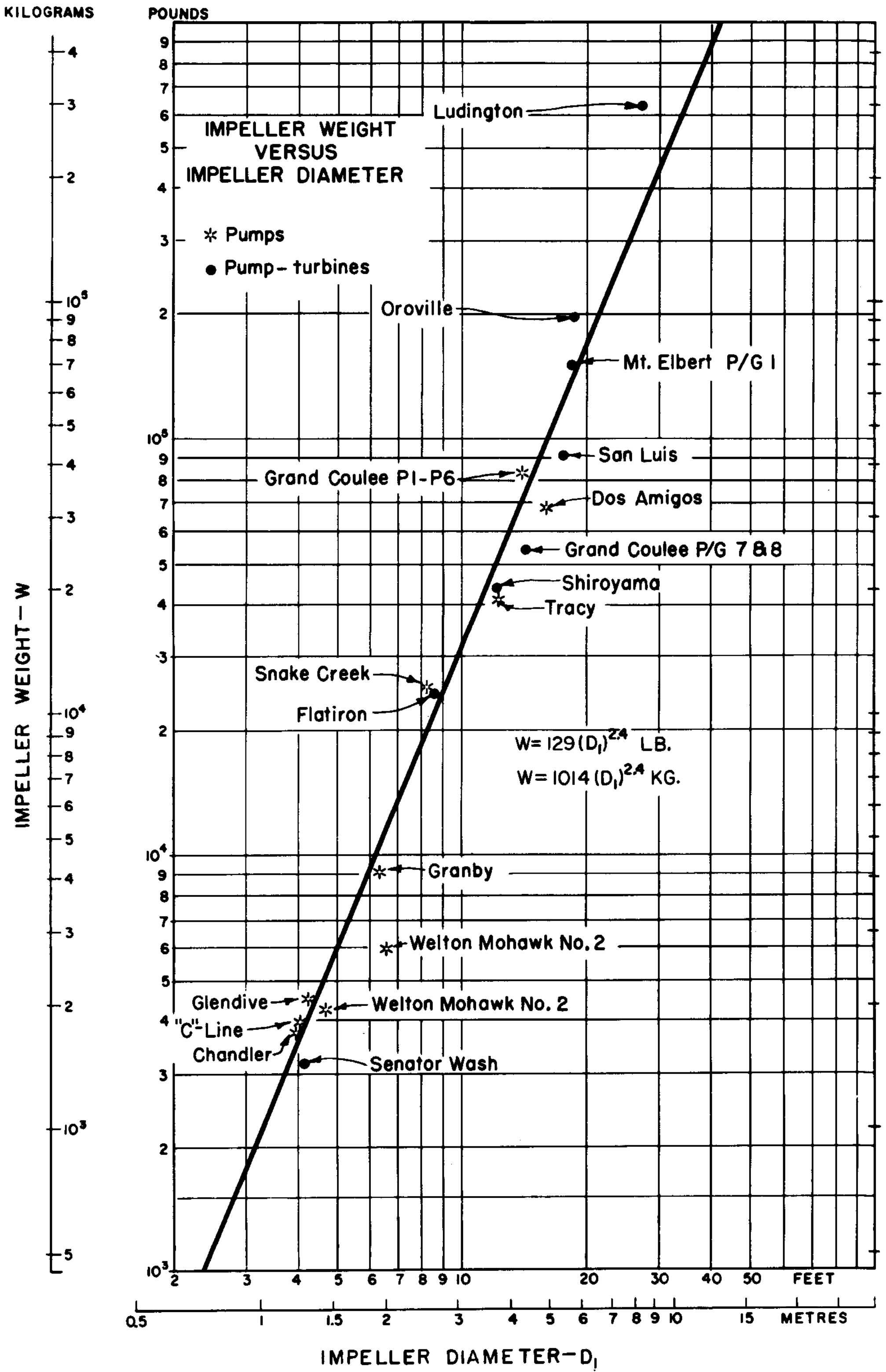


FIGURE 20.—*Impeller weight vs. impeller discharge diameter. 106-D-375.*

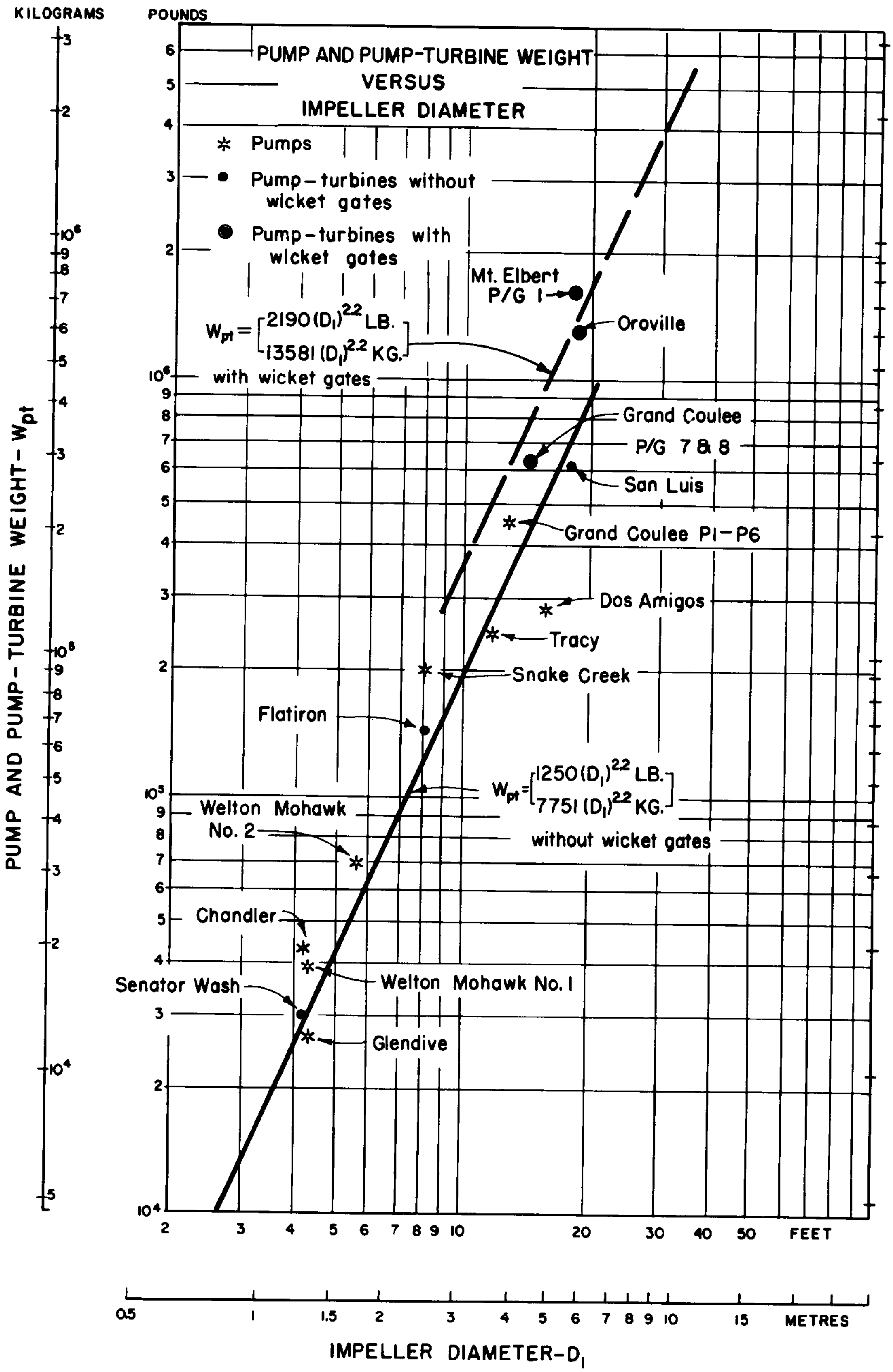


FIGURE 21.—*Pump and pump-turbine weight vs. impeller discharge diameter. 106-D-376.*

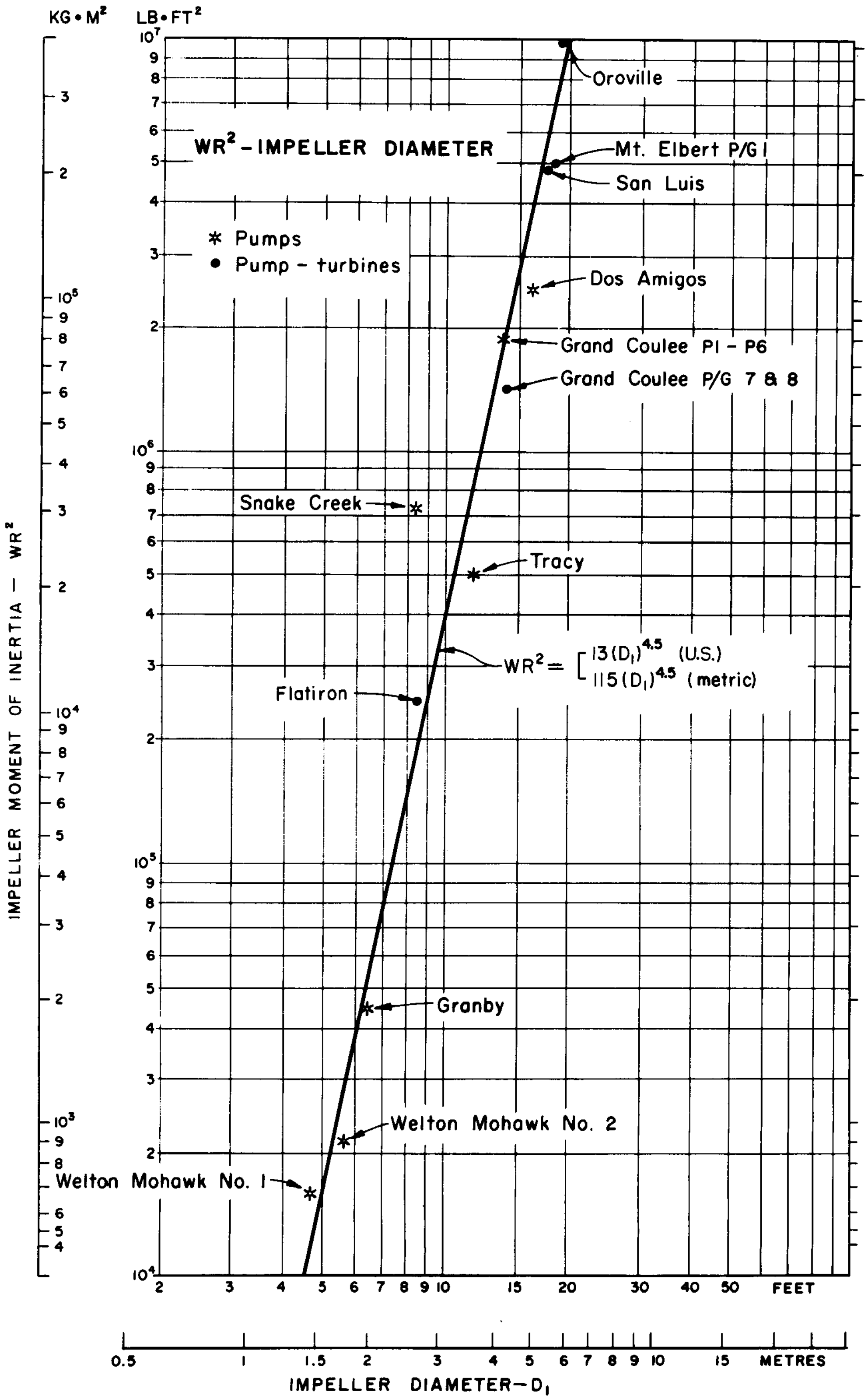


FIGURE 22.—*Impeller $WR^2$ vs. impeller discharge diameter. 106-D-377.*

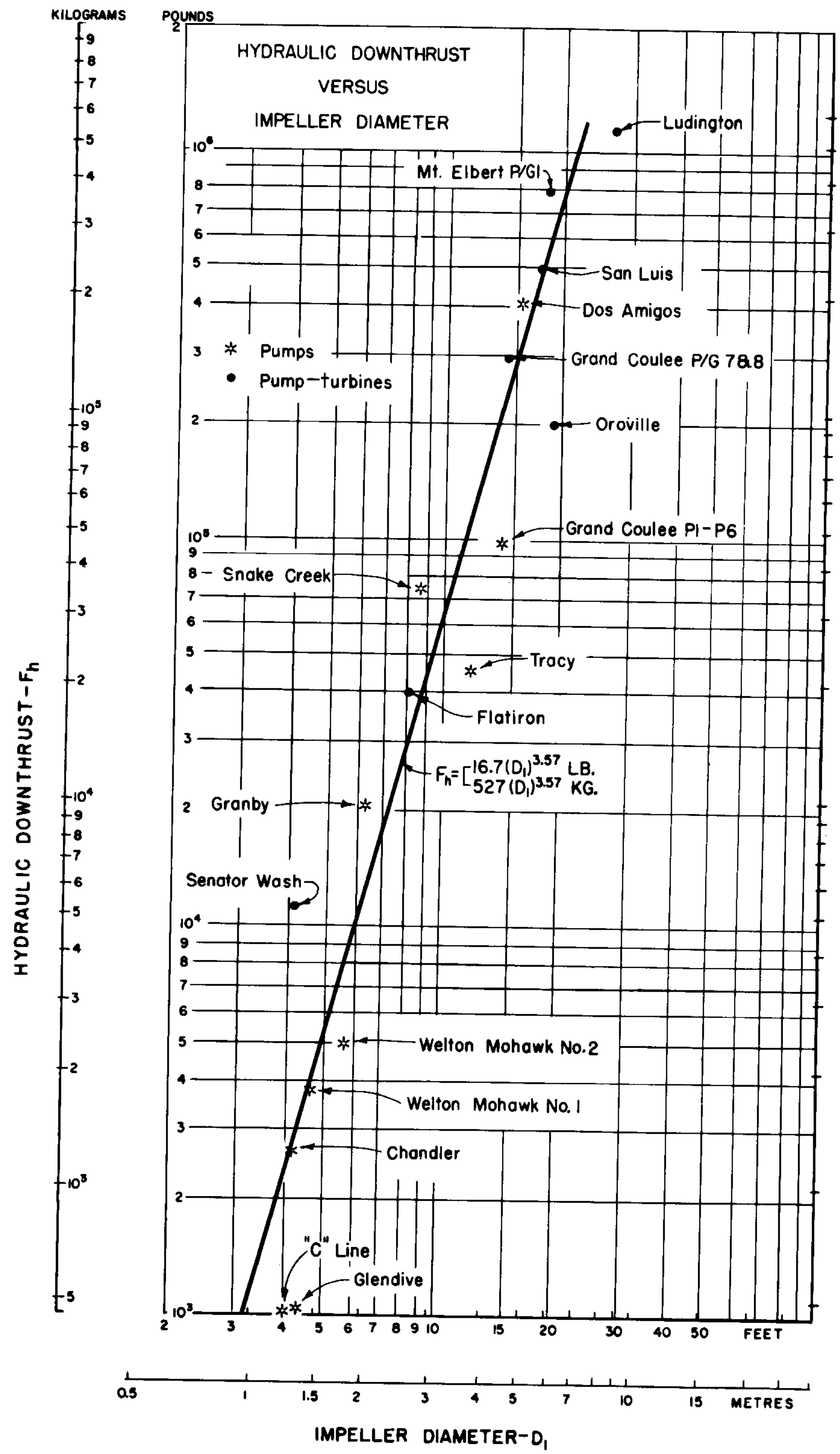


FIGURE 23.—*Hydraulic downthrust vs. impeller discharge diameter. 106-D-378.*

2. The turbine homologous equations are customarily written:

For constant diameter:

$$\frac{Q_2}{Q_1} = \left(\frac{H_2}{H_1}\right)^{1/2}$$

$$\frac{P_2}{P_1} = \left(\frac{H_2}{H_1}\right)^{3/2}$$

$$\frac{n_2}{n_1} = \left(\frac{H_2}{H_1}\right)^{1/2}$$

For constant head:

$$\frac{Q_2}{Q_1} = \left(\frac{D_2}{D_1}\right)^{1/2}$$

$$\frac{P_2}{P_1} = \left(\frac{D_2}{D_1}\right)^{2}$$

$$\frac{n_2}{n_1} = \frac{D_1}{D_2}$$

where, for different conditions:

$V_1$ and $V_2$=fluid velocity or peripheral velocity,
$Q_1$ and $Q_2$=discharge,
$P_1$ and $P_2$=turbine power output or pump power input,
$n_1$ and $n_2$=rotational speed,
$D_1$ and $D_2$=impeller/runner diameter, and
$H_1$ and $H_2$=head.

3. Pump ($n_{sp}$) and turbine ($n_{st}$) specific speed units conversion can be determined using the following equations:

$$n_{sp}=\frac{n(Q_{BE})^{1/2}}{(H_{BE})^{3/4}}$$

$$n_{st}=\frac{n(P_d)^{1/2}}{(h_d)^{5/4}}$$

$n_{sp}$, U.S. customary ft-gal/min units=51.7 $n_{sp}$, m-m³/s units
$n_{sp}$, U.S. customary ft-gal/min units=65 $n_{st}$, U.S. customary ft-hp units (approximate)
$n_{st}$, m-kW units=3.03 $n_{sp}$, m-m³/s units (approx.)
$n_{st}$, m-hp units=4.45 $n_{st}$, U.S. customary ft-hp units
$n_{st}$, m-kW units=3.81 $n_{st}$, U.S. customary ft-hp units

Note:

1 megawatt=1,000 kilowatts
kilowatts=kilovolt-amperes · power factor
1 horsepower (metric)=75 meter-kilograms per second
1 kilowatt=101.971 meter-kilograms per second
1 horsepower (metric)=1.014 horsepower (U.S.)
1 horsepower (U.S.)=550 foot-pounds per second
=0.7457 kilowatt
1 gallon (U.S.)=0.003 785 cubic meter
1 pound (mass)=2.204 622 kilogram
1 foot=0.3048 meter

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