

# TABLAS DE RIGIDECES

CALCULO E INGENIERIA

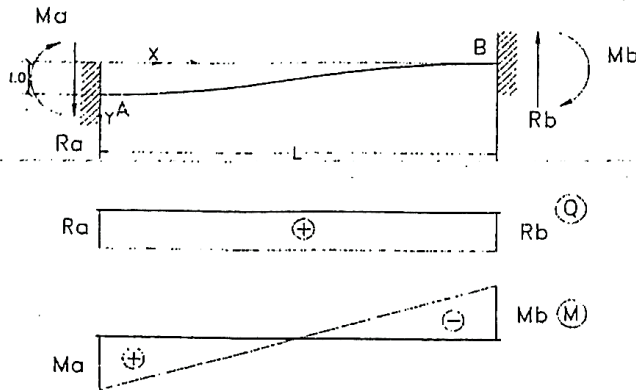
CONCRETO ARMADO

## SOLUCIONES DE BARRAS EMPOTRADA - EMPOTRADA Y EMPOTRADA-ARTICULADA

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Colaboración: Ing. Cynthia Soledad Ricco

BARRA EMP-EMP CARGADA CON UN DESPLAZAMIENTO "LO" EN A:



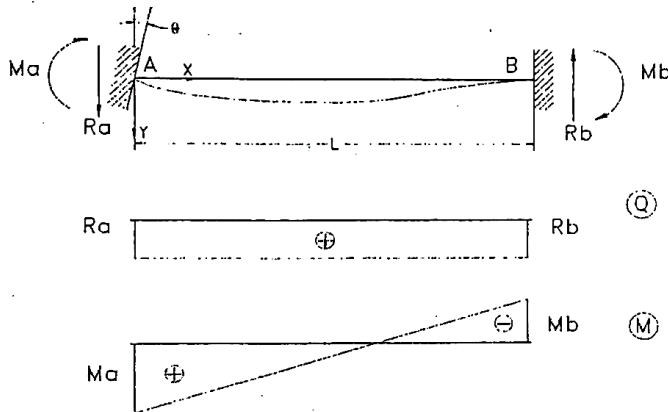
$$R_a = 12 \cdot \frac{E \cdot J}{L^3} \cdot L_0$$

$$R_b = 12 \cdot \frac{E \cdot J}{L^3} \cdot L_0$$

$$M_a = 6 \cdot \frac{E \cdot J}{L^2} \cdot L_0$$

$$M_b = 6 \cdot \frac{E \cdot J}{L^2} \cdot L_0$$

BARRA EMP-ARTIC CARGADA CON UN GIRO EN A:



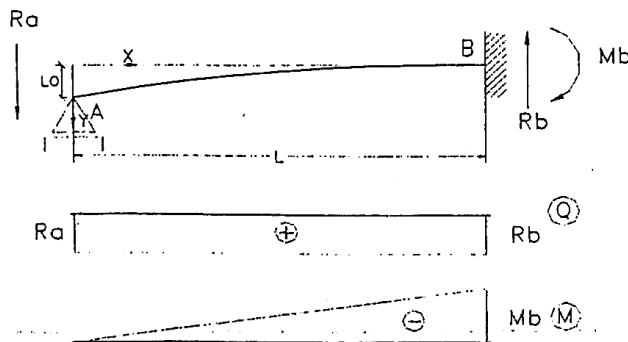
$$R_a = 6 \cdot \frac{E \cdot J}{L^2} \cdot \theta$$

$$R_b = 6 \cdot \frac{E \cdot J}{L^2} \cdot \theta$$

$$M_a = 4 \cdot \frac{E \cdot J}{L} \cdot \theta$$

$$M_b = 2 \cdot \frac{E \cdot J}{L} \cdot \theta$$

BARRA ARTIC-EMP CARGADA CON UN DESPLAZAMIENTO "LO" EN A:

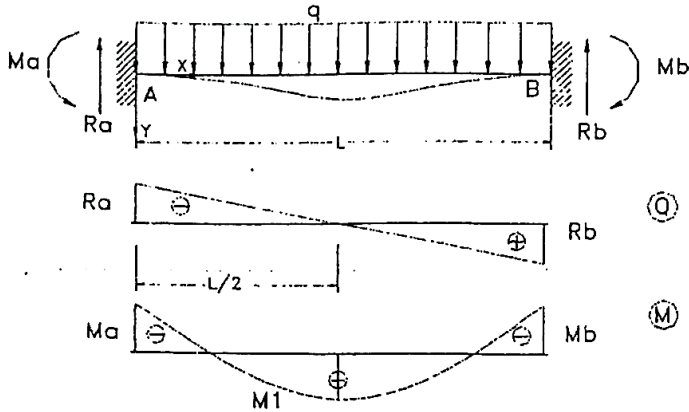


$$R_a = 3 \cdot \frac{E \cdot J}{L^3} \cdot L_0$$

$$R_b = 3 \cdot \frac{E \cdot J}{L^3} \cdot L_0$$

$$M_a = 3 \cdot \frac{E \cdot J}{L^2} \cdot L_0$$

BARRA EMP-EMP CARGADA CON UNA CARGA UNIFORME:



$$R_a = \frac{q}{2} * L$$

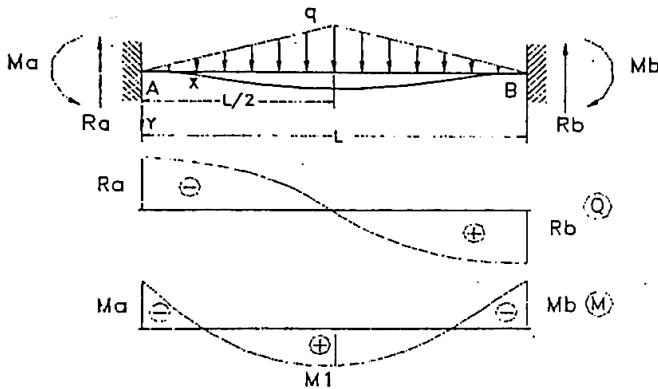
$$R_b = \frac{q}{2} * L$$

$$M_a = \frac{q}{12} * L^2$$

$$M_b = \frac{q}{12} * L^2$$

Si  $x_1 = \frac{L}{2} \rightarrow M_1 = \frac{q}{24} * L^2$

BARRA EMP-EMP CARGADA CON CARGA TRIANGULAR:



$$R_a = \frac{q}{4} * L$$

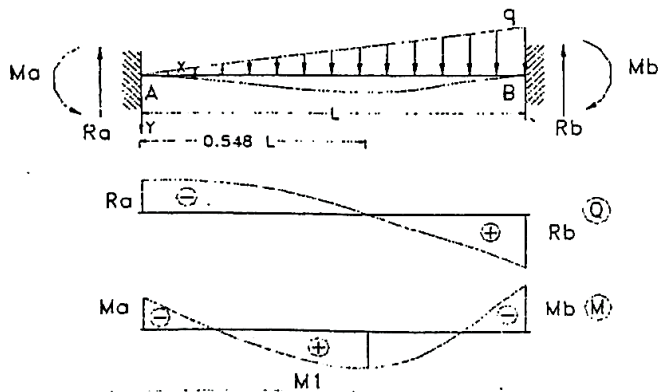
$$R_b = \frac{3q}{4} * L$$

$$M_a = \frac{5}{96} * q * L^2$$

$$M_b = \frac{5}{96} * q * L^2$$

Si  $x_1 = \frac{L}{2} \rightarrow M_1 = \frac{q}{32} * L^2$

BARRA EMP-EMP CARGADA CON UNA CARGA LINEAL:



$$R_a = \frac{3}{20} * q * L$$

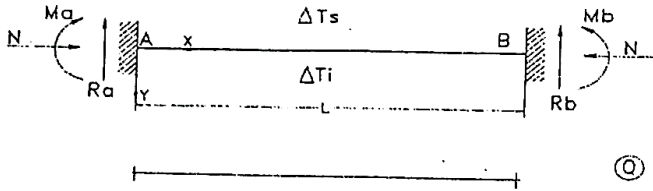
$$R_b = \frac{7}{20} * q * L$$

$$M_a = \frac{q}{30} * L^2$$

$$M_b = \frac{q}{20} * L^2$$

Si  $x_1 = 0.548 * L \rightarrow M_1 = \frac{q}{46.6} * L^2$

BARRA EMP-EMP CARGADA CON TEMPERATURA:



$R_a = 0$

$R_b = 0$

$M_a = \lambda * \left( \frac{\Delta T_i}{h} - \Delta T_s \right) * E * J$

$M_b = \lambda * \left( \frac{\Delta T_i}{h} - \Delta T_s \right) * E * J$

ⓐ

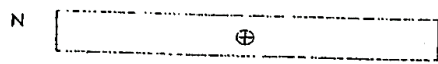
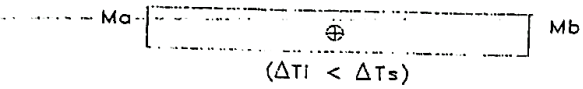
Ⓜ

$\lambda$  : coeficiente de dilatacion lineal del material

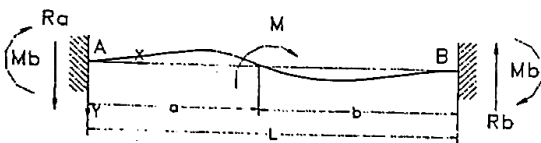
Ⓝ

$N = \lambda * \Delta T_G * E * A$

$\Delta T_G = \left( \frac{\Delta T_i}{2} + \Delta T_s \right) < 0$



BARRA EMP-EMP CARGADA CON UN MOMENTO M A UNA DISTANCIA a:



$R_a = 6 * M * \frac{a * b}{L^3}$

$R_b = 6 * M * \frac{a * b}{L^3}$

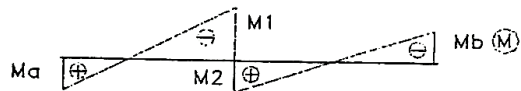
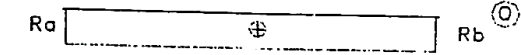
$M_a = M * \frac{b}{L^2} * (2 * a - b)$

$M_b = M * \frac{a}{L^2} * (2 * b - a)$

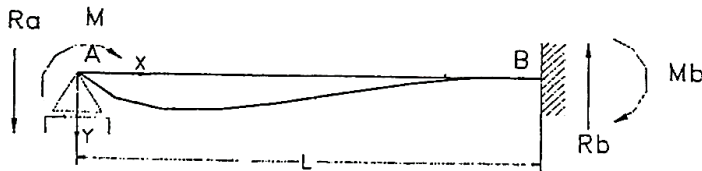
En  $x_1 = a \rightarrow$

$M_1 = M * \left( 1 - 4 * \frac{a}{L} + 9 * \frac{a^2}{L^2} - 6 * \frac{a^3}{L^3} \right)$

$M_2 = M * \left( 4 * \frac{a}{L} - 9 * \frac{a^2}{L^2} + 6 * \frac{a^3}{L^3} \right)$



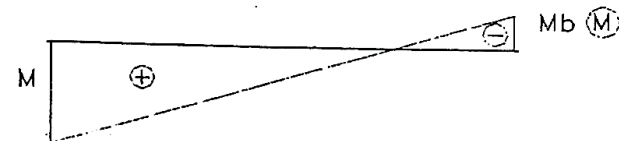
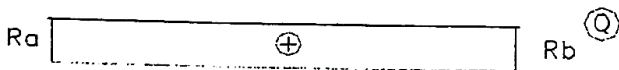
BARRA ARTIC-EMP CARGADA CON UN MOMENTO M EN A:



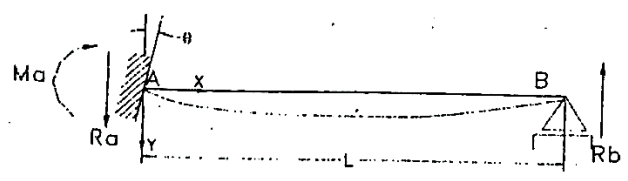
$R_a = \frac{3}{2} * \frac{M}{L}$

$R_b = \frac{3}{2} * \frac{M}{L}$

$M_b = \frac{M}{2}$



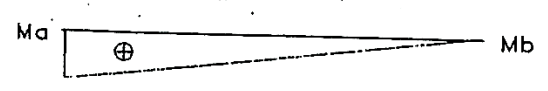
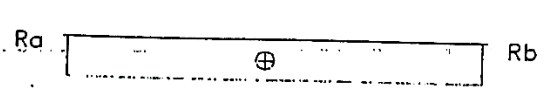
BARRA EMP-ARTIC CARGADA CON UN GIRO EN A:



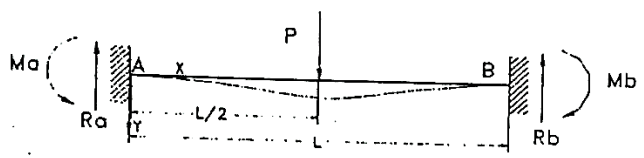
$$R_a = 3 \cdot \frac{E \cdot J}{L^2} \cdot \theta$$

$$R_b = 3 \cdot \frac{E \cdot J}{L^2} \cdot \theta$$

$$M_a = 3 \cdot \frac{E \cdot J}{L} \cdot \theta$$



BARRA EMP-EMP CARGADA CON UNA CARGA P EN L/2:

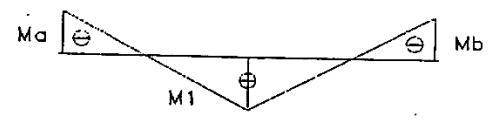
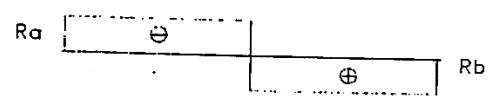


$$R_a = \frac{1}{2} \cdot P$$

$$R_b = \frac{1}{2} \cdot P$$

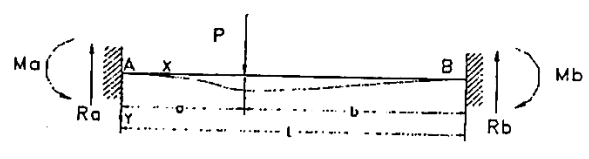
$$M_a = \frac{1}{8} \cdot P \cdot L$$

$$M_b = \frac{1}{8} \cdot P \cdot L$$



Si  $x_1 = \frac{L}{2} \rightarrow M_1 = \frac{1}{8} \cdot P \cdot L$

BARRA EMP-EMP CARGADA CON UNA CARGA P EN a:

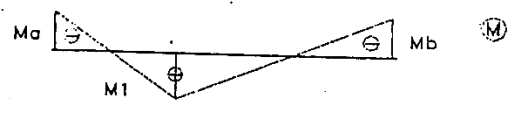
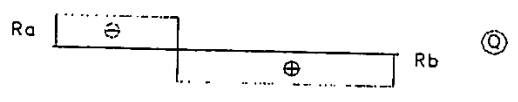


$$R_a = P \cdot \frac{b^2}{L^3} \cdot (3 \cdot a + b)$$

$$R_b = P \cdot \frac{a^2}{L^3} \cdot (3 \cdot b + a)$$

$$M_a = P \cdot a \cdot \frac{b^2}{L^2}$$

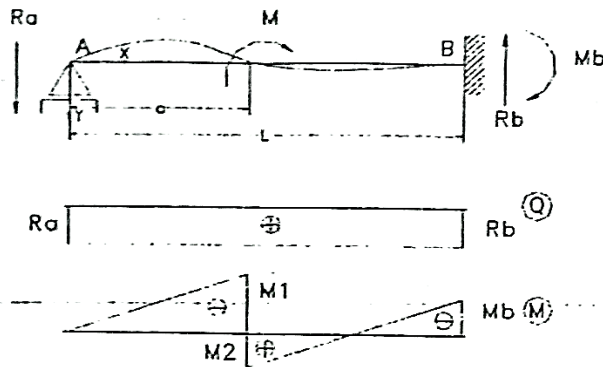
$$M_b = P \cdot b \cdot \frac{a^2}{L^2}$$



Si  $x_1 = a \rightarrow M_1 = 2 \cdot P \cdot \frac{a^2 \cdot b^2}{L^3}$

Si  $a < b \rightarrow |M_b| < |M_1| < |M_a|$   
 Si  $a > b \rightarrow |M_a| < |M_1| < |M_b|$   
 Si  $a = \frac{L}{3} \rightarrow M_a = \frac{4}{27} \cdot P \cdot L$   
 (máximo posible)

**BARRA ARTIC-EMP CARGADA CON UN MOMENTO M A UNA DISTANCIA a:**



$$R_a = \frac{3}{2} * \frac{M}{L} * (1 - \frac{a^2}{L^2})$$

$$R_b = \frac{3}{2} * \frac{M}{L} * (1 + \frac{a^2}{L^2})$$

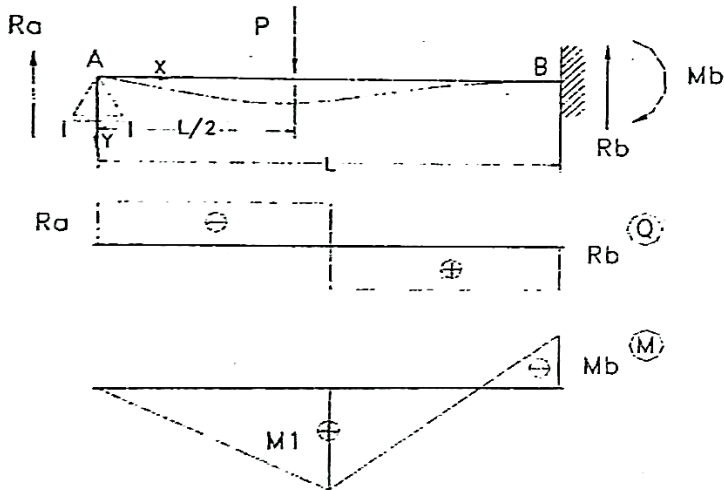
$$M_b = \frac{M}{2} * (1 - 3 * \frac{a^2}{L^2})$$

$$M_1 = \frac{3}{2} * \frac{M}{L} * a * (1 - \frac{a^2}{L^2})$$

$$M_2 = M * [1 - \frac{3}{2} * \frac{a}{L} * (1 - \frac{a^2}{L^2})]$$

Si  $a < 0.275 * L \rightarrow M_1 < |M_b|$   
 Si  $a = 0.577 * L \rightarrow M_b = 0$

**BARRA ARTIC-EMP CARGADA CON UNA CARGA P EN L/2:**

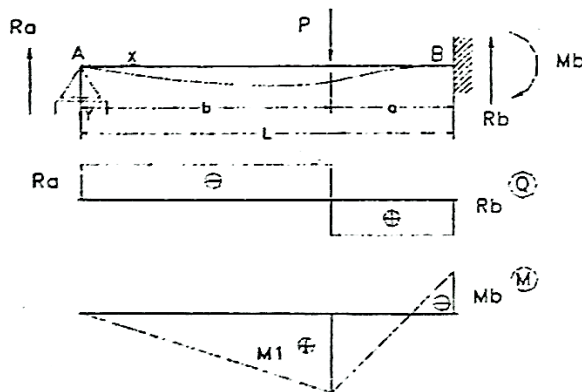


$$R_a = \frac{5}{16} * P$$

$$R_b = \frac{11}{16} * P$$

$$M_b = \frac{3}{16} * P * L$$

**BARRA ARTIC-EMP CARGADA CON UNA CARGA P A UNA DISTANCIA b:**



$$R_a = \frac{P}{2} * \frac{a^2}{L^2} * (3 - \frac{a}{L})$$

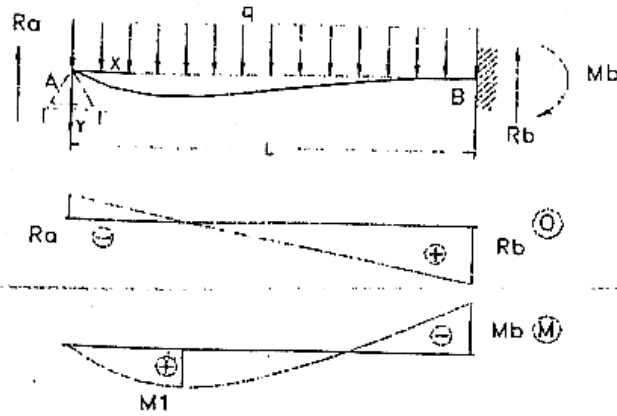
$$R_b = \frac{P}{2} * \frac{b}{L} * (3 - \frac{b^2}{L^2})$$

$$M_b = \frac{P * a * b}{2 * L^2} * (L + b)$$

$$M_1 = \frac{P * a^2 * b}{2 * L^2} * (3 - \frac{a}{L})$$

Si  $a = 0.634 * L \rightarrow M_1 = 0.174 * P * L$   
 Si  $a = 0.423 * L \rightarrow M_b = 0.193 * L$

BARRA ARTIC-EMP CARGADA CON UNA CARGA UNIFORME:



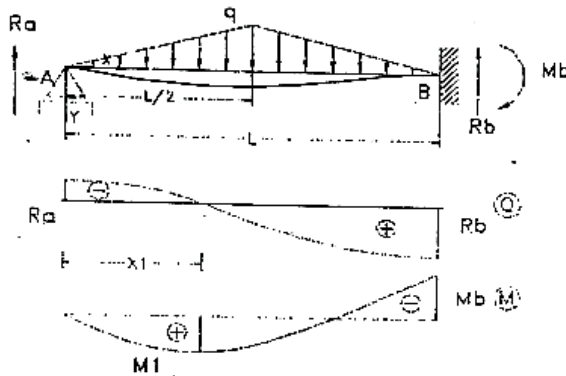
$$R_a = \frac{3}{8} * q * L$$

$$R_b = \frac{5}{8} * q * L$$

$$M_b = \frac{1}{8} * q * L^2$$

$$En \ x1 = \frac{3}{8} * L \rightarrow M1 = \frac{9}{128} * q * L^2$$

BARRA ARTIC-EMP CARGADA CON UNA CARGA LINEAL CON MAX EN L/2:



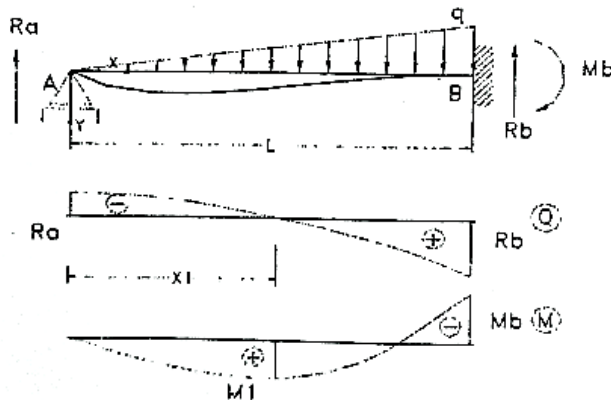
$$R_a = \frac{11}{64} * q * L$$

$$R_b = \frac{21}{64} * q * L$$

$$M_b = \frac{5}{64} * q * L^2$$

$$En \ x1 = 0.415 * L \rightarrow M1 = \frac{3}{64} * q * L^2$$

BARRA ARTIC-EMP CARGADA CON UNA CARGA LINEAL CON MAX EN B:



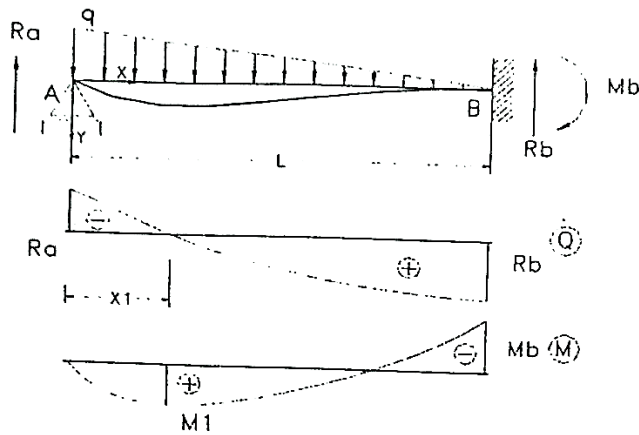
$$R_a = \frac{1}{10} * q * L$$

$$R_b = \frac{2}{5} * q * L$$

$$M_b = \frac{1}{15} * q * L^2$$

$$En \ x1 = 0.447 * L \rightarrow M1 = \frac{q * L^2}{15 * \sqrt{5}}$$

BARRA ARTIC-EMP CARGADA CON UNA CARGA LINEAL CON MAX EN A:



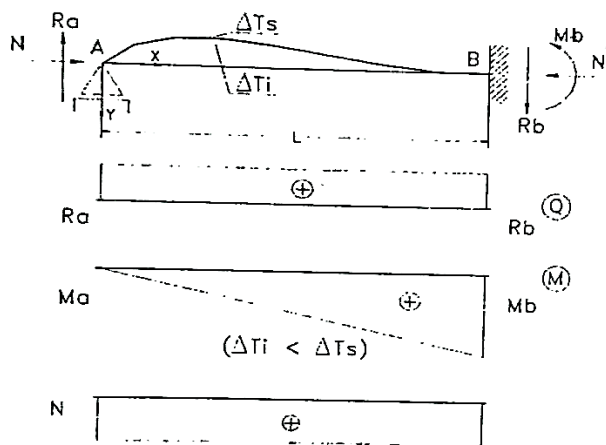
$$R_a = \frac{11}{40} \cdot q \cdot L$$

$$R_b = \frac{9}{40} \cdot q \cdot L$$

$$M_b = \frac{7}{120} \cdot q \cdot L^2$$

$$\text{En } x_1 = 0.329 \cdot L \rightarrow M_1 = \frac{q \cdot L^2}{23.6}$$

BARRA ARTIC-EMP CARGADA CON TEMPERATURA:



$$R_a = \frac{3}{2} \cdot \lambda \cdot \frac{\Delta T_i - \Delta T_s}{L} \cdot E \cdot J$$

$$R_b = \frac{3}{2} \cdot \lambda \cdot \frac{\Delta T_i - \Delta T_s}{L} \cdot E \cdot J$$

$$M_b = \frac{3}{2} \cdot \lambda \cdot (\Delta T_i - \Delta T_s) \cdot E \cdot J$$

$\lambda$  : coeficiente de dilatación lineal del material

$$N = \lambda \cdot \Delta T_G \cdot E \cdot A$$

$$\Delta T_G = \frac{(\Delta T_i + \Delta T_s)}{2} < 0$$