TP4

2 (b) $\int_C (xy + y + z)\,ds = \int_0^1 f(r(t))\,\|r'(t)\|\,dt =$

$C: r(t) = (2t, t, 2-2t) \quad 0 \leq t \leq 1$

$r'(t) = (2, 1, -2) \quad \|r'(t)\| = \sqrt{4+1+4} = 3$

$= \int_0^1 \left((2t)(t) + t + (2-2t)\right) \cdot 3\,dt =$

$= 3\int_0^1 (2t^2 + t + 2 - 2t)\,dt = 3\int_0^1 (2t^2 - t + 2)\,dt =$

$= 3\left(\frac{2t^3}{3} - \frac{t^2}{2} + 2t\right)\Big|_0^1 = 3\left(\left(\frac{2}{3} - \frac{1}{2} + 2\right) - (0)\right)$

$= \boxed{13/2}$

9 (a) $r(t) = (\cos t, \sin t, -\cos t) \quad 0 \leq t \leq \pi$

$\begin{cases} x = \cos t & dx = -\sin t\,dt \\ y = \sin t & dy = \cos t\,dt \\ z = -\cos t & dz = \sin t \end{cases}$

(a) $\int_C xz\,dx = \int_0^{\pi} -\cos^2 t \cdot -\sin t\,dt =$

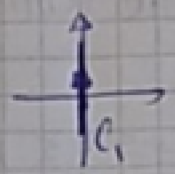
$= -\int_1^{-1} u^2\,du = -\frac{u^3}{3}\Big|_1^{-1} = -\left(-\frac{1}{3} - \left(\frac{1}{3}\right)\right)$

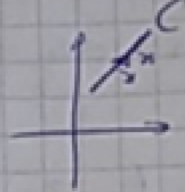
$u = \cos t$

$du = -\sin t\,dt$

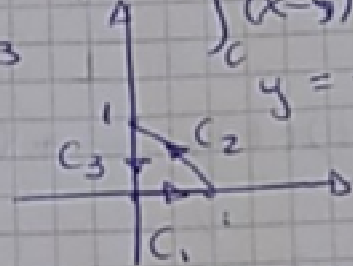
$= \boxed{\frac{2}{3}}$

(10) (a) negativa

(b) 

ó 

(14) (a) $C : C_1 \cup C_2 \cup C_3$



$\int_C (x-y)\,dx + (x+y)\,dy = \boxed{1}$

$y = -x+1$ $\quad = \frac{1}{2} + 1 + (-\frac{1}{2})$

$C_1 : r_1(t) = (t, 0)$ $\quad \begin{cases} x = t \\ y = 0 \end{cases}$ $\quad dx = 1\,dt$

$0 \le t \le 1$ $\quad dy = 0$

$r_1'(t) = (1, 0)$

$$\int_{C_1} (x-y)\,dx + (x+y)\,dy = \int_0^1 (t-0)\,dt + (t+0)0 .$$

$$= \frac{t^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$$

$C_2 : r_2(t) = (-t+1, t)$ $\quad 0 \le t \le 1$

$$\begin{cases} x = -t+1 & dx = -1\,dt \\ y = t & dy = dt . \end{cases}$$

$$\int_{C_2} (x-y)\,dx + (x+y)\,dy = \int_0^1 (-t+1-t)\cdot -dt + (-t+1+t)\,dt$$

$$= \int_0^1 (+2t - 1 + 1)\,dt = \frac{2t^2}{2}\Big|_0^1 = 1$$

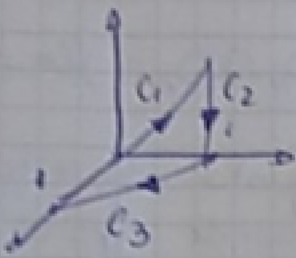
$C_3 : r_3(t) = (0, -t)$ $\quad -1 \le t \le 0 .$

$x = 0$ $\quad dx = 0$

$y = -t$ $\quad dy = -dt$

$$\int_{C_3} (x-y)\,dx + (x+y)\,dy = \int_{-1}^0 t\cdot 0 + (-t)\cdot -dt = \frac{t^2}{2}\Big|_{-1}^0 = \boxed{-1/2}$$

(22) $F = (2x, 2z, 2y)$ $\quad \int_C F\cdot T\,ds = \int_{C_1} F\cdot T\,ds + \int_{C_2} F\cdot T\,ds + \int_{C_3} F\cdot T\,ds$



$C_1: r_1(t) = (\cos t, \sin t, t) \quad 0\le t\le \pi/2$

$r_1'(t) = (-\sin t, \cos t, 1)$

$$\int_{C_1} F\cdot T\,ds = \int_0^{\pi/2} F(r(t))\cdot r'(t)\,dt = \int_0^{\pi/2} (2\cos t, 2t, 2\sin t)\cdot(-\sin t, \cos t, 1)\,dt$$

$$= \int_0^{\pi/2} (-2\cos t\sin t + 2t\cos t + 2\sin t)\,dt =$$

$$= \int_0^{\pi/2} -2\cos t\sin t\,dt + \int_0^{\pi/2} 2t\cos t\,dt + \int_0^{\pi/2} 2\sin t\,dt =$$

$u=\cos t \quad\quad u=t \quad dv=\cos t\,dt$

$$= \int_1^0 2u\,du + \left[t\cdot(\sin t) - \int_0^{\pi/2} \sin t\,dt\right|_0^{\pi/2} + 2(-\cos t\Big|_0^{\pi/2}$$

$$= u^2\Big|_1^0 + \left[\pi/2\cdot 1 - (-\cos t\Big|_0^{\pi/2} - 2(0-1)\right.$$

$$= -1 + \pi/2 + (0-1) + 2 = \boxed{\pi/2}$$

$C_2: r_2(t) = (0, 1, \pi/2(1-t)) \quad 0\le t\le 1 \quad r_2'(t) = (0, 0, -\pi/2)$

$$\int_{C_2} F\cdot T\,ds = \int_0^1 (2\cdot 0, \pi(1-t), 2)\cdot(0,0,-\pi/2)\,dt = \int_0^1 -\pi\,dt = -\pi t\Big|_0^1 = \boxed{-\pi}$$

$C_3: r_3(t) = (t, 1-t, 0) \quad 0\le t\le 1 \quad r_3'(t) = (1, -1, 0)$

$$\int_{C_3} F\cdot T\,ds = \int_0^1 (2t, 2\cdot 0, 2(1-t))\cdot(1,-1,0)\,dt = \int_0^1 2t\,dt = t^2\Big|_0^1 = \boxed{1}$$

$$\int_C F\cdot T\,ds = \pi/2 + (-\pi) + 1 = \boxed{1-\pi/2}$$

(27) $F = \left(3x^2 ; \frac{z^2}{y} ; 2z\ln y\right)$

$\quad\quad M \quad\quad N \quad\quad R.$

$D_M = \mathbb{R}^3$

(a) $D_N = \{(x,y,z) : y \neq 0\}$ $\quad D_M \cap D_N \cap D_R = \{(x,y,z) \cdot y > 0\}$

$D_R = \{(x,y,z) : y > 0\}$

Conjunto abierto (su borde $y = 0 \notin$) conexo, simplemente conexo

(b) $M_y = N_x \quad M_z = R_x \quad N_z = R_y$

$0 = 0 \quad 0 = 0 \quad \frac{2z}{y} = \frac{2z}{y}$ ✓

$F = \nabla f.$

(c) $f_x = 3x^2 \quad f(x,y,z) = \int 3x^2 dx = x^3 + \alpha(y,z)$

$f_y = \frac{z^2}{y} \quad f(x,y,z) = \int \frac{z^2}{y} dy = z^2 \cdot \ln y + \beta(x,z)$

$f_z = 2z\ln y \quad f(x,y,z) = \int 2z\ln y \, dz = z^2 \cdot \ln y + \delta(x,y)$

$f(x,y,z) = x^3 + z^2 \ln y + c$

(d) $\int_C F T ds = \int_a^b F(r(t)) \cdot r'(t) dt = f(r(b)) - f(r(a))$

$C : r(t) \quad a \leq t \leq b.$