TEORÍA ELASTICIDAD

TP#2 CALCULO DE CORRIMIENTOS

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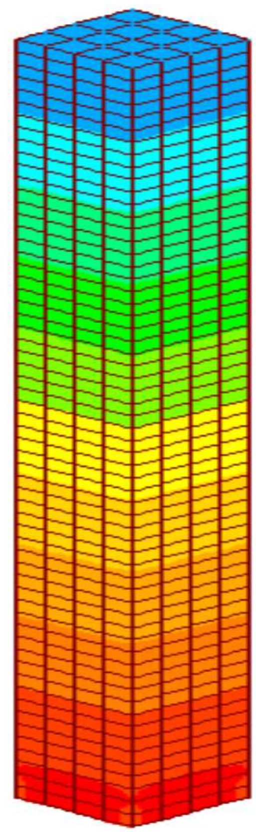
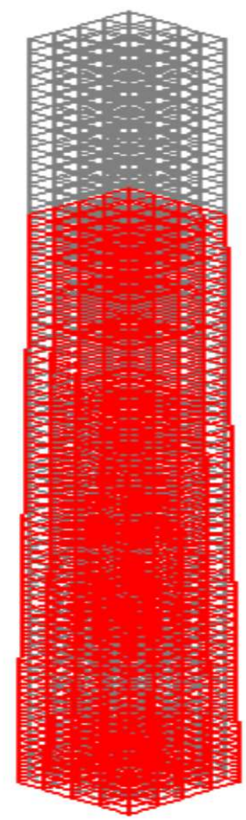
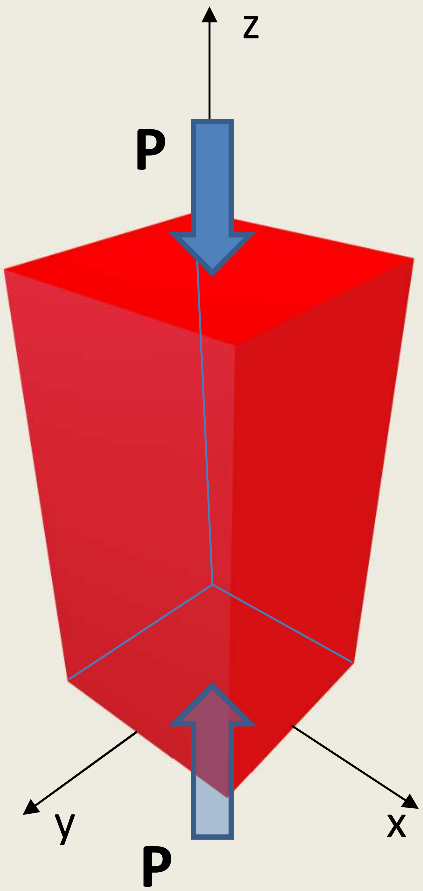
ANALISIS ESTRUCTURAL II





CALCULO DE CORRIMIENTOS

Hipótesis de Tensiones Prisma en Compresión mas Peso Propio



Campo tensiones

$$\sigma_x = 0$$

$$\sigma_y = 0$$

$$\sigma_z = \frac{P}{A} + \gamma(h - z)$$

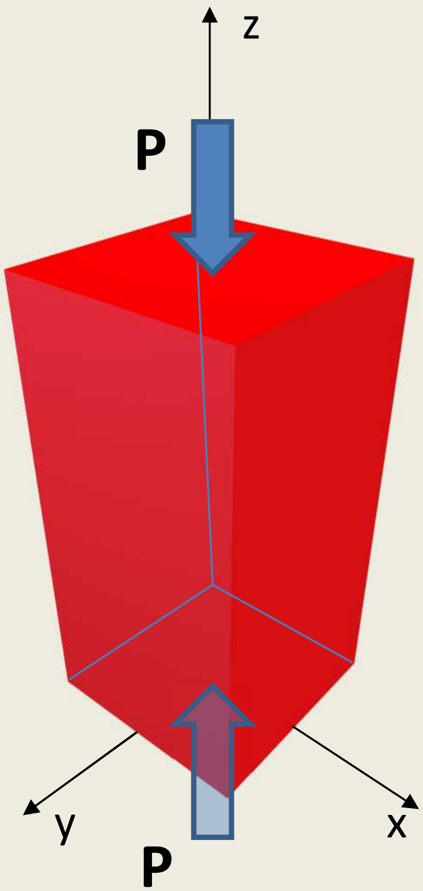
$$\tau_{xy} = 0$$

$$\tau_{yz} = 0$$

$$\tau_{zx} = 0$$

CALCULO DE CORRIMIENTOS

Tensiones y Deformaciones



Campo tensiones

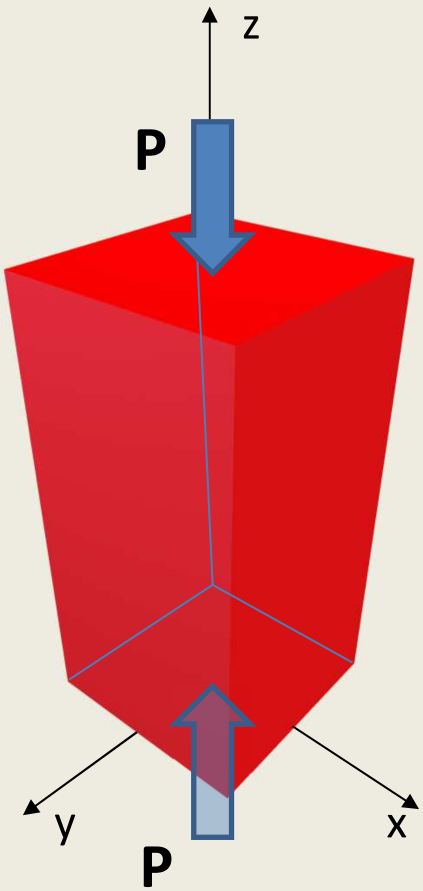
$$\sigma_x = 0$$
$$\sigma_y = 0$$
$$\sigma_z = \frac{P}{A} + \gamma(h - z)$$
$$\tau_{xy} = 0$$
$$\tau_{yz} = 0$$
$$\tau_{zx} = 0$$

Campo deformaciones

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$
$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$
$$\varepsilon_z = \frac{\partial w}{\partial z} = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\tau_{xy}}{G}$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\tau_{yz}}{G} = 0$$
$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \frac{\tau_{zx}}{G} = 0$$

CALCULO DE CORRIMIENTOS

Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\varepsilon_z = \frac{\partial w}{\partial z} = \frac{\sigma_z}{E} - \frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$

$$\sigma_z = P_0 + \gamma(h - z)\text{ , llamando } P_0 = \frac{P}{A}$$

$$\frac{\partial w}{\partial z} = \frac{P_0}{E} + \frac{\gamma}{E}h - \frac{\gamma}{E}z$$

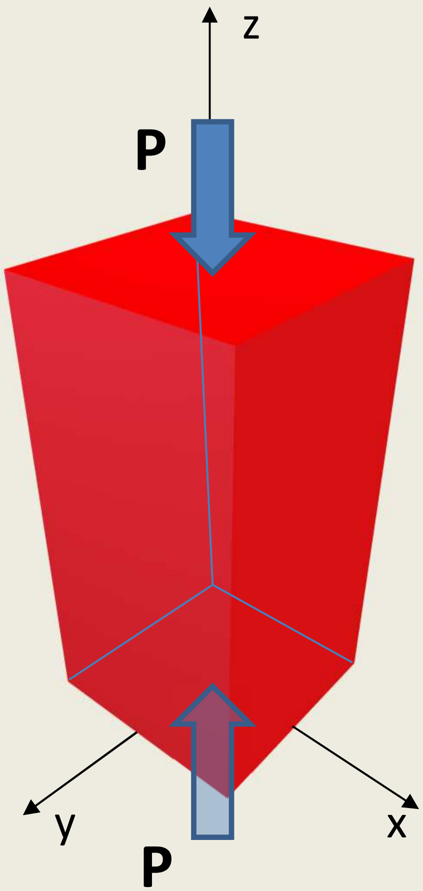
$$\partial w = \left(\frac{P_0}{E} + \frac{\gamma}{E}h - \frac{\gamma}{E}z\right)\partial z$$

$$w = \int \left(\frac{P_0}{E} + \frac{\gamma}{E}h - \frac{\gamma}{E}z\right)dz$$

$$w = \frac{P_0}{E}z + \frac{\gamma}{E}h\,z - \frac{\gamma}{2\,E}z^2 + F_1(x, y)$$

CALCULO DE CORRIMIENTOS

Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \frac{\tau_{zx}}{G} = 0$$

$$\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0 \;\therefore\; \frac{\partial u}{\partial z} = -\frac{\partial w}{\partial x}$$

$$\partial u = -\frac{\partial w}{\partial x} dz$$

$$u = \int -\frac{\partial w}{\partial x} dz$$

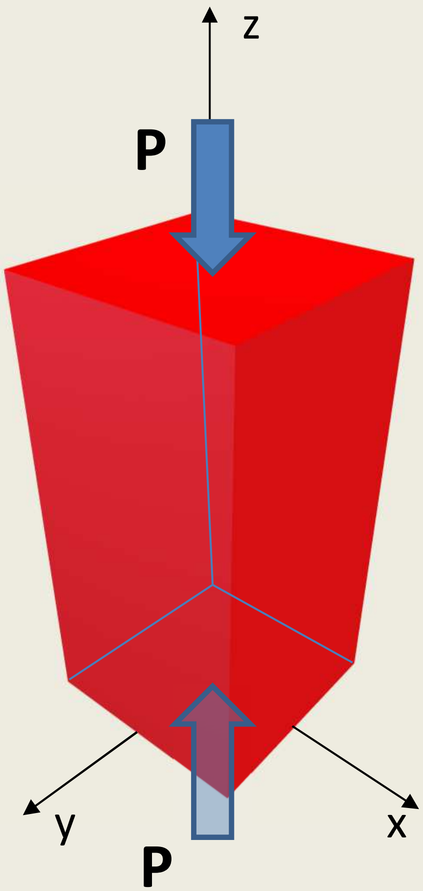
$$w = \frac{P_0}{E} z + \frac{\gamma}{E} h\, z - \frac{\gamma}{2\,E} z^2 + F_1(x, y)$$

$$u = \int -\frac{\partial}{\partial x} F_1(x, y)\, dz$$

$$u = -\frac{\partial\, F_1(x, y)}{\partial x}\, z + F_2(x, y)$$

CALCULO DE CORRIMIENTOS

Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\tau_{yz}}{G} = 0$$

$$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \quad \therefore \quad \frac{\partial v}{\partial z} = -\frac{\partial w}{\partial y}$$

$$dv = -\frac{\partial w}{\partial y} dz$$

$$v = \int -\frac{\partial w}{\partial y} dz$$

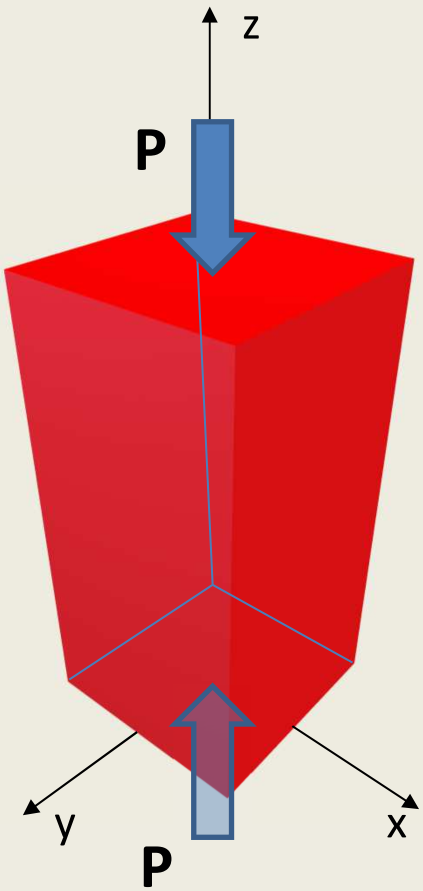
$$w = \frac{P_0}{E} z + \frac{\gamma}{E} h\, z - \frac{\gamma}{2\,E} z^2 + F_1(x, y)$$

$$v = \int -\frac{\partial}{\partial y} F_1(x, y)\, dz$$

$$v = -\frac{\partial\, F_1(x, y)}{\partial y}\, z + F_3(x, y)$$

CALCULO DE CORRIMIENTOS

Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\sigma_x}{E} - \frac{\nu}{E}\left(\sigma_y + \sigma_z\right)$$

$$\frac{\partial u}{\partial x} = -\frac{\nu}{E}\left(P_0 + \gamma(h - z)\right)$$

$$\frac{\partial u}{\partial x} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h + \frac{\nu}{E}\gamma z$$

$$u = -\frac{\partial\, F_1(x,y)}{\partial x}\, z + F_2(x,y)$$

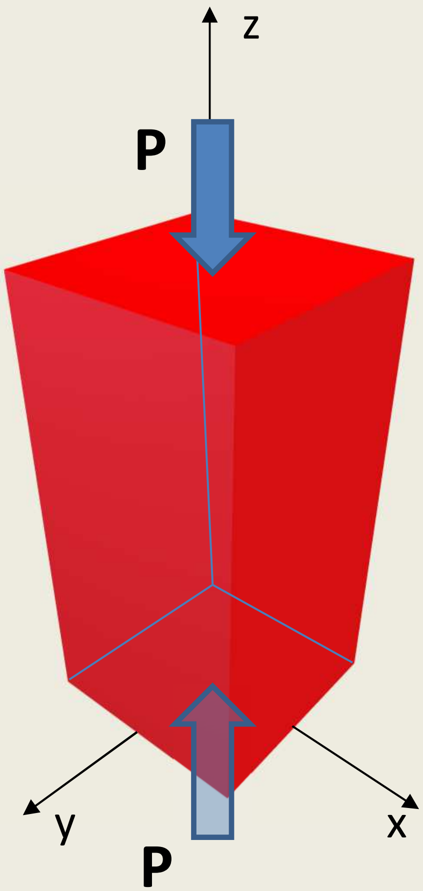
$$\frac{\partial u}{\partial x} = -\frac{\partial^2\, F_1(x,y)}{\partial x^2}\, z + \frac{\partial\, F_2(x,y)}{\partial x}$$

$$\frac{\partial^2\, F_1(x,y)}{\partial x^2} = -\frac{\nu}{E}\gamma$$

$$\frac{\partial\, F_2(x,y)}{\partial x} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h$$

CALCULO DE CORRIMIENTOS

Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$\frac{\partial v}{\partial y} = -\frac{\nu}{E}\left(P_0 + \gamma(h-z)\right)$$

$$\frac{\partial v}{\partial y} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h + \frac{\nu}{E}\gamma z$$

$$v = -\frac{\partial\, F_1(x,y)}{\partial y}\, z + F_3(x,y)$$

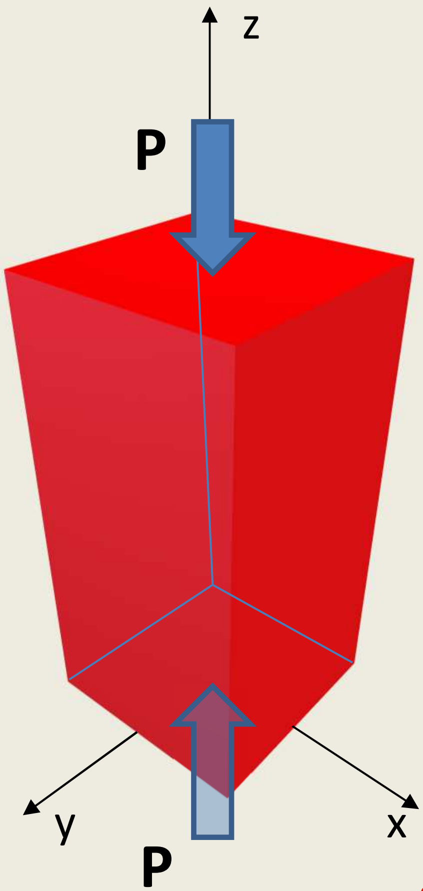
$$\frac{\partial v}{\partial y} = -\frac{\partial^2\, F_1(x,y)}{\partial y^2}\, z + \frac{\partial\, F_3(x,y)}{\partial y}$$

$$\frac{\partial^2\, F_1(x,y)}{\partial y^2} = -\frac{\nu}{E}\gamma$$

$$\frac{\partial\, F_3(x,y)}{\partial y} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h$$

CALCULO DE CORRIMIENTOS

Desarrollo



Condición

De Relaciones Anteriores

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\tau_{xy}}{G} = 0$$

$$u = -\frac{\partial\, F_1(x,y)}{\partial x} z + F_2(x,y)$$

$$v = -\frac{\partial\, F_1(x,y)}{\partial y} z + F_3(x,y)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial^2\, F_1(x,y)}{\partial x \partial y} z + \frac{\partial\, F_2(x,y)}{\partial y}$$

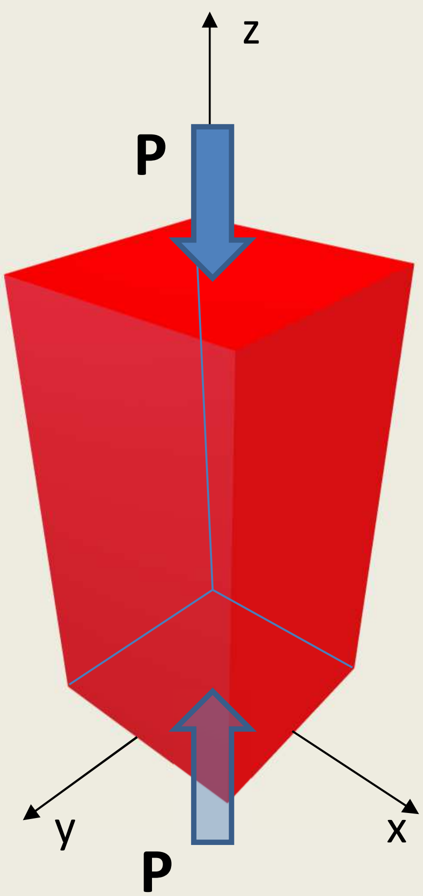
$$\frac{\partial v}{\partial x} = -\frac{\partial^2\, F_1(x,y)}{\partial x \partial y} z + \frac{\partial\, F_3(x,y)}{\partial x}$$

$$-2\frac{\partial^2\, F_1(x,y)}{\partial x \partial y} z + \frac{\partial\, F_2(x,y)}{\partial y} + \frac{\partial\, F_3(x,y)}{\partial x} = 0$$

$$\frac{\partial\, F_2(x,y)}{\partial y} = -\frac{\partial\, F_3(x,y)}{\partial x}$$

CALCULO DE CORRIMIENTOS

Desarrollo



De Relaciones Anteriores

$$\frac{\partial\, F_2(x,y)}{\partial x} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h$$

$$\frac{\partial\, F_3(x,y)}{\partial y} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h$$

$$\frac{\partial\, F_2(x,y)}{\partial y} = -\frac{\partial\, F_3(x,y)}{\partial x}$$

Integrando

$$F_2(x,y) = -\frac{\nu}{E}P_0\, x - \frac{\nu}{E}\gamma h\, x + F_4(y)$$

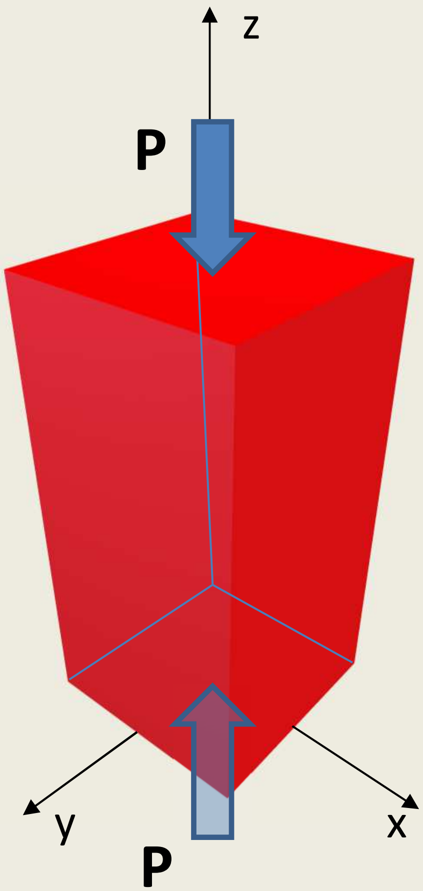
$$F_3(x,y) = -\frac{\nu}{E}P_0\, y - \frac{\nu}{E}\gamma h\, y + F_5(x)$$

$$F_4(y) = \;\; Ay + B$$

$$F_5(x) = -Ax + C$$

CALCULO DE CORRIMIENTOS

Desarrollo



Reemplazando

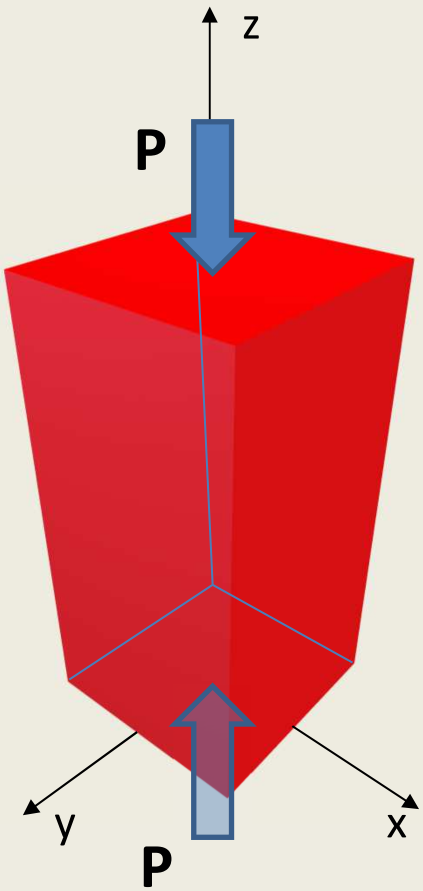
$$F_2(x,y) = -\frac{\nu}{E} P_0\, x - \frac{\nu}{E}\gamma h\, x + Ay + B$$
$$F_3(x,y) = \frac{\nu}{E} P_0\, y - \frac{\nu}{E}\gamma h\, y - Ax + C$$

Integrando F_1

$$\frac{\partial^2\, F_1(x,y)}{\partial x^2} = -\frac{\nu}{E}\gamma \quad , \quad \frac{\partial^2\, F_1(x,y)}{\partial y^2} = -\frac{\nu}{E}\gamma$$
$$\frac{\partial F_1(x,y)}{\partial x} = -\frac{\nu}{E}\gamma\, x + F_6(y)\ ,\ F_1^{*} = -\frac{\nu}{E}\gamma\, \frac{x^2}{2} + F_6(y)\, x + F_7(y)$$
$$\frac{\partial F_1(x,y)}{\partial y} = -\frac{\nu}{E}\gamma\, y + F_8(x)\ ,\ F_1^{**} = -\frac{\nu}{E}\gamma\, \frac{y^2}{2} + F_8(x)\, y + F_9(x)$$

CALCULO DE CORRIMIENTOS

Desarrollo



Sumando $F_1^{*} + F_1^{**}$

$$F_1^{*} + F_1^{**} = -\frac{\nu}{E}\gamma\,\frac{x^2}{2} + F_6(y)\,x + F_7(y) - \frac{\nu}{E}\gamma\,\frac{y^2}{2} + F_8(x)\,y + F_9(x)$$

$$F_1^{*} + F_1^{**} = -\frac{\nu}{E}\gamma\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + F_6(y)\,x + F_7(y) + F_8(x)\,y + F_9(x)$$

La condición

$$\frac{\partial^2 F_1(x,y)}{\partial x \partial y} = 0$$

$$\frac{\partial^2 F_1^{*}}{\partial x \partial y} = \frac{\partial F_6(y)}{\partial y} \Rightarrow F_6(y) = ctte = H$$

$$\frac{\partial^2 F_1^{**}}{\partial x \partial y} = \frac{\partial F_8(x)}{\partial x} \Rightarrow F_8(x) = ctte = J$$

CALCULO DE CORRIMIENTOS

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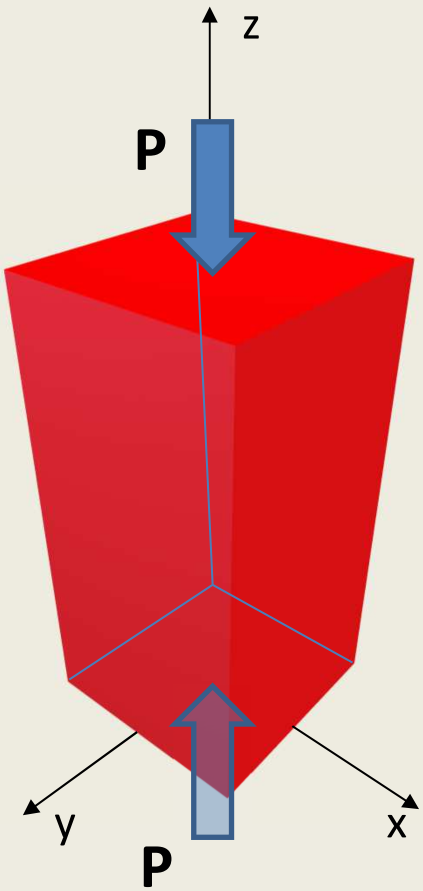


Reemplazando en F_1

$$F_1 = -\frac{\nu}{E}\gamma\left(\frac{x^2}{2}+\frac{y^2}{2}\right) + H\,x + J\,y + F_7(y) + F_9(x)$$

Derivando

$$\frac{\partial F_1(x,y)}{\partial x} = \frac{\partial F_1^*}{\partial x}$$

$$-\frac{\nu}{E}\gamma x + H + \frac{\partial F_9(x)}{\partial x} = -\frac{\nu}{E}\gamma x + H \Rightarrow \frac{\partial F_9(x)}{\partial x} = 0 \Rightarrow F_9(x) = ctte = K$$

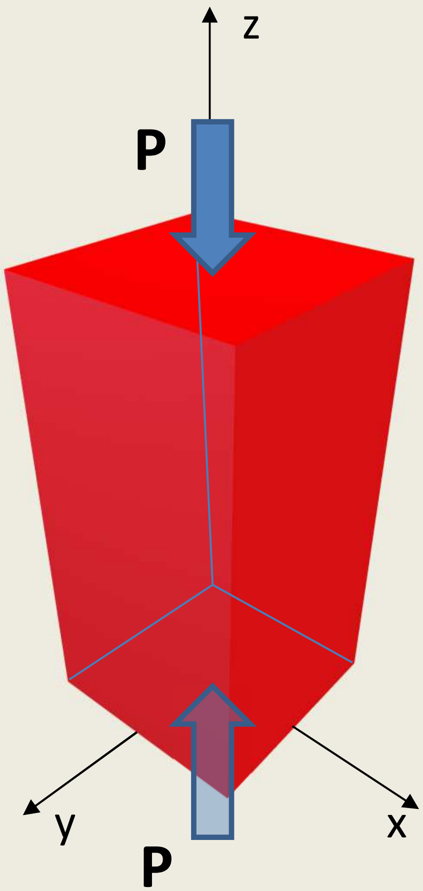
$$\frac{\partial F_1(x,y)}{\partial y} = \frac{\partial F_1^{**}}{\partial y}$$

$$-\frac{\nu}{E}\gamma y + J + \frac{\partial F_7(y)}{\partial y} = -\frac{\nu}{E}\gamma y + J \Rightarrow \frac{\partial F_7(y)}{\partial y} = 0 \Rightarrow F_7(y) = ctte = L$$

$$M = K + L$$

CALCULO DE CORRIMIENTOS

Desarrollo



Reemplazando en F_1

$$F_1 = -\frac{\nu}{E}\gamma\left(\frac{x^2}{2}+\frac{y^2}{2}\right) + H\,x + J\,y + M$$

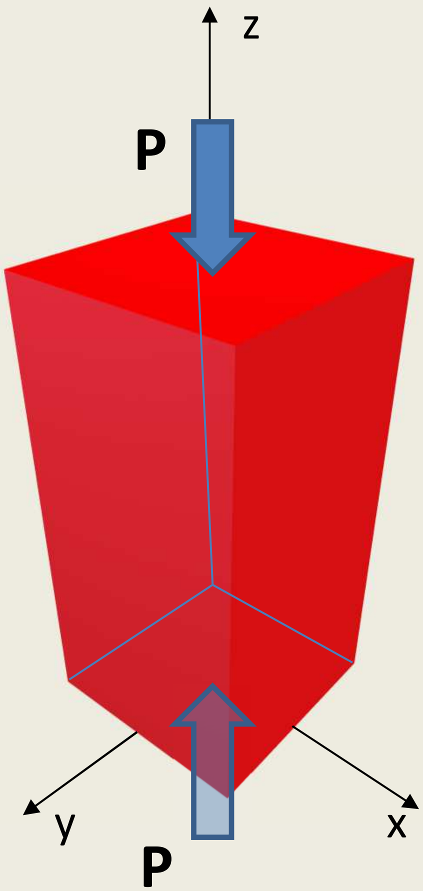
Reemplazando en la expresiones de w, u, v

$$w = \frac{P_0}{E}z + \frac{\gamma}{E}h\,z - \frac{\gamma}{2\,E}z^2 + F_1(x,y)$$

$$w = \frac{1}{E}\left(P_0 z + \gamma\,h\,z - \gamma\frac{z^2}{2}\right) - \frac{\nu}{2E}\,\gamma(x^2+y^2) + Hx + Jy + M$$

CALCULO DE CORRIMIENTOS

Desarrollo



Reemplazando en F_1

$$F_1 = -\frac{\nu}{E}\gamma\left(\frac{x^2}{2}+\frac{y^2}{2}\right) + H\,x + J\,y + M$$

Reemplazando en la expresiones de w, u, v

$$u = -\frac{\partial\, F_1(x,y)}{\partial x}\, z + F_2(x,y)$$

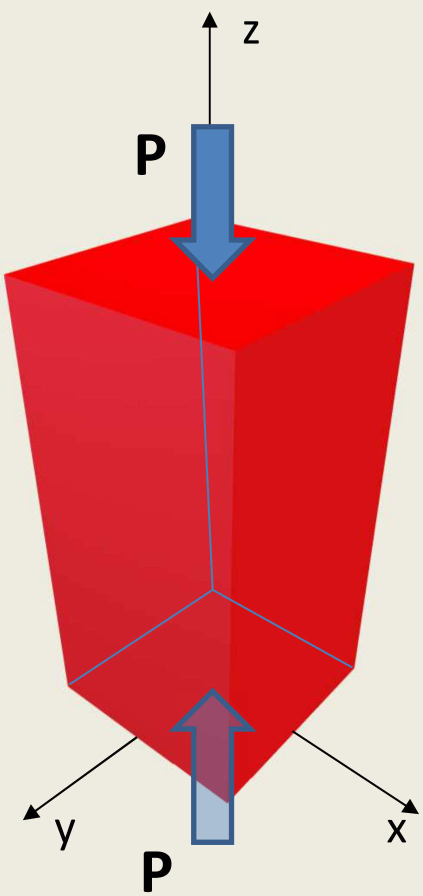
$$u = \frac{\nu}{E}\gamma\, x\, z - H\, z - \frac{\nu}{E}P_0\, x - \frac{\nu}{E}\gamma h\, x + A\, y + B$$

$$v = -\frac{\partial\, F_1(x,y)}{\partial y}\, z + F_3(x,y)$$

$$v = \frac{\nu}{E}\gamma\, y\, z - J\, z - \frac{\nu}{E}P_0\, y - \frac{\nu}{E}\gamma h\, y - A\, x + C$$

CALCULO DE CORRIMIENTOS

Desarrollo



Expresiones Generales de w, u, v

$$w = \frac{1}{E}\left(P_0 z + \gamma\, h\, z - \gamma \frac{z^2}{2}\right) - \frac{\nu}{2E}\,\gamma(x^2 + y^2) + Hx + Jy + M$$

$$u = \frac{\nu}{E}\gamma\, x\, z - H\, z - \frac{\nu}{E} P_0\, x - \frac{\nu}{E}\gamma h\, x + A\, y + B$$

$$v = \frac{\nu}{E}\gamma\, y\, z - J\, z - \frac{\nu}{E} P_0\, y - \frac{\nu}{E}\gamma h\, y - A\, x + C$$

Cond. Contorno $x = y = z = 0 \;\Rightarrow\; u = v = w = 0$

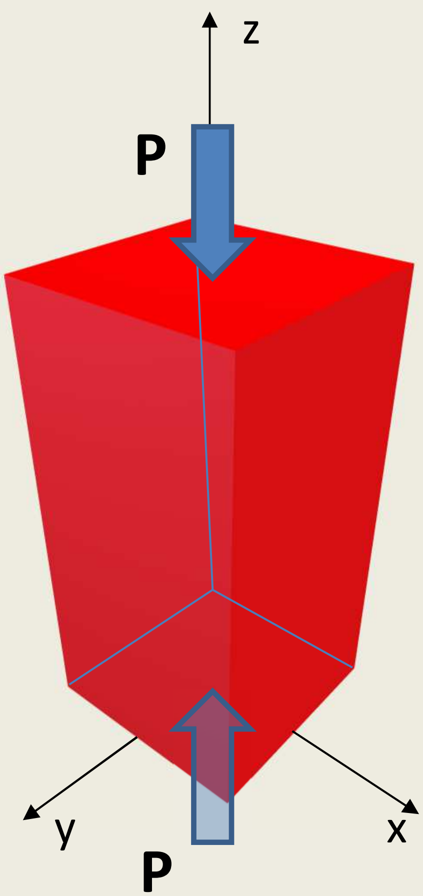
$$w = 0 \Rightarrow \quad M = 0$$

$$u = 0 \Rightarrow \quad B = 0$$

$$v = 0 \Rightarrow \quad C = 0$$

CALCULO DE CORRIMIENTOS

Desarrollo



Expresiones Generales de w, u, v

$$w = \frac{1}{E}\left(P_0 z + \gamma\, h\, z - \gamma \frac{z^2}{2}\right) - \frac{\nu}{2E}\,\gamma(x^2 + y^2) + Hx + Jy + M$$

$$u = \frac{\nu}{E}\gamma\, x\, z - H\, z - \frac{\nu}{E} P_0\, x - \frac{\nu}{E}\gamma h\, x + A\, y + B$$

$$v = \frac{\nu}{E}\gamma\, y\, z - J\, z - \frac{\nu}{E} P_0\, y - \frac{\nu}{E}\gamma h\, y - A\, x + C$$

C. Cont. $x = y = z = 0 \;\Rightarrow\; g_{xy} = g_{yz} = g_{zx} = 0$

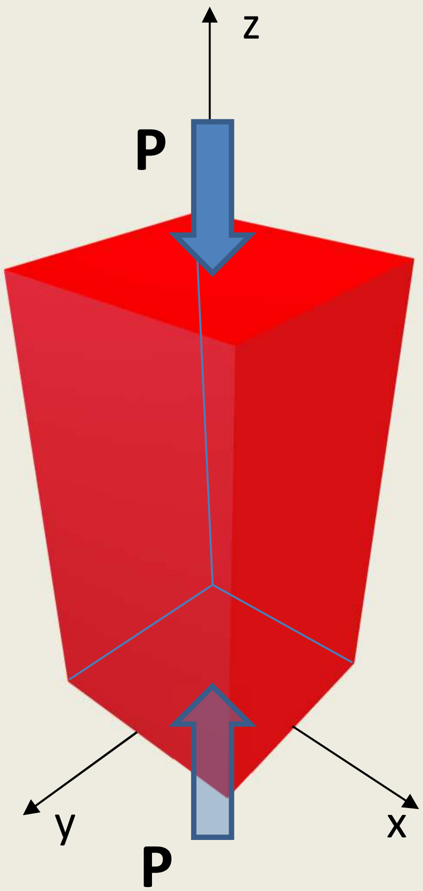
$$g_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \;\Rightarrow\; \frac{1}{2}(-A - A) = 0 \;\Rightarrow\; A = 0$$

$$g_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = 0 \;\Rightarrow\; \frac{1}{2}(+J + J) = 0 \;\Rightarrow\; J = 0$$

$$g_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0 \;\Rightarrow\; \frac{1}{2}(-H - H) = 0 \;\Rightarrow\; H = 0$$

CALCULO DE CORRIMIENTOS

Desarrollo



Expresiones Particulares de w, u, v

$$u = -\frac{\nu}{E} P_0\, x + \frac{\nu}{E}\gamma\, x\, z - \frac{\nu}{E}\gamma\, h\, x$$

$$v = -\frac{\nu}{E} P_0\, y + \frac{\nu}{E}\gamma\, y\, z - \frac{\nu}{E}\gamma\, h\, y$$

$$w = \frac{1}{E} P_0\, z + \frac{1}{E}\,\gamma\, h\, z\; - \frac{1}{E}\,\gamma\, \frac{z^2}{2} - \frac{\nu}{2E}\,\gamma\,(x^2 + y^2)$$

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Mg. Ing. DANIEL E. LÓPEZ

ANALISIS ESTRUCTURAL II

Fin