TP3

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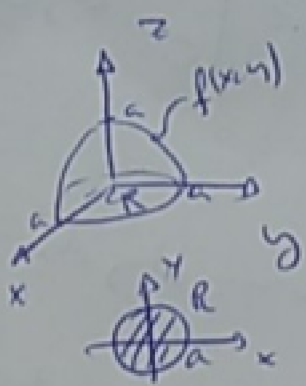


$$v.p = \frac{1}{A_R}\iint_R f(x,y)\,dx\,dy =$$

$$= \frac{1}{\pi a^2}\iint_R \sqrt{a^2-x^2-y^2}\,dx\,dy =$$

$$= \frac{1}{\pi a^2}\int_0^{2\pi}\int_0^{a}\sqrt{a^2-r^2}\cdot r\,dr\,d\theta =$$

$u = a^2 - r^2$

$du = -2r\,dr$

$$= \frac{1}{\pi a^2}\int_0^{2\pi}\int_{a^2}^{0}\sqrt{u}\cdot -\frac{1}{2}\,du\,d\theta =$$

$$R = \begin{cases} 0 \le r \le a \\ 0 \le \theta \le 2\pi \end{cases}$$

$dA = r\,dr\,d\theta$

$$= \frac{1}{\pi a^2}\int_0^{2\pi} -1/2\,\frac{u^{3/2}}{3/2}\Big|_{a^2}^{0}\,d\theta =$$

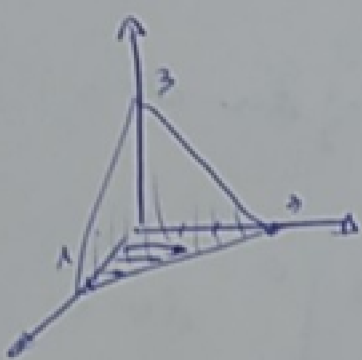
$$= \frac{1}{\pi a^2}\int_0^{2\pi} -1/3\,(0 - a^3)\,d\theta =$$

$$= \frac{1}{\pi a^2}\,\frac{1}{3}a^3\int_0^{2\pi} d\theta = \frac{a}{3\pi}\,\theta\Big|_0^{2\pi}$$

$$= \frac{2\pi a}{3\pi} - 0 = \boxed{\frac{2a}{3}}$$

37 b. $\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz\,dy\,dx = \int_0^1 \int_0^{3-3x} z \Big|_0^{3-3x-y} dy\,dx =$

$S: \begin{cases} 0 \le z \le 3-3x-y \\ 0 \le y \le 3-3x \\ 0 \le x \le 1 \end{cases}$



$= \int_0^1 \int_0^{3-3x} (3-3x-y)\,dy\,dx =$

$= \int_0^1 3y - 3xy - \frac{y^2}{2} \Big|_0^{3-3x} dx$

$= \int_0^1 \left\{3(3-3x) - 3x(3-3x) - \frac{1}{2}(3-3x)^2 - 0\right\} \cdot dx$

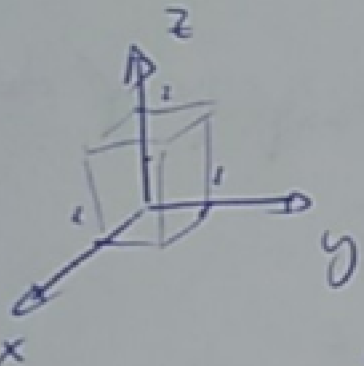
$= \int_0^1 9 - 9x - 9x + 9x^2 - \frac{1}{2}(9 - 18x + 9x^2)\, dx =$

$= \int_0^1 \left(\frac{9}{2} - 9x + \frac{9}{2}x^2\right) dx = \frac{9}{2}x - \frac{9x^2}{2} + \frac{9}{2}\frac{x^3}{3}\Big|_0^1$

$= \frac{9}{2} - \frac{9}{2} + \frac{9}{6} - 0 = \frac{9}{6} = \boxed{\frac{3}{2}}$

40 (b) $f(x,y,z) = x+y-z$ sólido 1º octante.

acotado por $x=1 \quad y=1 \quad z=2$:



$$S = \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1 \\ 0 \le z \le 2 \end{cases}$$

$$v.p = \frac{1}{V_S} \iiint_S f(x,y,z)\,dx\,dy\,dz =$$

$$= \frac{1}{2} \int_0^2 \int_0^1 \int_0^1 (x+y-z)\,dx\,dy\,dz =$$

$$= \frac{1}{2} \int_0^2 \int_0^1 \frac{x^2}{2} + xy - xz \Big|_0^1 dy\,dz =$$

$$= \frac{1}{2} \int_0^2 \int_0^1 \left(\left(\frac{1}{2} + y - z\right) - 0\right) dy\,dz =$$

$$= \frac{1}{2} \int_0^2 \frac{1}{2}y + \frac{y^2}{2} - yz \Big|_0^1 dz =$$

$$= \frac{1}{2} \int_0^2 \left(\left(\frac{1}{2} + \frac{1}{2} - z\right) - 0\right)) dz =$$

$$= \frac{1}{2} \int_0^2 (1-z)\,dz = \frac{1}{2} \left(z - \frac{z^2}{2}\Big|_0^2\right.$$

$$= \frac{1}{2} \left(\left(2 - \frac{4}{2}\right) - 0\right) = \frac{1}{2}(2-2) = \boxed{0}$$

60 (a) $S = \begin{cases} \cos\varphi \le \rho \le 2 \\ 0 \le \varphi \le \pi/2 \\ 0 \le \theta \le 2\pi . \end{cases}$

$$V = \iiint_S dV = \int_0^{2\pi}\int_0^{\pi/2}\int_{\cos\varphi}^{2} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta =$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \frac{\rho^3}{3}\operatorname{sen}\varphi \Big|_{\cos\varphi}^{2} d\varphi \, d\theta =$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \left(\frac{8}{3}\sin\varphi - \frac{\cos^3\varphi}{3}\sin\varphi\right) d\varphi \, d\theta =$$

$u = \cos\varphi$

$du = -\sin\varphi \, d\varphi$

$\varphi = 0 \quad u = 1$

$\varphi = \pi/2 \quad u = 0$

$$= \int_0^{2\pi} \frac{8}{3}(-\cos\varphi \Big|_0^{\pi/2} d\theta + \int_0^{2\pi}\int_1^0 \frac{u^3}{3} du \, d\theta =$$

$$= \int_0^{2\pi} \frac{8}{3}(0 - (-1)) d\theta + \int_0^{2\pi} \frac{u^4}{12}\Big|_1^0 d\theta =$$

$$= \int_0^{2\pi} 8/3 \, d\theta + \int_0^{2\pi} (0 - 1/12) \, d\theta =$$

$$= \frac{8}{3}\theta \Big|_0^{2\pi} - \frac{1}{12}\theta \Big|_0^{2\pi} =$$

$$= \frac{16}{3}\pi - 0 - \left(\frac{\pi}{6} - 0\right) = \boxed{\frac{31}{6}\pi}$$

60 (b)

$$S = \begin{cases} 0 \le \rho \le 2\cos\varphi \\ \pi/4 \le \varphi \le \pi/2 \\ 0 \le \theta \le 2\pi \end{cases}$$

$$V = \iiint_S dV = \int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2\cos\varphi} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta =$$

$$= \int_0^{2\pi}\int_{\pi/4}^{\pi/2} \frac{\rho^3}{3}\sin\varphi \Big|_0^{2\cos\varphi} d\varphi \, d\theta =$$

$u = \cos\varphi$

$du = -\sin\varphi \, d\varphi$

$\varphi = \pi/4 \quad \varphi = \pi/2$

$u = \sqrt{2}/2 \quad u = 0$

$$= \int_0^{2\pi}\int_{\pi/4}^{\pi/2} \left(\frac{8\cos^3\varphi}{3} \cdot \sin\varphi - 0\right) d\varphi \, d\theta =$$

$$= \int_0^{2\pi}\int_{\sqrt{2}/2}^{0} \frac{8u^3}{3} \cdot -du \, d\theta = \int_0^{2\pi} 0 - \left[\frac{-8}{12}\left(\sqrt{2}/2\right)^4\right] d\theta =$$

$$= \int_0^{2\pi} \frac{2}{3}\frac{1}{4} d\theta = \frac{1}{6}\theta \Big|_0^{2\pi} = \frac{\pi}{6}.$$

62. (a) $V = \iiint\limits_S dV = \int_0^{2\pi}\int_0^1\int_{r^4-1}^{4-4r^2} r\,dz\,dr\,d\theta$

$$S = \begin{cases} 0 \le r \le 1 \\ 0 \le \theta \le 2\pi \\ r^4-1 \le z \le 4-4r^2 \end{cases}$$

(b) $V = \iiint\limits_S dV = \int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{1-r} r\,dz\,dr\,d\theta$

$$S = \begin{cases} 0 \le r \le 1 \\ 0 \le \theta \le 2\pi \\ -\sqrt{1-r^2} \le z \le 1-r \end{cases}$$

(c) $V = \iiint\limits_S dV = \int_{\pi/2}^{3\pi/2}\int_0^{-3\cos\theta}\int_0^{r} r\,dz\,dr\,d\theta$

$$S = \begin{cases} 0 \le r \le -3\cos\theta \\ \frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \\ 0 \le z \le r \end{cases}$$