(15) (a) $u_x = u_y$

si $u = X(x)\,Y(y)$

$$X'Y = X \cdot Y'$$

$$\frac{X'}{X}(x) = \frac{Y'}{Y}(y) \text{ luego.}$$

$$\frac{X'}{X} = \lambda = \frac{Y'}{Y}$$

$$\frac{X'}{X} = \lambda \qquad \frac{Y'}{Y} = \lambda$$

$$X' = \lambda X \qquad Y' = \lambda Y$$

$$X' - \lambda X = 0$$

$$X(x) = e^{-\int -\lambda dx}\left[\int e^{\int \lambda dx} \cdot 0\, dx + c\right]$$

$$= e^{\lambda x}\left[0 + c\right]$$

$$X(x) = C_1 e^{\lambda x} \qquad Y(y) = C_2 e^{\lambda y}$$

$$u = C_1 e^{\lambda x} \cdot C_2 e^{\lambda y}$$

$$u(x,y) = A\, e^{\lambda(x+y)}$$

15

c) $k\,u_{xx} = u_{tt} \qquad k>0.$

$u = X(x)Y(t)$

$u_x = X'Y \qquad u_y = XY'$

$u_{xx} = X''Y \qquad u_{yy} = XY''$

$$kX''Y = XY''$$

$$k\frac{X''}{X} = \frac{Y''}{Y} = \lambda$$

$$k\frac{X''}{X} = \lambda$$

$$kX'' - \lambda X = 0$$

$$kr^2 - \lambda = 0$$

• $\lambda > 0$

$r^2 = \lambda/k = \alpha^2$

$r = \pm\alpha$

$X = C_1 e^{\alpha x} + C_2 e^{-\alpha x}$

• $\lambda = 0$

$X = C_1 e^{0x} + C_2 x e^{0x}$

$= C_1 + C_2 x$

• $\lambda < 0$

$r^2 = -\alpha^2$

$r = \pm\alpha i$

$X = C_1\cos\alpha x + C_2\sin\alpha x$

$$\frac{Y''}{Y} = \lambda$$

$\lambda > 0$.

$Y'' - \lambda Y = 0$.

$r^2 - \lambda = 0$

$r_1 = \sqrt{\lambda} \quad r_2 = -\sqrt{\lambda}$

$Y = C_1 e^{\sqrt{\lambda} t} + C_2 e^{-\sqrt{\lambda} t}$

$\lambda < 0$.

$r^2 - \lambda = 0$

$r_{1,2} = \pm \alpha i$

$Y = C_1 \cos \alpha t + C_2 \sin \alpha t$

$\lambda = 0$.

$r_{1,2} = 0$.

$Y = C_1 e^{0t} + C_2 e^{0y} t$

$\boxed{Y = C_1 + C_2 t}$

$U = X \cdot Y$

$\lambda > 0$

$U = (C_1 e^{\sqrt{\lambda} x} + C_2 e^{-\sqrt{\lambda} x})(A e^{\sqrt{\lambda} t} + B e^{-\sqrt{\lambda} t})$

$\lambda = 0$

$U = (C_1 + C_2 x)(A + Bt)$

$\lambda < 0 \qquad \alpha =$

$U = (C_1 \cos \alpha x + C_2 \sin \alpha x)(A \cos \alpha t + B \sin \alpha t)$

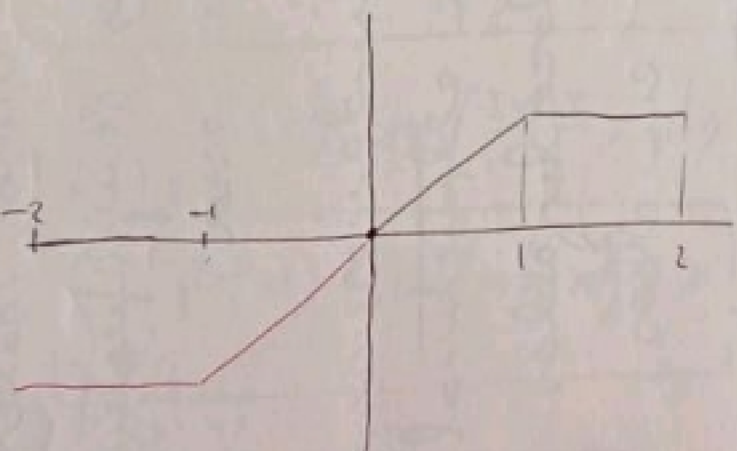
b)

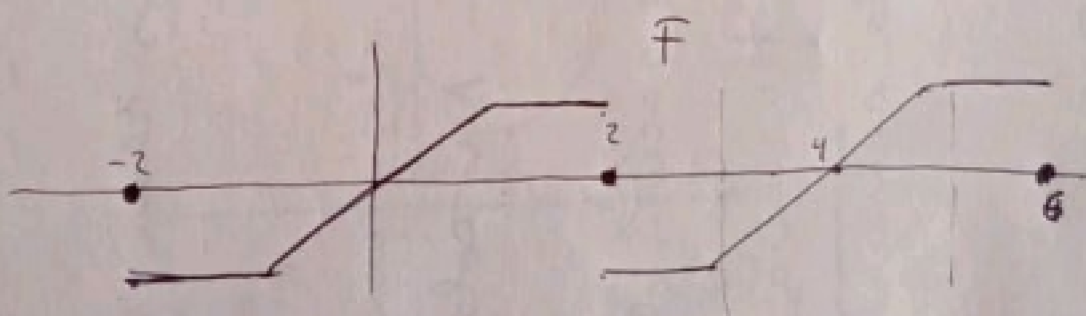
$$A = -\frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}\right)$$

$$B = \frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{2}{n\pi}(-1)^n$$

$$b_n = -\left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}\right) - \frac{2}{n\pi}(-1)^n$$

$$F(x) = \sum_{n=1}^{\infty}\left[-\left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}\right) - \frac{2}{n\pi}(-1)^n\right]\sin\left(\frac{n\pi}{2}x\right)$$





$$A = \frac{2}{n\pi} x \sin\left(\frac{n\pi}{2}x\right)\Big|_0^1 - \frac{2}{n\pi}\int_0^1 \sin\left(\frac{n\pi}{2}x\right)dx$$

$$= \frac{2}{n\pi} x \sin\left(\frac{n\pi}{2}x\right)\Big|_0^1 + \left(\frac{2}{n\pi}\right)^2 \cos\left(\frac{n\pi}{2}x\right)\Big|_0^1$$

$$= \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \left(\frac{2}{n\pi}\right)^2\left(\cos\left(\frac{n\pi}{2}\right) - 1\right)$$

$$B = \left(\frac{2}{n\pi}\right)\sin\left(\frac{n\pi}{2}x\right)\Big|_1^2 = -\left(\frac{2}{n\pi}\right)\sin\left(\frac{n\pi}{2}\right)$$

$$a_n = \left(\frac{2}{n\pi}\right)^2\left(\cos\left(\frac{n\pi}{2}\right) - 1\right)$$

$$f(x) = \frac{3}{4} + \sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\right)^2\left(\cos\left(\frac{n\pi}{2}\right) - 1\right)\cos\left(\frac{n\pi}{2}x\right)$$

Serie de senos

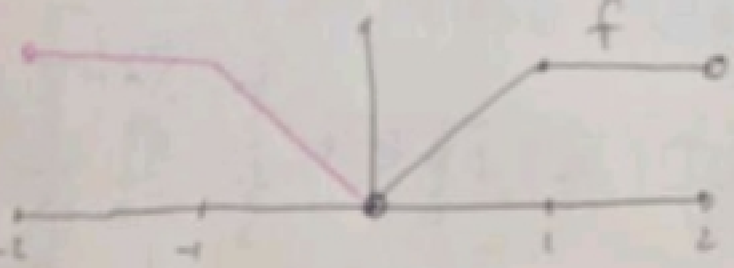
$$b_n = \frac{2}{p}\int_0^p f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

$$= \int_0^2 f(x)\sin\left(\frac{n\pi}{2}x\right)dx$$

$$= \underbrace{\int_0^1 x\sin\left(\frac{n\pi}{2}x\right)dx}_{A} + \underbrace{\int_1^2 \sin\left(\frac{n\pi}{2}x\right)dx}_{B}$$

(11.b) $f(x)=\begin{cases} x & 0<x<1 \\ 1 & 1\le x<2 \end{cases}$

$(0,2)$



Serie de cosenos

$a_0,\ a_n$

$$a_0=\frac{2}{p}\int_0^p f(x)\,dx=\int_0^2 f(x)\,dx=$$

$$\int_0^1 x\,dx+\int_1^2 1\,dx=\frac{x^2}{2}\Big|_0^1+x\Big|_1^2$$

$$=\frac{1}{2}+1=\boxed{\frac{3}{2}}\to\boxed{\frac{3}{4}}$$

$$a_n=\frac{2}{p}\int_0^p f(x)\cos\left(\frac{n\pi}{p}x\right)dx=\int_0^2 f(x)\cos\left(\frac{n\pi}{2}x\right)dx$$

$$=\underbrace{\int_0^1 x\cos\left(\frac{n\pi}{2}x\right)dx}_{A}+\underbrace{\int_1^2 \cos\left(\frac{n\pi}{2}x\right)dx}_{B}$$

$u=x \qquad dv=\cos\left(\frac{n\pi}{2}x\right)dx$

$du=dx \qquad v=\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}x\right)$

(14) $f(x) = \begin{cases} 1/2 & 0 < x < 1 \\ x^2 & 1 \le x < 2 \end{cases}$ $\quad (0, L)$ $\quad (0, 2)$

$F(2) \quad F(-1) \quad F(5/2)$

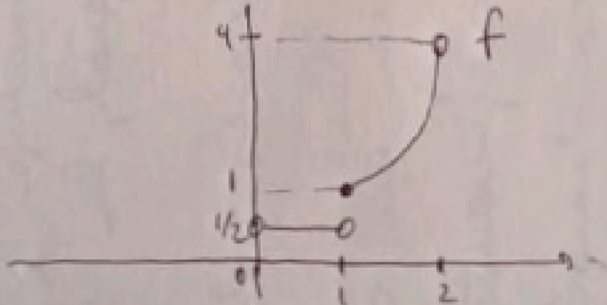
$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \boxed{\int_0^2 f(x)\,dx}$$

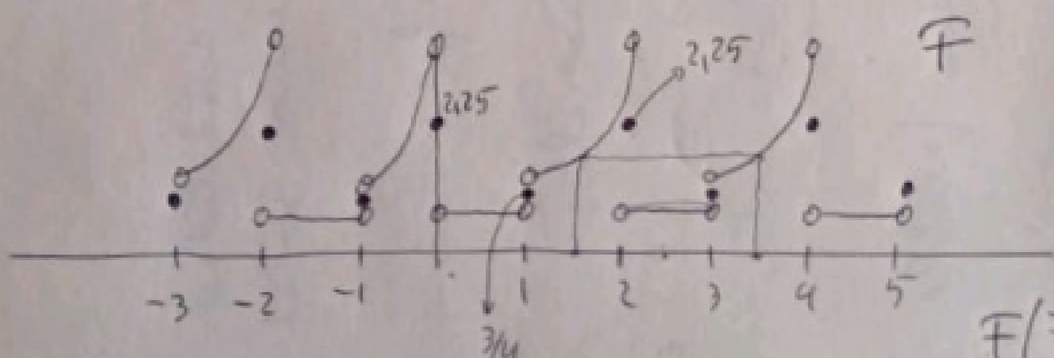
$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2n\pi}{L}x\right)dx$$

$$= \boxed{\int_0^2 f(x)\cos(n\pi x)\,dx}$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2n\pi}{L}x\right)dx$$

$$= \boxed{\int_0^2 f(x)\sin(n\pi x)\,dx}$$





$F(7/2) = f(3/2)$

$F(2) = 2{,}25 \quad F(-1) = 3/4 \quad F(5/2) = 1/2$