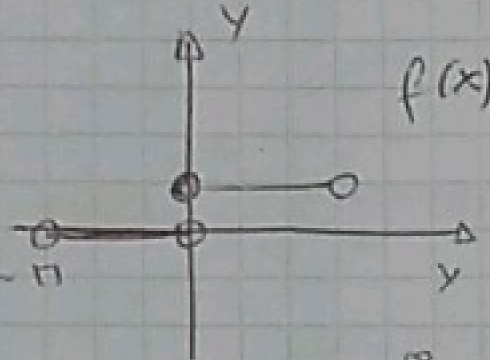
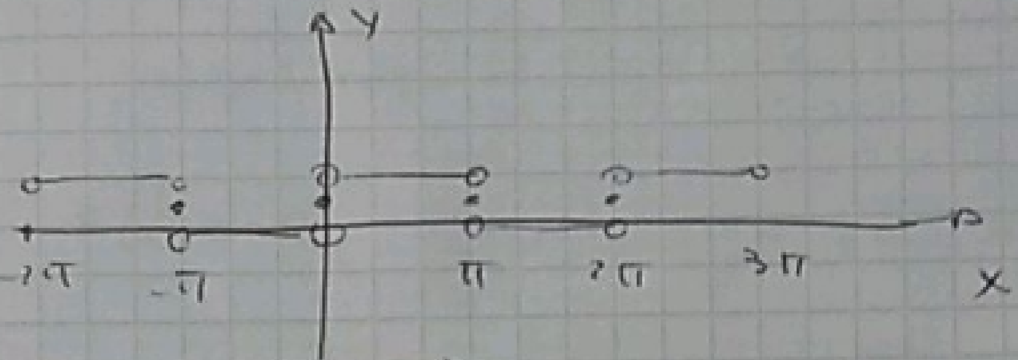
TP6

4(b) $f(x)=\begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \leq x < \pi \end{cases}$





$$F(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{p}+b_n \operatorname{sen}\frac{n\pi x}{p}\right)$$

$$a_0=\frac{1}{p}\int_{-p}^{p} f(x)\,dx=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx=\frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx+\int_{0}^{\pi} 1\,dx\right]$$

$$=\frac{1}{\pi}\left[\,x\Big|_0^{\pi}\right]=\frac{1}{\pi}\cdot\pi=\boxed{1}$$

$$a_n=\frac{1}{p}\int_{-p}^{p} f(x)\cdot\cos\frac{n\pi x}{\pi}\,dx=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx=\frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx+\int_{0}^{\pi}\cos nx\,dx\right]$$

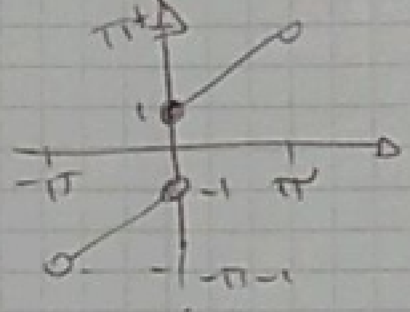
$$=\frac{1}{\pi}\left[\frac{\operatorname{sen} nx}{n}\Big|_0^{\pi}\right]=\boxed{0}$$

$$b_n=\frac{1}{p}\int_{-p}^{p} f(x)\operatorname{sen}\frac{n\pi x}{\pi}\,dx=\frac{1}{\pi}\left[\int_{-\pi}^{0} 0\cdot\operatorname{sen} nx\,dx+\int_{0}^{\pi}\operatorname{sen} nx\,dx\right]$$

$$=\frac{1}{\pi}\left[-\frac{\cos nx}{n}\Big|_0^{\pi}\right]=\frac{1}{\pi}\left(\frac{-(-1)^n+1}{n}\right)=\boxed{\frac{1-(-1)^n}{n\pi}}$$

$$f(x)\cong F(x)=\frac{1}{2}+\sum_{n=1}^{\infty}\left(0\cdot\cos nx+\frac{1-(-1)^n}{n\pi}\operatorname{sen} nx\right)$$

10 ⓐ $f(x) = \begin{cases} x-1 & -\pi < x < 0 \\ x+1 & 0 \le x < \pi \end{cases}$



$f(x) \simeq F(x)$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{p} + b_n \operatorname{sen}\frac{n\pi x}{p}\right)$$

$$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{p}\int_0^p f(x) \operatorname{sen}\frac{n\pi x}{p}\,dx$$

$$b_n = \frac{2}{\pi}\int_0^\pi (x+1)\operatorname{sen}\frac{n\pi x}{\pi}\,dx = \frac{2}{\pi}\left[\int_0^\pi x \operatorname{sen} nx\,dx + \int_0^\pi \operatorname{sen} nx\,dx\right]$$

$$= \frac{2}{\pi}\left[x\cdot\left(-\frac{\cos nx}{n}\right)\Big|_0^\pi - \int_0^\pi -\frac{\cos nx}{n}\,dx - \frac{\cos nx}{n}\Big|_0^\pi\right] =$$

$u = x \quad dv = \operatorname{sen} nx\,dx$

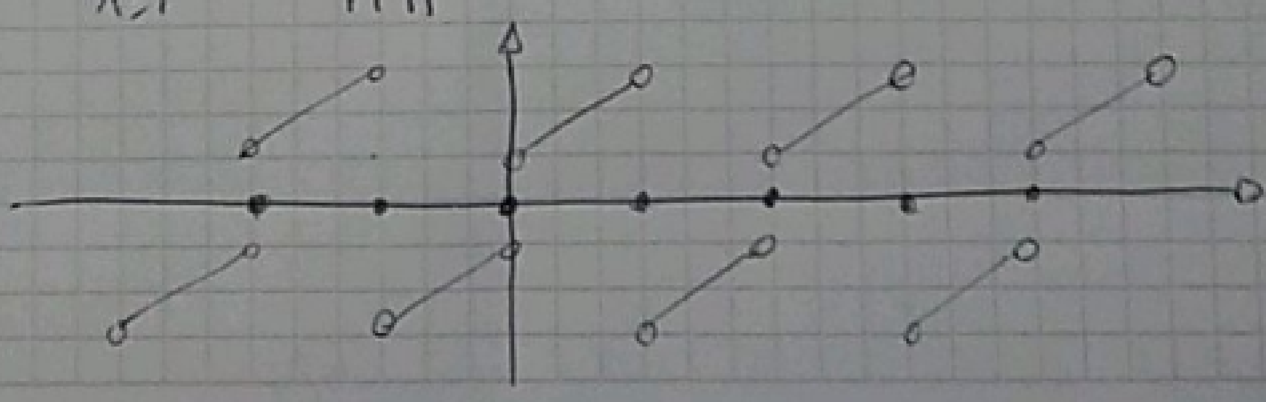
$du = dx \quad v = -\frac{\cos nx}{n}$

$$= \frac{2}{\pi}\left[+\pi\cdot -\frac{(-1)^n}{n} - 0 + \cancel{\frac{\operatorname{sen} nx}{n^2}\Big|_0^\pi} - \frac{(-1)^n}{n} + \frac{1}{n}\right]$$

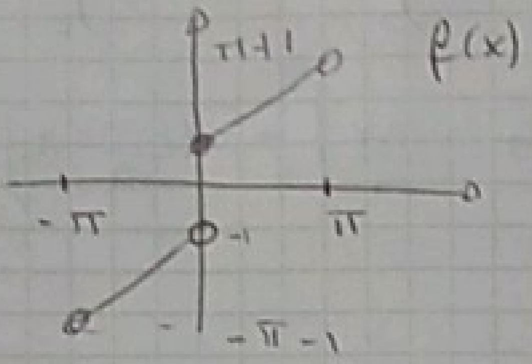
0

$$= \frac{2}{\pi}\left[\frac{-\pi\cdot(-1)^n - (-1)^n + 1}{n}\right]$$

$$F(x) = \sum_{n=1}^{\infty} \frac{2\left(1-(-1)^n(\pi+1)\right)}{n\pi}\operatorname{sen} nx$$



10 ⓐ $f(x) = \begin{cases} x-1 & -\pi < x < 0 \\ x+1 & 0 \leq x < \pi \end{cases}$ $(-\pi, \pi)$



$F = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{p} + b_n \sin \frac{n\pi x}{p} \right)$

$a_0 = \frac{1}{p} \int_{-p}^{p} f(x)\,dx$

$a_n = \frac{1}{p} \int_{-p}^{p} f(x) \cos\left(\frac{n\pi x}{p}\right) dx$

$b_n = \frac{1}{p} \int_{-p}^{p} f(x) \sin\left(\frac{n\pi x}{p}\right) dx$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi} \left[\int_{-\pi}^{0} (x-1)\,dx + \int_{0}^{\pi} (x+1)\,dx \right] =$$

$$= \frac{1}{\pi} \left[\frac{x^2}{2} - x \Big|_{-\pi}^{0} + \frac{x^2}{2} + x \Big|_{0}^{\pi} \right] = \frac{1}{\pi} \left[-\left(\frac{\pi^2}{2} + \pi \right) + \frac{\pi^2}{2} + \pi - 0 \right]$$

$$= 0.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos \frac{n\pi x}{\pi}\,dx = \frac{1}{\pi} \left[\int_{-\pi}^{0} (x-1) \cos nx\,dx + \int_{0}^{\pi} (x+1) \cos nx\,dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} x \cos nx\,dx - \int_{-\pi}^{0} \cos nx\,dx + \int_{0}^{\pi} x \cos nx\,dx + \int_{0}^{\pi} \cos nx\,dx \right].$$

$$= \frac{1}{\pi} \left[x \cdot \frac{\sin nx}{n} \Big|_{-\pi}^{0} - \int_{-\pi}^{0} \frac{\sin nx}{n}\,dx - \frac{\sin nx}{n} \Big|_{-\pi}^{0} + x \frac{\sin nx}{n} \Big|_{0}^{\pi} - \int_{0}^{\pi} \frac{\sin nx}{n}\,dx + \frac{\sin nx}{n} \Big|_{0}^{\pi} \right]$$

$u = x \quad dv = \cos nx\,dx$

$du = dx \quad v = \frac{\sin nx}{n}$

$$= \frac{1}{\pi} \left[0 + \frac{\cos nx}{n^2} \Big|_{-\pi}^{0} + 0 + 0 + \frac{\cos nx}{n^2} \Big|_{0}^{\pi} + 0 \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{n^2} - \frac{(-1)^n}{n^2} + \frac{(-1)^n - 1}{n^2} \right] = \frac{1}{\pi} [0] = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin\frac{n\pi x}{\pi}\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0}(x-1)\sin nx\,dx + \int_{0}^{\pi}(x+1)\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} x\sin nx\,dx - \int_{-\pi}^{0}\sin nx\,dx + \int_{0}^{\pi} x\sin nx\,dx + \int_{0}^{\pi}\sin nx\,dx\right] =$$

$$= \frac{1}{\pi}\left[-x\frac{\cos nx}{n}\Big|_{-\pi}^{0} + \int_{-\pi}^{0}\frac{\cos nx}{n}dx + \frac{\cos nx}{n}\Big|_{-\pi}^{0} + -x\frac{\cos nx}{n} + \int_{0}^{\pi}\frac{\cos nx}{n}dx - \frac{\cos nx}{n}\Big|_{0}^{\pi}\right.$$

$u = x \quad dv = \sin nx\,dx$

$du = dx \quad v = -\frac{\cos nx}{n}$

$$= \frac{1}{\pi}\left[0 - \frac{\pi(-1)^n}{n} + \frac{\sin nx}{n^2}\Big|_{-\pi}^{0} + \frac{1}{n} - \frac{(-1)^n}{n} - \frac{\pi(-1)^n}{n} + \frac{\sin nx}{n^2}\Big|_{0}^{\pi} - \frac{(-1)^n}{n} + \frac{1}{n}\right]$$

$$= \frac{1}{\pi}\left[-\pi\frac{(-1)^n}{n} + 0 + 0 - \frac{(-1)^n}{n} - \frac{\pi(-1)^n}{n} + 0 - \frac{(-1)^n}{n} + \frac{1}{n}\right]$$

$$\frac{2 - 2(-1)^n(\pi+1)}{n\pi}$$