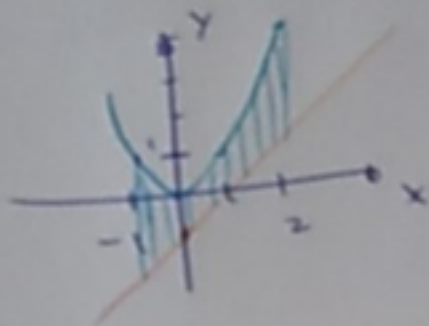


T.P.3

4 (a)

$$\begin{cases} -1 \leq x \leq 2 \\ x-1 \leq y \leq x^2 \end{cases}$$

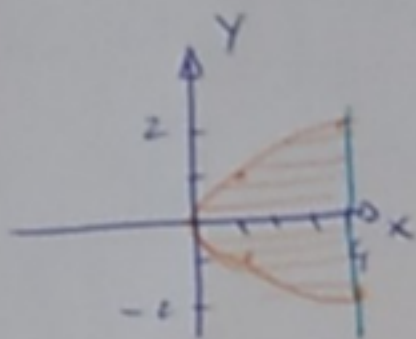


$$\begin{aligned} y &= x - 1 \\ y &= x^2 \end{aligned}$$

(b)

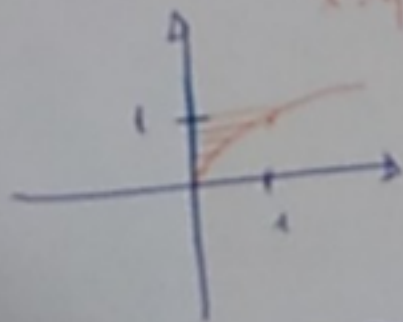
$$\begin{cases} -2 \leq y \leq 2 \\ y^2 \leq x \leq 4 \end{cases}$$

$$\begin{aligned} x &= y^2 \\ x &= 4 \end{aligned}$$



$$6 \text{ (c)} \int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx dy = \int_0^1 \int_0^{y^3} 3y^2 e^u du dy =$$

$$\begin{cases} 0 \leq y \leq 1 \\ 0 \leq x \leq y^2 \end{cases}$$



$$\begin{aligned} x &= 0 \\ x &= y^2 \end{aligned}$$

$$\begin{aligned} u &= xy & \begin{cases} x=0 & u=0 \\ x=y^2 & u=y^3 \end{cases} \\ du &= y dx \end{aligned}$$

$$= \int_0^1 3y^2 e^u \Big|_0^{y^3} dy =$$

$$= \int_0^1 3y^2 (e^{y^3} - e^0) dy =$$

$$= \int_0^1 (3y^2 e^{y^3} - 3y^2) dy =$$

$$= \int_0^1 3y^2 e^{y^3} dy - \int_0^1 3y^2 dy =$$

$$= (1-0) = e^1 - 1 - 1 = \boxed{e^1 - 2}$$

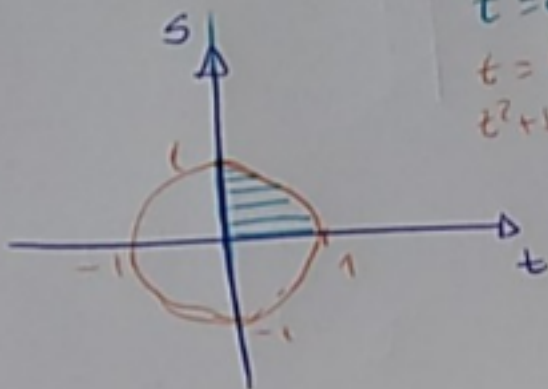
$$\begin{aligned} u &= y^3 & \begin{cases} y=0 & u=0 \\ y=1 & u=1 \end{cases} \\ du &= 3y^2 dy \end{aligned}$$

$$= \int_0^1 e^u du - 3y^3/3 \Big|_0^1 = e^u \Big|_0^1 =$$

TP3

$$8a) \int_0^1 \int_0^{\sqrt{1-s^2}} 8t \, dt \, ds = \int_0^1 \frac{8t^2}{2} \Big|_0^{\sqrt{1-s^2}} ds =$$

$$\begin{cases} 0 \leq t \leq \sqrt{1-s^2} \\ 0 \leq s \leq 1 \end{cases} = \int_0^1 4[(\sqrt{1-s^2})^2 - 0^2] ds =$$



$$\begin{aligned} t &= 0 \\ t &= \sqrt{1-s^2} \\ t^2 + s^2 &= 1 \end{aligned}$$

$$= \int_0^1 (4 - 4s^2) ds =$$

$$= 4s - \frac{4s^3}{3} \Big|_0^1 =$$

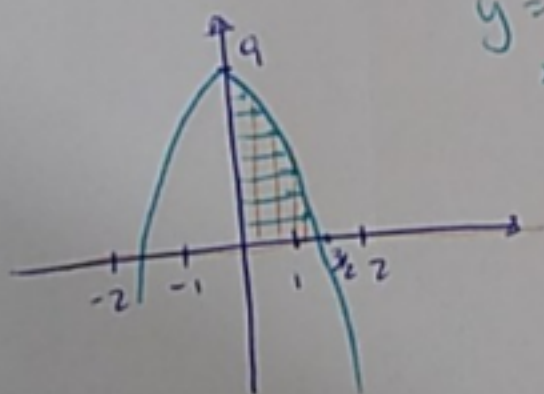
$$= (4 - 4/3) - (0 - 0)$$

$$= \boxed{\frac{8}{3}}$$

$$9b) \int_0^{3/2} \int_0^{9-4x^2} 16x \, dy \, dx = \int_0^9 \int_0^{\sqrt{\frac{9-y}{4}}} 16x \, dx \, dy$$

$$R = \begin{cases} 0 \leq y \leq 9-4x^2 \\ 0 \leq x \leq 3/2 \end{cases}$$

$$R = \begin{cases} 0 \leq x \leq \sqrt{\frac{9-y}{4}} \\ 0 \leq y \leq 9 \end{cases}$$



$$\begin{aligned} y &= 9-4x^2 \\ x &= \sqrt{\frac{9-y}{4}} \end{aligned}$$