UNCUYO
UNIVERSIDAD NACIONAL DE CUYO

FACULTAD DE INGENIERÍA

ESTATICA Y RESISTENCIA DE MATERIALES
INGENIERIA INDUSTRIAL Y MECATRONICA

ALGUNOS EJEMPLOS DEL TP5!!!

Ejercicio N°2:

Calcular analiticamente las coordenadas del centro de gravedad de la siguiente figura

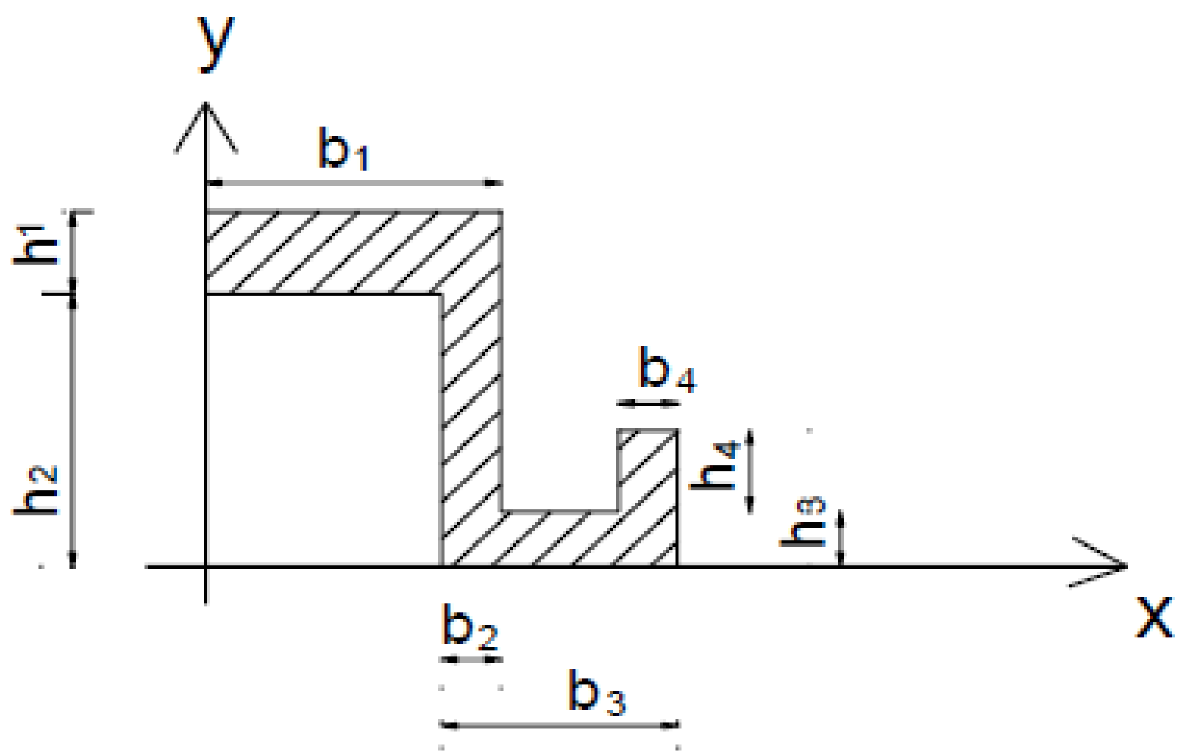
Datos:

$b_1 = 10cm$ $h_1 = 3cm$

$b_2 = 2cm$ $h_2 = 10cm$

$b_3 = 8cm$ $h_3 = 2cm$

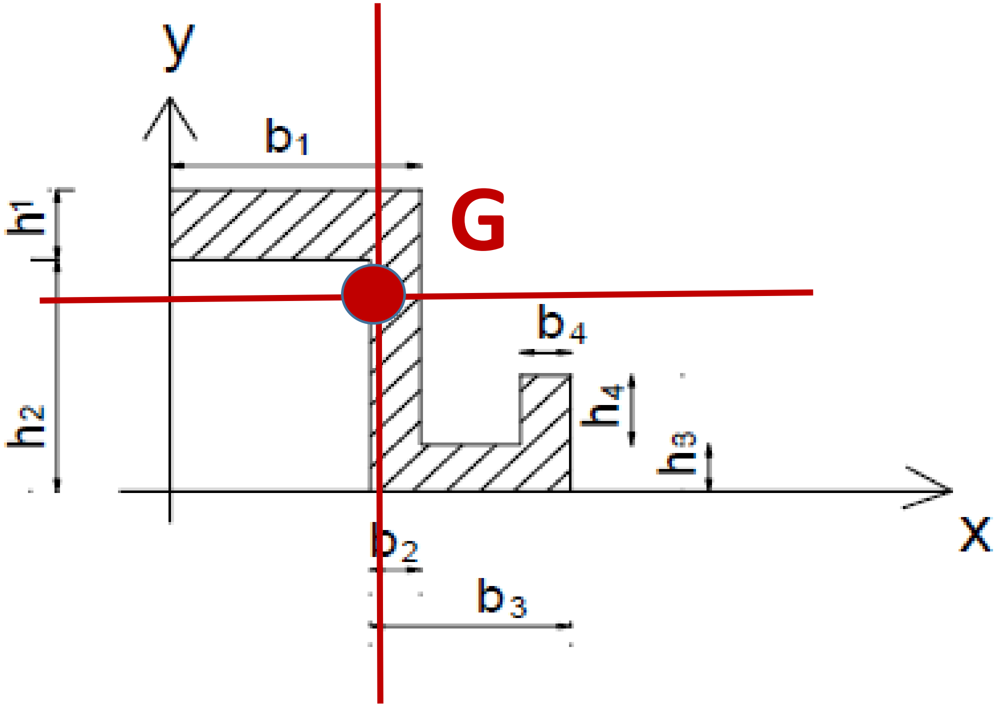
$b_4 = 2cm$ $h_4 = 3cm$



Solución:

$X_G = 8,5cm$

$Y_G = 8,08cm$ **!!**



$$A1 = 10x3 = 30\ cm^2 (x_{g1} = 5\ cm\ -\ y_{g1} = 11.50\ cm)$$

$$A2 = 10x2 = 20\ cm^2 (x_{g2} = 9\ cm\ -\ y_{g2} = 5\ cm)$$

$$A3 = 6x2 = 12\ cm^2 (x_{g3} = 13cm\ -\ y_{g3} = 1\ cm)$$

$$A4 = 3x2 = 6\ cm^2 (x_{g4} = 15cm\ -\ y_{g4} = 3.50\ cm)$$

$$\sum Ai = 30 + 20 + 12 + 6 = 68\ cm2$$

$$X_G = \frac{(30 * 5) + (20 * 9) + (12 * 13) + (6 * 15)}{68cm2} = 8.50\ cm$$

$$Y_G = \frac{(30 * 11.5) + (20 * 5) + (12 * 1) + (6 * 3.50)}{68cm2} = 7.03\ cm$$

Ejercicio N°3:

Calcular analiticamente las coordenadas del centro de gravedad de la siguiente figura

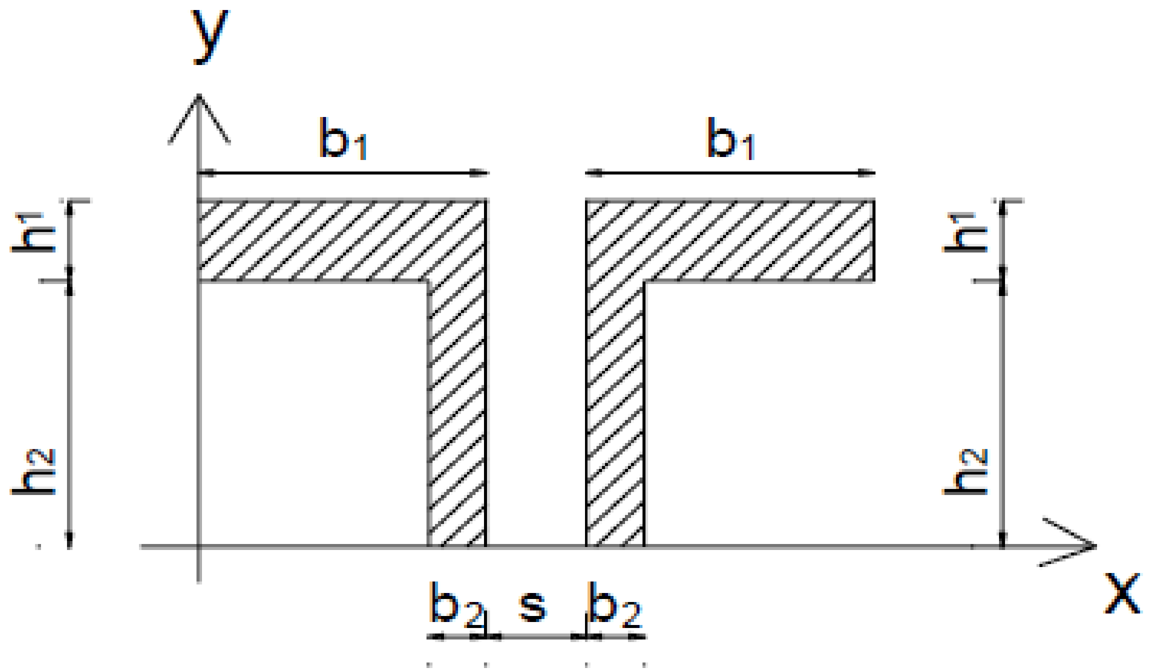
Datos:

b_1 = 10cm h_1 = 3cm S= 5cm

b_2 = 2cm h_2 = 10cm

b_3 = 8cm h_3 = 2cm

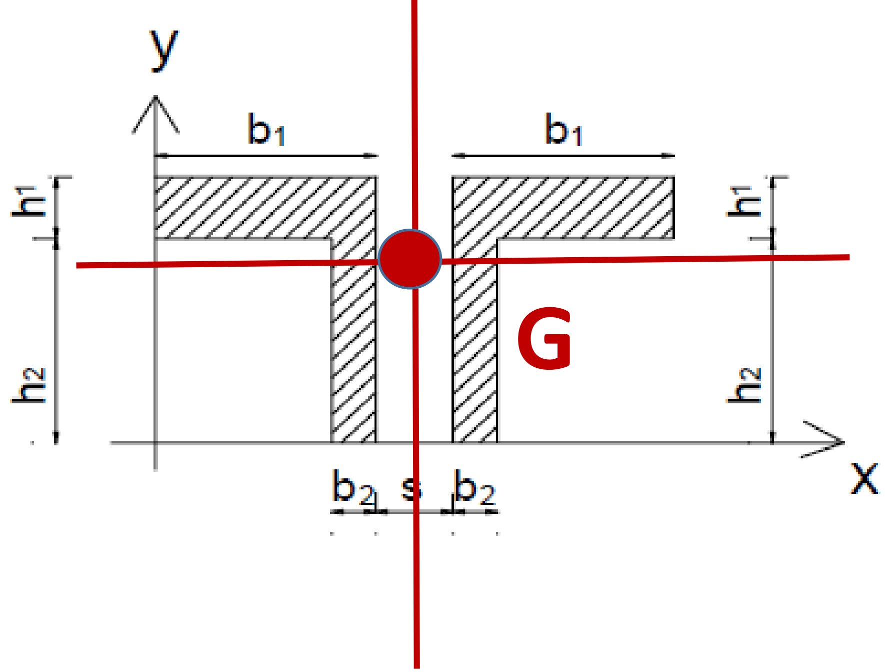
b_4 = 2cm h_4 = 3cm



Solución:

X_G = 12,5cm

Y_G = 10,16cm !!



$$A1 = 10x3 = 30\ cm^2(x_{g1} = 5\ cm\ -\ y_{g1} = 11.50\ cm)$$

$$A2 = 10x2 = 20\ cm^2(x_{g2} = 9\ cm\ -\ y_{g2} = 5\ cm)$$

$$A3 = 10x3 = 30\ cm^2(x_{g3} = 20cm\ -\ y_{g3} = 11.50\ cm)$$

$$A4 = 10x2 = 20\ cm^2(x_{g4} = 16cm\ -\ y_{g4} = 5.00\ cm)$$

$$\sum Ai = 30 + 20 + 30 + 20 = 100\ cm2$$

$$X_G = \frac{(30 * 5) + (20 * 9) + (30 * 20) + (20 * 16)}{100cm2} = 12.50\ cm$$

$$Y_G = \frac{(30 * 11.5) + (20 * 5) + (30 * 11.5) + (20 * 5)}{100cm2} = 8.90\ cm$$

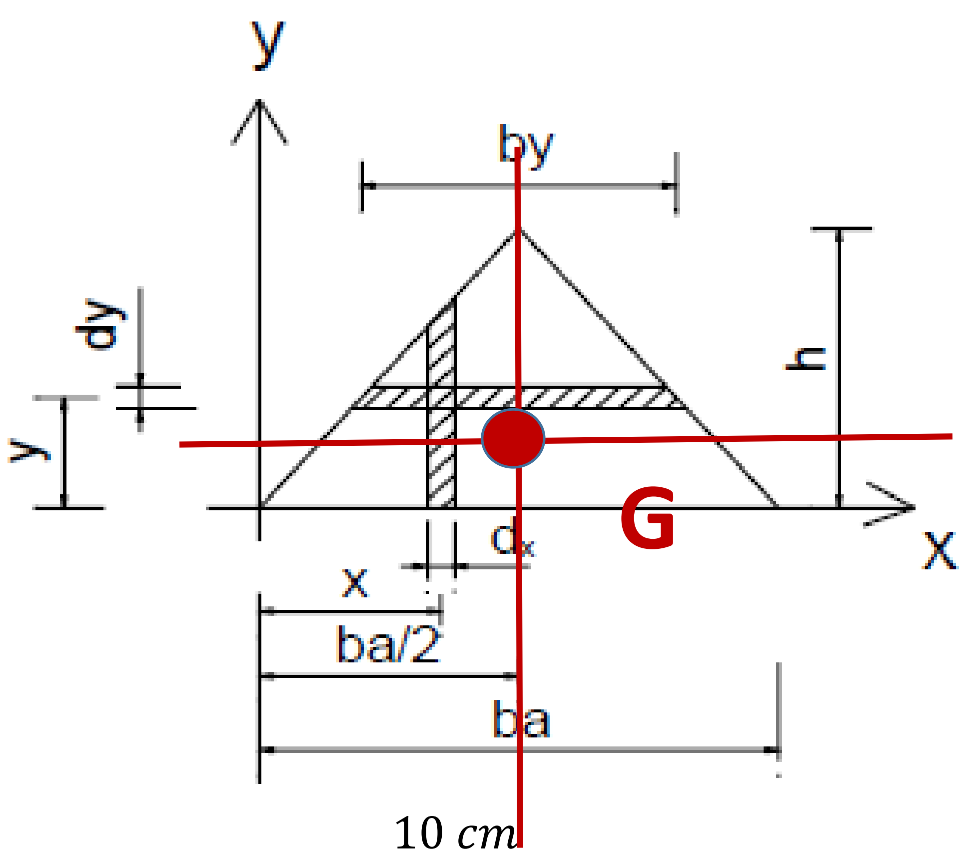
Ejercicio N°4:

Calcular analiticamente las coordenadas del centro de gravedad de la siguiente figura.

Datos:

$h = 10cm$

$b_a = 15cm$



$y_G = 3{,}33cm$

$x_G = 7{,}5cm$

$$X_G = \frac{15cm}{2} = 7.50\ cm \qquad Y_G = \frac{10\ cm}{3} = 3.33\ cm$$

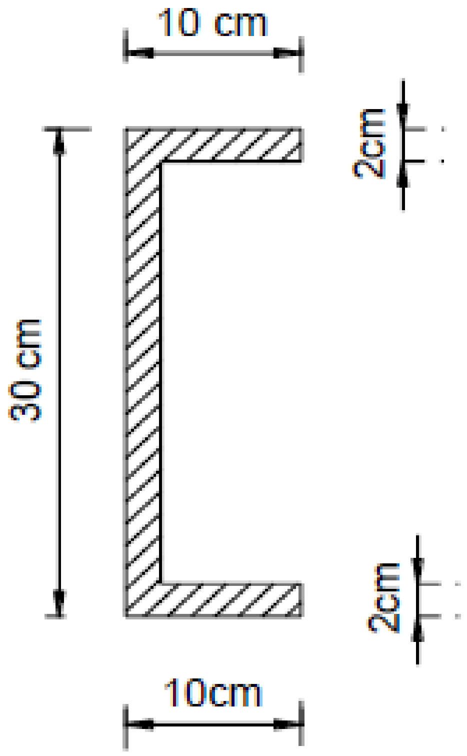
Ejercicio N°7:

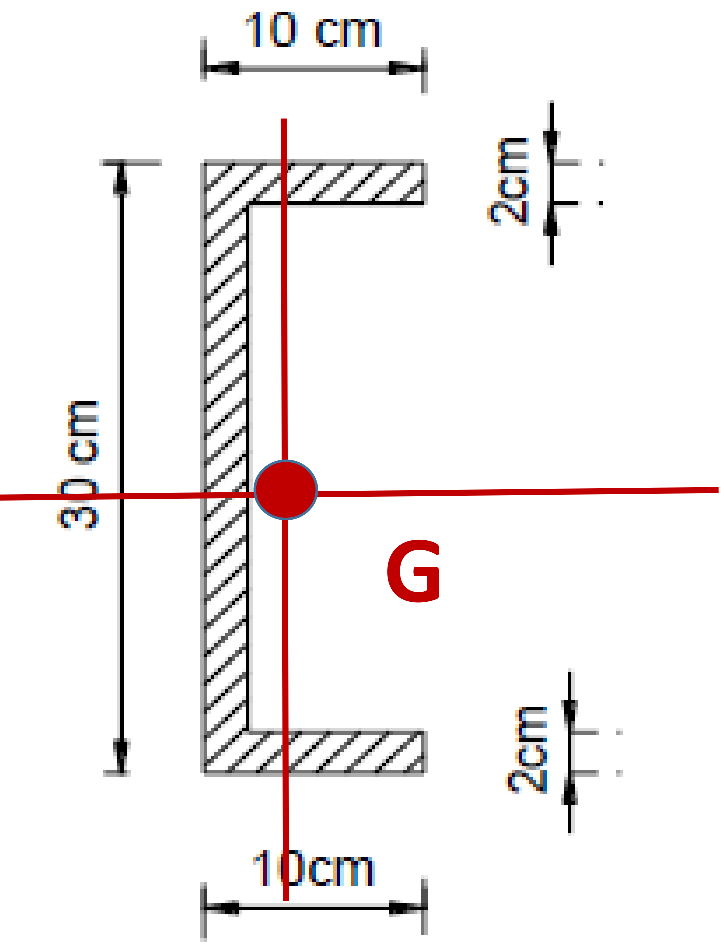
Calcular los momentos de inercia respecto de los ejes baricéntricos del perfil de la figura.

Solución:

$Jxx_G = 10790,33 cm^4$

$Jyy_G = 720,07 cm^4$





$$A1 = 10x2 = 20\ cm^2 (x_{g1} = 5\ cm\ -\ y_{g1} = 29\ cm)$$

$$A2 = 28x2 = 56\ cm^2 (x_{g2} = 1\ cm\ -\ y_{g2} = 15\ cm)$$

$$A3 = 10x2 = 20\ cm^2 (x_{g3} = 5cm\ -\ y_{g3} = 1\ cm)$$

$$\sum Ai = 20 + 56 + 20 = 96\ cm2$$

$$X_G = \frac{(20 * 5) + (56 * 1) + (20 * 5)}{96cm2} = 2.67\ cm$$

$$Y_G = \frac{(20 * 29) + (56 * 15) + (20 * 1)}{96m2} = 15\ cm$$

$$A1 = 20\ cm^2\ -\ I_{x1} = \frac{10x2^3}{12} = 6.67\ \text{-}\ I_{y1} = \frac{2x\ 10^3}{12} = 166.67$$
$$y_{g1} = (29 - 15) = 14cm\ -\ x_{g1} = 5 - 2.66 = 2.34cm$$

$$A2 = 56\ cm^2\ -\ I_{x2} = \frac{2x26^3}{12} = 2929.33\ \text{-}\ I_{y2} = \frac{26x\ 2^3}{12} = 17.33$$
$$y_{g2} = (15 - 15) = 0cm\ -\ x_{g2} = 2.67 - 1.00 = 1.67cm$$

$$A3 = 20\ cm^2\ -\ I_{x1} = \frac{10x2^3}{12} = 6.67\ \text{-}\ I_{y1} = \frac{2x\ 10^3}{12} = 166.67$$
$$y_{g1} = (15 - 1) = 14cm\ -\ x_{g1} = 5 - 2.66 = 2.34cm$$

$$A1 = 20\ cm^2\ -\ I_{x1} = \frac{10x2^3}{12} = 6.67\ \text{-}\ I_{y1} = \frac{2x\ 10^3}{12} = 166.67$$
$$y_{g1} = (29 - 15) = 14cm\ -\ x_{g1} = 5 - 2.66 = 2.34cm$$

$$A2 = 56\ cm^2\ -\ I_{x2} = \frac{2x26^3}{12} = 2929.33\ \text{-}\ I_{y2} = \frac{26x\ 2^3}{12} = 17.33$$
$$y_{g2} = (15 - 15) = 0cm\ -\ x_{g2} = 2.67 - 1.00 = 1.67cm$$

$$A3 = 20\ cm^2\ -\ I_{x1} = \frac{10x2^3}{12} = 6.67\ \text{-}\ I_{y1} = \frac{2x\ 10^3}{12} = 166.67$$
$$y_{g1} = (15 - 1) = 14cm\ -\ x_{g1} = 5 - 2.66 = 2.34cm$$

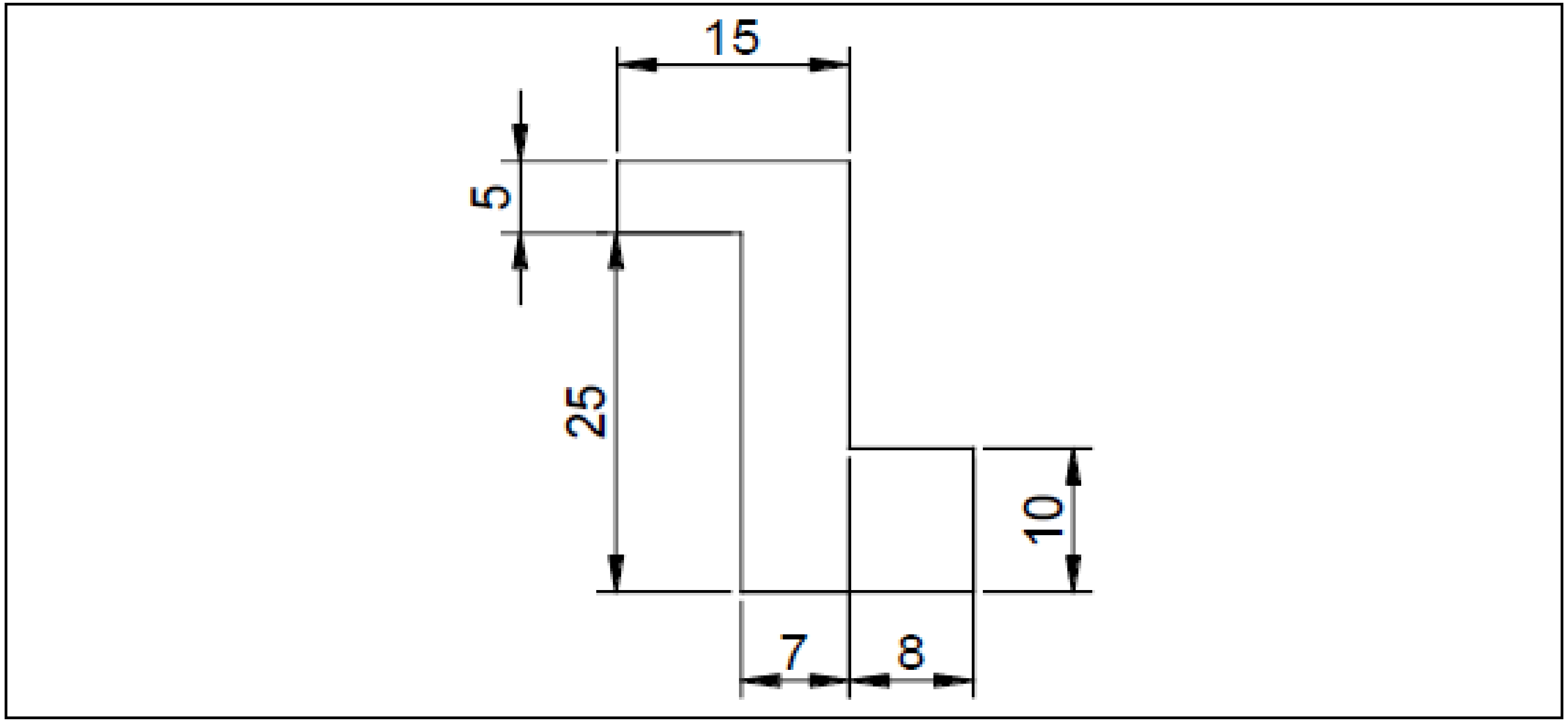
$$I_{xxg} = [6.67 + (20 * 14^2)] + [2929.33 + (56 * 0^2)] + [6.67 + (20 * 14^2] = \mathbf{10782.67\ cm^4}$$

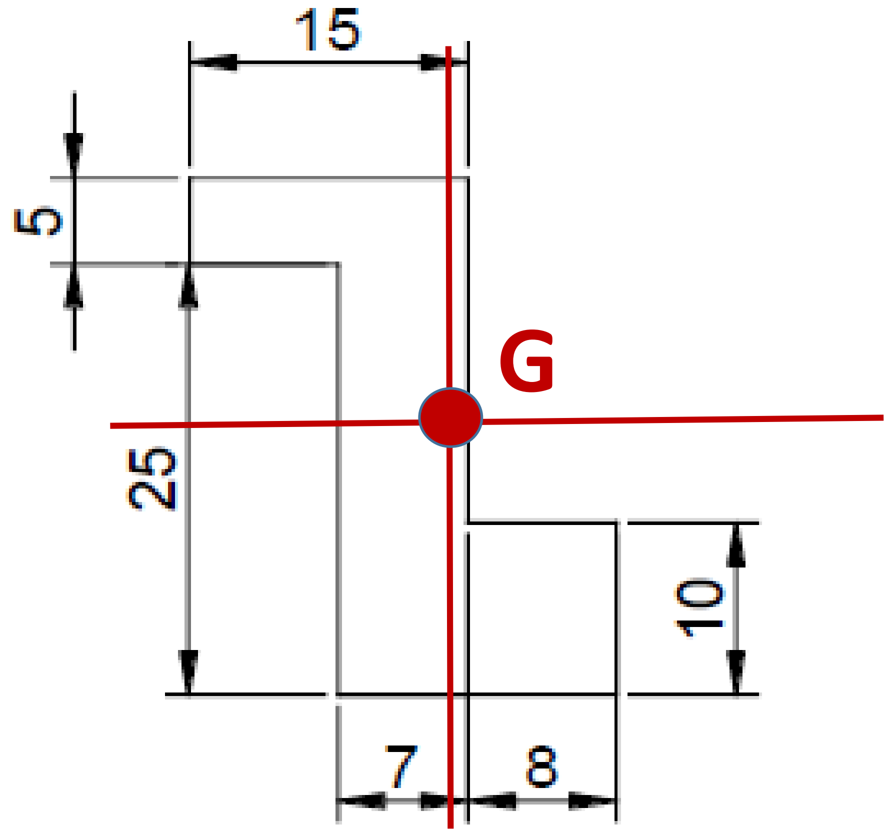
$$I_{yyg} = [166.67 + (20 * 2.34^2)] + [17.33 + (56 * 1.67^2)] + [166.67 + (20 * 2.34^2] = \mathbf{3258.93\ cm^4}$$

Ejercicio N°9:

Calcular para el perfil de la figura la posición de lo ejes principales de inercia y los valores de los momentos máximos y mínimos respecto de los ejes baricéntricos

Verificar los resultados obtenidos analíticamente, mediante la Circunsferencia de MOHR y de LAND





$$A1 = 15x5 = 75\ cm^2 (x_{g1} = 7.5\ cm\ -\ y_{g1} = 27.5\ cm)$$

$$A2 = 25x7 = 175\ cm^2 (x_{g2} = 11.5\ cm\ -\ y_{g2} = 12.5\ cm)$$

$$A3 = 8x10 = 80\ cm^2 (x_{g3} = 19cm\ -\ y_{g3} = 5\ cm)$$

$$\sum Ai = 75 + 175 + 80 = 330\ cm2$$

$$X_G = \frac{(75 * 7.5) + (175 * 11.5) + (80 * 19)}{330cm2} = 12.41\ cm$$

$$Y_G = \frac{(75 * 27.5) + (175 * 12.5) + (80 * 5)}{330m2} = 14.10\ cm$$

$$A1 = 75\ cm^2\ -\ I_{x1} = \frac{15x5^3}{12} = 156.25\ \text{-}\ I_{y1} = \frac{5x\ 15^3}{12} = 1406.25$$

$$y_{g1} = (27.5 - 14.10) = 13.4cm\ -\ x_{g1} = 12.41 - 7.50 = 4.91cm$$

$$A2 = 175\ cm^2\ -\ I_{x2} = \frac{7x25^3}{12} = 9114.58\ \text{-}\ I_{y2} = \frac{25x\ 7}{12} = 714.58$$

$$y_{g2} = (12.41 - 11.5) = 0.91cm\ -\ x_{g2} = 14.10 - 12.50 = 1.60cm$$

$$A3 = 80\ cm^2\ -\ I_{x1} = \frac{8x10^3}{12} = 666.67\ \text{-}\ I_{y1} = \frac{10x\ 8^3}{12} = 426.67$$

$$y_{g1} = (14.10 - 5) = 9.10cm\ -\ x_{g1} = 19 - 12.41 = 6.59cm$$

$$A1 = 75\ cm^2\ -\ I_{x1} = \frac{15x5^3}{12} = 156.25\ \text{-}\ I_{y1} = \frac{5x\ 15^3}{12} = 1406.25$$

$$y_{g1} = (27.5 - 14.10) = 13.4cm\ -\ x_{g1} = 12.41 - 7.50 = 4.91cm$$

$$A2 = 175\ cm^2\ -\ I_{x2} = \frac{7x25^3}{12} = 9114.58\ \text{-}\ I_{y2} = \frac{25x\ 7}{12} = 714.58$$

$$y_{g2} = (12.41 - 11.5) = 0.91cm\ -\ x_{g2} = 14.10 - 12.50 = 1.60cm$$

$$A3 = 80\ cm^2\ -\ I_{x1} = \frac{8x10^3}{12} = 666.67\ \text{-}\ I_{y1} = \frac{10x\ 8^3}{12} = 426.67$$

$$y_{g1} = (14.10 - 5) = 9.10cm\ -\ x_{g1} = 19 - 12.41 = 6.59cm$$

$$I_{xxg} = [156.25 + (75 * 13.4^2)] + [9114.58 + (175 * 0.91^2)] + [666.67 + (80 * 9.10^2] = \mathbf{30174.22\ cm^4}$$

$$I_{yyg} = [1406.25 + (75 * 4.91^2)] + [714.58 + (175 * 6.59^2)] + [426.67 + (80 * 6.59^2] = \mathbf{15429.77\ cm^4}$$

Momento Centrifugo => $\boldsymbol{I_{xyG} = I_{x0y0} + Ai * xi * yi}$

$$I_{yyg} = [0 + (75 * (-4.91)(13.4))] + [0 + 175 * (0.91)(-1.60))] + [0 + (80 * (6.59)(-9.10)] = \mathbf{-9986.87\ cm^4}$$

$$I_{\substack{max\\min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^{\ 2}}$$

$$I_{xxg} = \mathbf{30174.22\ cm^4}$$

$$I_{yyg} = \mathbf{15429.77\ cm^4}$$

$$I_{yyg} = \mathbf{-9986.87\ cm^4}$$

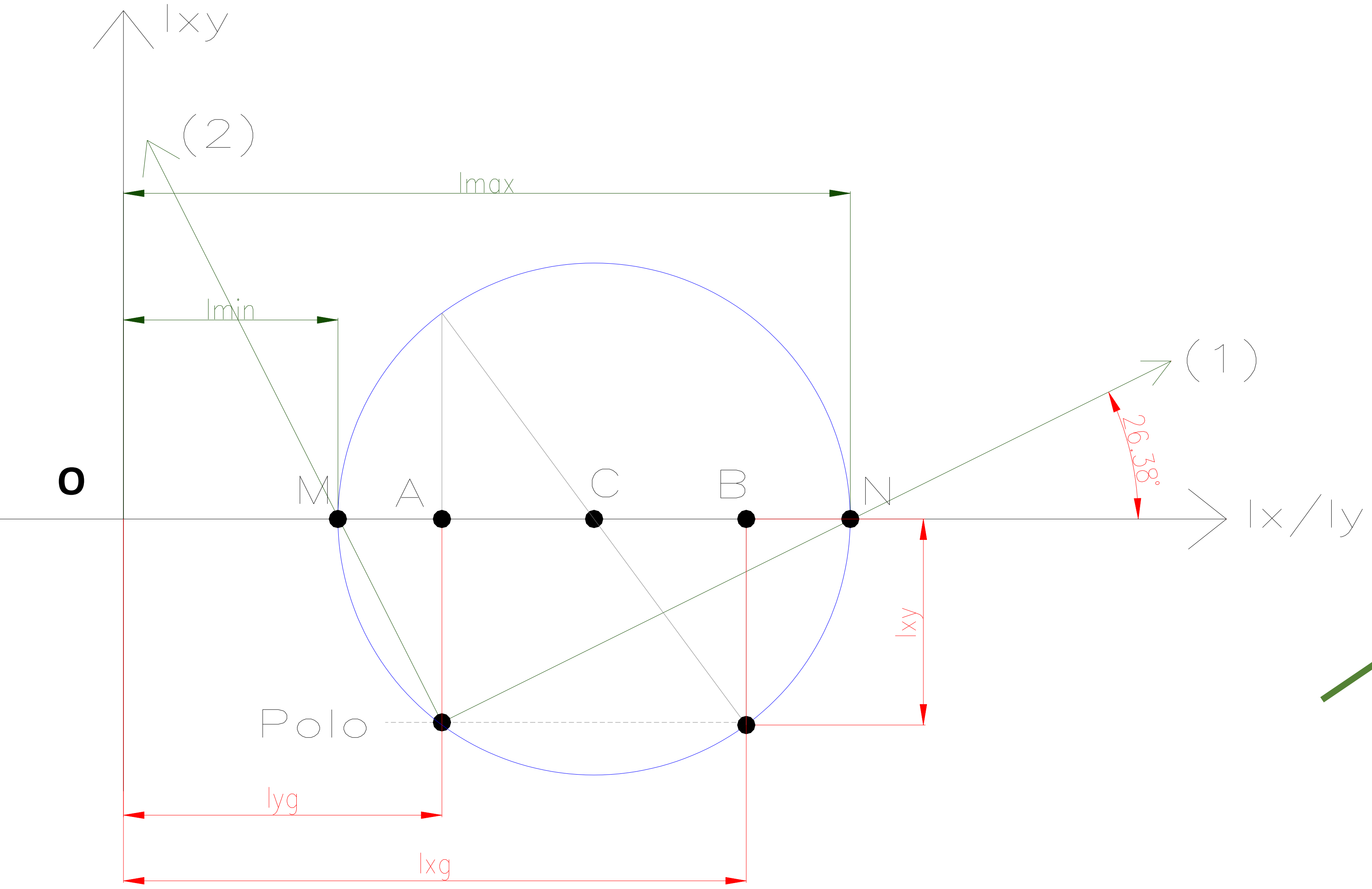
$$I_{max} = \frac{30174.22 + 15429.77}{2} + \sqrt{(\frac{30174.22 - 15429.77}{2})^2 + 9986.87^2} = 22801.99 + 12413.11 = \mathbf{35215.19\ cm^4}$$

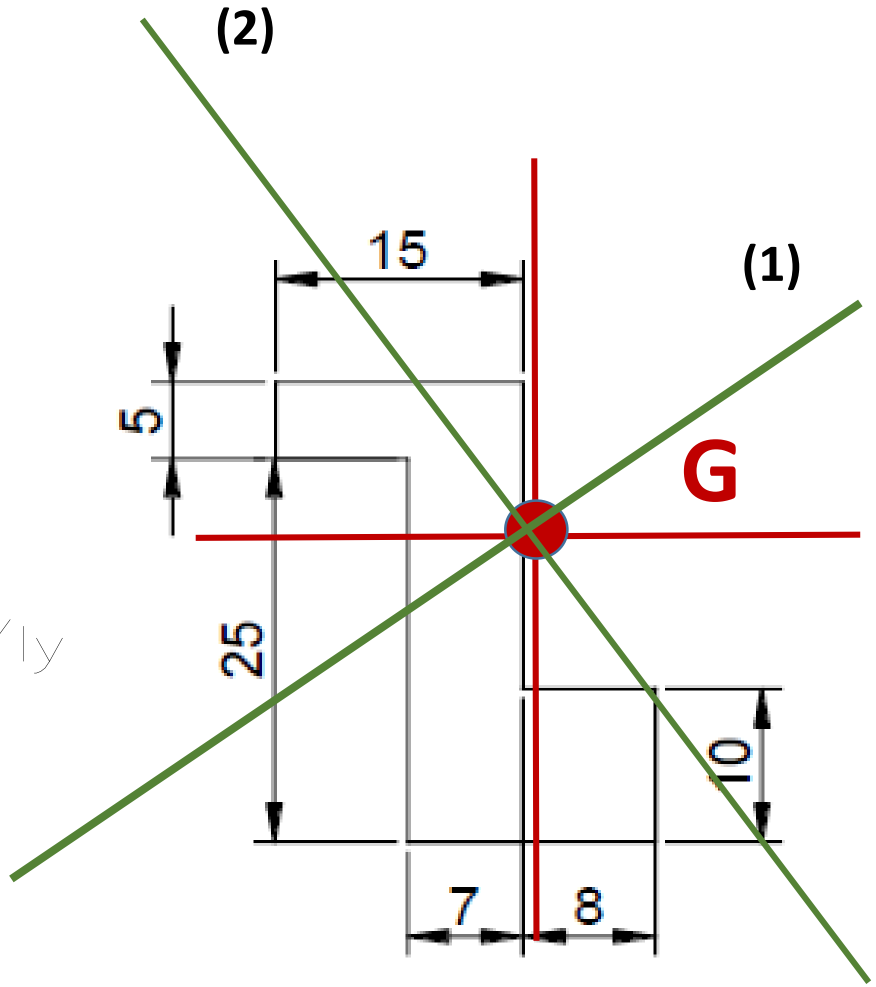
$$I_{min} = \frac{30174.22+15429.77}{2} - \sqrt{(\frac{30174.22-15429.77}{2})^2 + 9986.87^2} = 22801.99 - 12413.11 = \mathbf{10388.88\ cm^4}$$

$$tg(2\cdot\theta_o) = -\frac{2\cdot I_{xy}}{I_x - I_y}$$

$$tg\ 2\theta = -\frac{2 * (-9986.87)}{(30174.22 - 15429.77)} = 1.355 \Rightarrow \boldsymbol{\theta = 26.78°}$$

CIRCULO DE MOHR





CIRCULO DE LAND