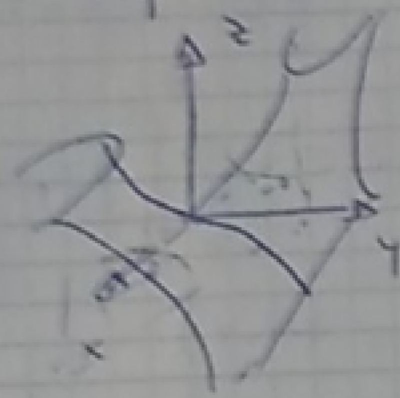
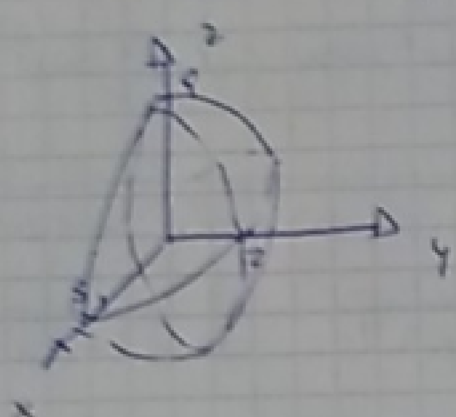
TP4 B

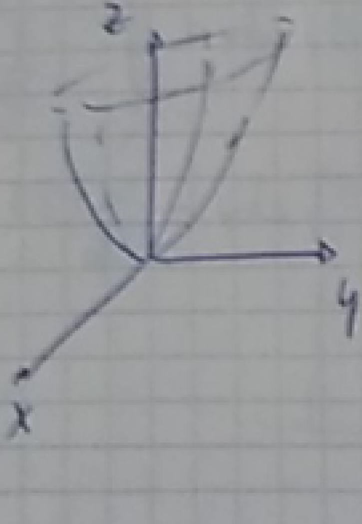
1) a) $\frac{x^2}{4} - \frac{y^2}{64} = z$



b) $\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{81} = 1$



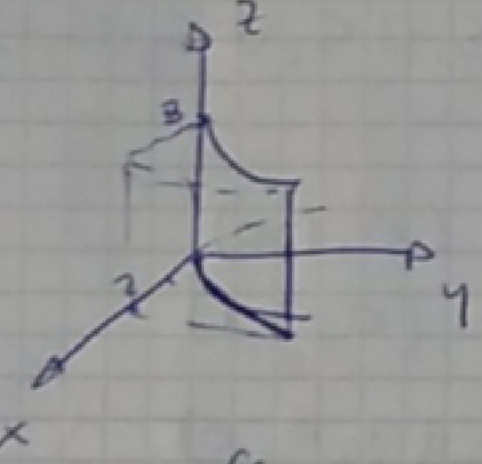
c) $\frac{x^2}{16} + \frac{y^2}{49} = z$



11 TP4 B

(a) $f(x,y,z) = x \qquad S: y = x^2 \quad 0 \le x \le 2$

$\qquad\qquad\qquad\qquad\qquad\qquad 0 \le z \le 3$



$$\begin{cases} r(x,z) = (x, x^2, z) \\ 0 \le x \le 2 \\ 0 \le z \le 3 \end{cases}$$

$r_x = (1, 2x, 0) \qquad r_x \times r_y = (2x, -1, 0)$

$r_z = (0,0,1) \qquad \|r_x \times r_y\| = \sqrt{4x^2+1}$

$$\iint_S f(x,y,z)\,dS = \iint_S f(x,y)\,\|r_x \times r_y\|\,dx\,dy =$$

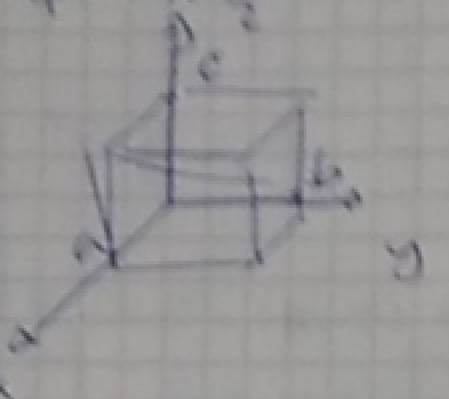
$$= \int_0^3 \int_0^2 x \cdot \sqrt{4x^2+1}\,dx\,dz = \int_0^3 \int_1^{17} u^{1/2} \cdot \frac{1}{8}\,du\,dz =$$

$$\left.\begin{aligned} u &= 4x^2+1 \\ du &= 8x\,dx \\ u(0) &= 1 \\ u(2) &= 17 \end{aligned}\right\} \quad = \int_0^3 \frac{u^{3/2}}{3/2} \cdot \frac{1}{8} \Big|_1^{17} dz = \int_0^3 \frac{1}{12}\,(17^{3/2}-1)\,dz =$$

$$= \frac{(17^{3/2}-1)}{12} \cdot z \Big|_0^3 = \frac{(17^{3/2}-1)}{4}$$

TP4 B.

11 (d) $f(x,y,z) = xyz$ $\qquad \begin{cases} x=0 & x=a \\ y=0 & y=b \\ z=0 & z=c. \end{cases}$



$S = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$

$S_1 : \begin{cases} r_1(0, y, z) \\ 0 \leq y \leq b \quad 0 \leq z \leq c \end{cases}$ $\qquad r_{1y} = (0,1,0)$

$r_{1z} = (0,0,1)$

$r_{1y} \times r_{1z} = (1,0,0)$

$\| r_{1y} \times r_{1z} \| = 1$

$$\iint_{S_1} f \, d\sigma = \int_0^c \int_0^b 0yz \cdot 1 \, dy \, dz = \boxed{0}$$

$S_2 : \begin{cases} r_2(a, y, z) \\ 0 \leq y \leq b \quad 0 \leq z \leq c \end{cases}$ $\qquad r_{2y} = (0,1,0)$

$r_{2z} = (0,0,1)$

$r_{2y} \times r_{2z} = (1,0,0)$

$\| r_{2y} \times r_{2z} \| = 1$

$$\iint_{S_2} f \, d\sigma = \int_0^c \int_0^b ayz \, dy \, dz$$

$$= \int_0^c z a \frac{y^2}{2} \Big|_0^b dz = \int_0^c \frac{ab^2}{2} z \, dz = \frac{ab^2}{2} \frac{z^2}{2} \Big|_0^c$$

$$= \boxed{\frac{ab^2 c^2}{4}}.$$

$S_2: \begin{cases} r_2(x,0,z) \\ 0\le x\le a \quad 0\le z\le c \end{cases}$ $\quad r_{2x}=(1,0,0)$

$r_{2z}=(0,0,1)$

$r_{2x}\times r_{2z}=(0,-1,0)$

$\|r_{2x}\times r_{2z}\|=1$

$$\iint_{S_2} f\,d\sigma=\int_0^c\int_0^a x\cdot 0\cdot z\,dx\,dz=\boxed{0}$$

$S_4: \begin{cases} r_4(x,b,z) \\ 0\le x\le a \quad 0\le z\le c \end{cases}$ $\quad r_{4x}=(1,0,0)$

$r_{4z}=(0,0,1)$

$r_{4x}\times r_{4z}=(0,-1,0)$

$\|r_{4x}\times r_{4z}\|=1$

$$\iint_{S_4} f\,d\sigma=\int_0^c\int_0^a x\cdot bz\,dx\,dz=\int_0^c\left(\frac{x^2}{2}bz\Big|_0^a\right)dz=$$

$$=\int_0^c \frac{a^2}{2}bz\,dz=\frac{a^2b}{2}\frac{z^2}{2}\Big|_0^c=\boxed{\frac{a^2bc^2}{4}}.$$

$S_5: \begin{cases} r_5(x,y,0) \\ 0\le x\le a \quad 0\le y\le b \end{cases}$ $\quad r_{5x}=(1,0,0)$ $\quad r_{5x}\times r_{5y}=(0,0,1)$

$r_{5y}=(0,1,0)$ $\quad \|r_{5x}\times r_{5y}\|=1$

$$\iint_{S_5} f\,d\sigma=\int_0^b\int_0^a xy\,0\,dx\,dy=\boxed{0}.$$

$S_6: \begin{cases} r_6(x,y,c) \\ 0\le x\le a \quad 0\le y\le b \end{cases}$ $\quad r_{6x}=(1,0,0)$ $\quad r_{6x}\times r_{6y}=(0,0,1)$

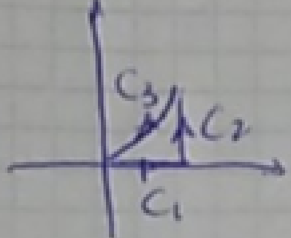
$r_{6y}=(0,1,0)$ $\quad \|r_{6x}\times r_{6y}\|=1$

$$\iint_{S_6} f\,d\sigma=\int_0^b\int_0^a xyc\,dx\,dy=\int_0^b \frac{x^2}{2}yc\Big|_0^a dy=\int_0^b\frac{a^2c}{2}y\,dy=$$

$$=\frac{a^2c}{2}\frac{y^2}{2}\Big|_0^b=\frac{a^2b^2c}{4}$$

$$\iint_S f\,d\sigma=0+\frac{ab^2c^2}{4}+0+\frac{a^2bc^2}{4}+0+\frac{a^2b^2c}{4}=\boxed{\frac{ab^2c^2+a^2bc^2+a^2b^2c}{4}}$$

(32) TP4A $F=(2xy^3;-4x^2y^2)$



$$T=\int_C F T\,ds=\int_{C_1} F T\,ds+\int_{C_2} F T\,ds+\int_{C_3} F T\,ds=0+\frac{4}{3}-\frac{14}{11}$$
$$=\boxed{\frac{2}{33}}$$

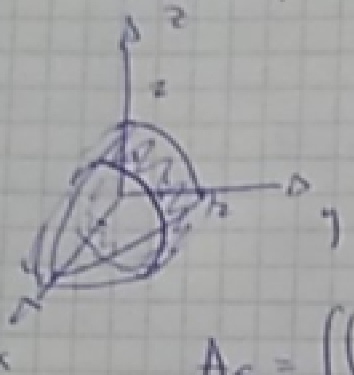
C_1 $\quad r_1(t)=(t,0) \quad 0\le t\le 1$
$r_1'(t)=(1,0)$
$$\int_{C_1} F T\,ds=\int_0^1 (0,0)\cdot(1,0)\,dt=\boxed{0}$$

C_2 $\quad r_2(t)=(1,t) \quad 0\le t\le 1$
$r_2'(t)=(0,1)$
$$\int_{C_2} F T\,ds=\int_0^1 (2t^3;4t^2)\cdot(0,1)\,dt=\int_0^1 4t^2\,dt=$$
$$=\frac{4t^3}{3}\Big|_0^1=\boxed{4/3}$$

C_3 $\quad r_3(t)=(-t,-t^3) \quad -1\le t\le 0$
$r_3'(t)=(-1,-3t^2)$
$$\int_{C_3} F T\,ds=\int_{-1}^0 (2t^{10};4t^8)\cdot(-1,-3t^2)\,dt=$$
$$=\int_{-1}^0 (-2t^{10}-12t^{10})\,dt=-14\frac{t^{11}}{11}\Big|_{-1}^0$$
$$=0-\left(\frac{14}{11}\right)=\boxed{-\frac{14}{11}}$$

TP4B

10 $\quad x = 4 - y^2 - z^2 \qquad 1 \le y^2 + z^2 \le 4$



$r(u,v) = (4-u^2, u\cos v, u\,\mathrm{sen}\,v)$

$\begin{cases} 1 \le u \le 2 \\ 0 \le v \le 2\pi \end{cases}$

$$A_S = \iint_S d\sigma = \int_0^{2\pi}\int_1^2 \|r_u \times r_v\|\,du\,dv$$

$r_u = (-2u, \cos v, \mathrm{sen}\,v)$

$r_v = (0, -u\,\mathrm{sen}\,v, u\cos v)$

$r_u \times r_v = (u \,;\, +2u^2\cos v \,;\, 2u^2\,\mathrm{sen}\,v)$

$$\|r_u \times r_v\| = \sqrt{u^2 + 4u^4\cos^2 v + 4u^4\,\mathrm{sen}^2 v}$$

$$= \sqrt{u^2 + 4u^4} = u\cdot\sqrt{1+4u^2}$$

$$A_S = \int_0^{2\pi}\int_1^2 u\sqrt{1+4u^2}\,du\,dv =$$

$t = 1+4u^2$

$dt = 8u\,du$

$t(1) = 5$

$t(2) = 17$

$$= \int_0^{2\pi}\int_5^{17} t^{1/2}\cdot\frac{1}{8}\,dt\,dv = \int_0^{2\pi} \frac{t^{3/2}}{3/2}\cdot\frac{1}{4}\Big|_5^{17} dv =$$

$$= \int_0^{2\pi} \frac{1}{6}\left(17^{3/2} - 5^{3/2}\right) dv = \frac{1}{6}\left(17^{3/2} - 5^{3/2}\right)\theta\Big|_0^{2\pi}$$

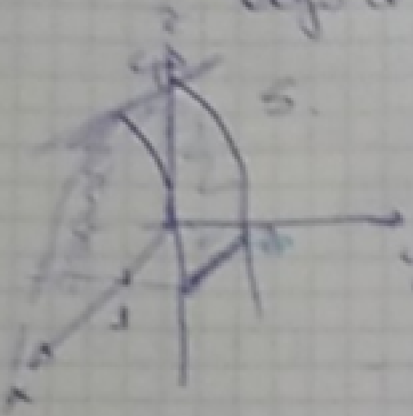
$$= \boxed{\frac{\left(17^{3/2} - 5^{3/2}\right)\pi}{3}}$$

TP4 B.

12 (a) $F(x,y,z) = (z^2, x, -3z)$

$S:$ $z = 4 - y^2$ cortado por $\begin{cases} x = 0 \\ x = 1 \\ z = 0. \end{cases}$

elijo el eje x



$S.$ $\begin{cases} r(x,y) = (x, y, 4-y^2) \\ 0 \leq x \leq 1 \\ -2 \leq y \leq 2 \end{cases}$

$r_x = (1, 0, 0)$ $\quad r_x \times r_y = (0, 2y, 1)$

$r_y = (0, 1, -2y)$ $\quad r(\frac{1}{2}, 2) = (\frac{1}{2}, 2, 0)$

$r_x \times r_y (0, 1, 1)$

$$\iint_S F \cdot n \, d\sigma = \iint_S F(r(u,v)) \cdot (r_u \times r_v) \, du \, dv$$

$$= \int_{-2}^{2}\int_0^1 ((4-y^2)^2, x, -3(4-y^2)) \cdot (0, 2y, 1) \, dx \, dy =$$

$$= \int_{-2}^{2}\int_0^1 (0 + x2y + (-3)(4-y^2) \cdot 1) \, dx \, dy =$$

$$= \int_{-2}^{2} x^2 y + 3y^2 x - 12x \Big|_0^1 \, dy =$$

$$= \int_{-2}^{2} (y + 3y^2 - 12) \, dy = \frac{y^2}{2} + \frac{3y^3}{3} - 12y \Big|_{-2}^{2} =$$

$$= (2 + 8 - 24) - (2 - 8 + 24) = \boxed{-32}$$