TEORÍA ELASTICIDAD

TP#5 APLICACIONES 2D

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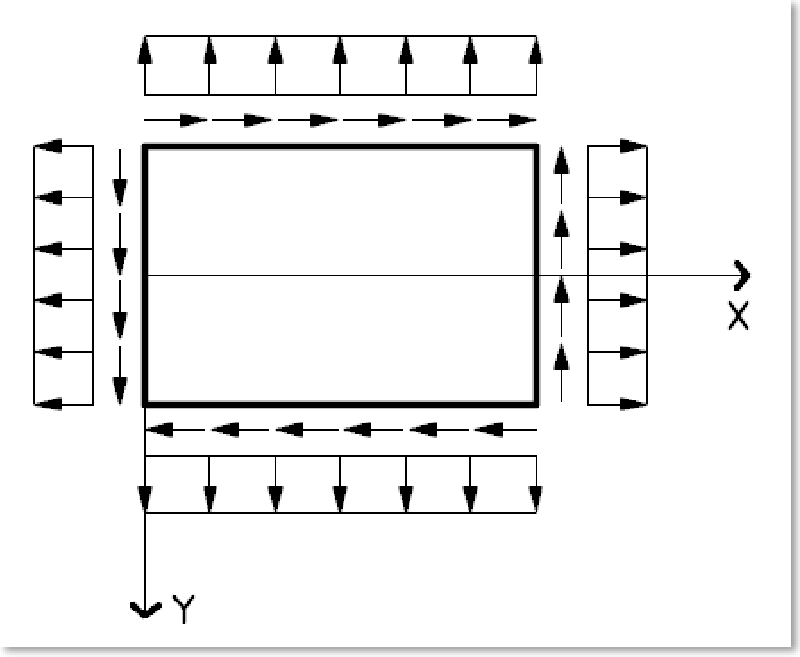
ANALISIS ESTRUCTURAL II



FUNCIÓN DE TENSIÓN

Ec. de Compatibilidad y Tensiones para la función ϕ

E.C.

$$\nabla^4 \emptyset = \frac{\partial^4 \emptyset}{\partial x^4} + \frac{2\,\partial^4 \emptyset}{\partial x^2 \partial y^2} + \frac{\partial^4 \emptyset}{\partial y^4}$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2}$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2}$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y}$$

Vale con X=Y=Z=0

FUNCIÓN DE TENSIÓN

Polinomio 2° Grado

$$\emptyset_2 = a_2\, x^2 + b_2\, x\, y + c_2\, y^2$$

E.C.

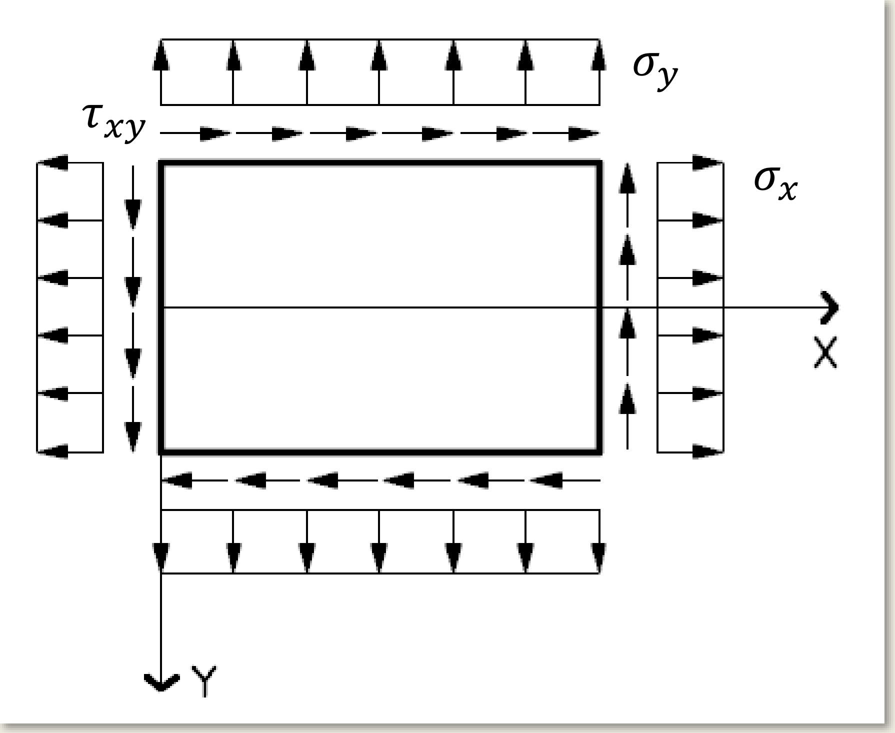
$$\nabla^4 \emptyset_2 = 0$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = 2\, c_2$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = 2\, a_2$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = -b_2$$



FUNCIÓN DE TENSIÓN

Polinomio 2° Grado

$$\emptyset_2 = a_2\, x^2 + b_2\, x\, y + c_2\, y^2$$

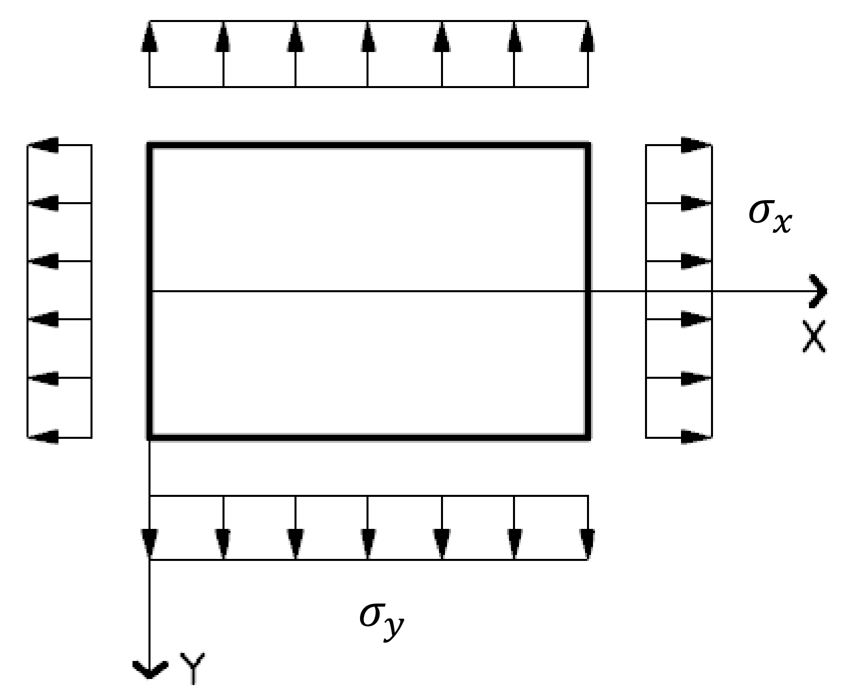
$$b_2 = 0$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = 2\, c_2$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = 2\, a_2$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = 0$$



FUNCIÓN DE TENSIÓN

Polinomio 2° Grado

$$\emptyset_2 = a_2\, x^2 + b_2\, x\, y + c_2\, y^2$$

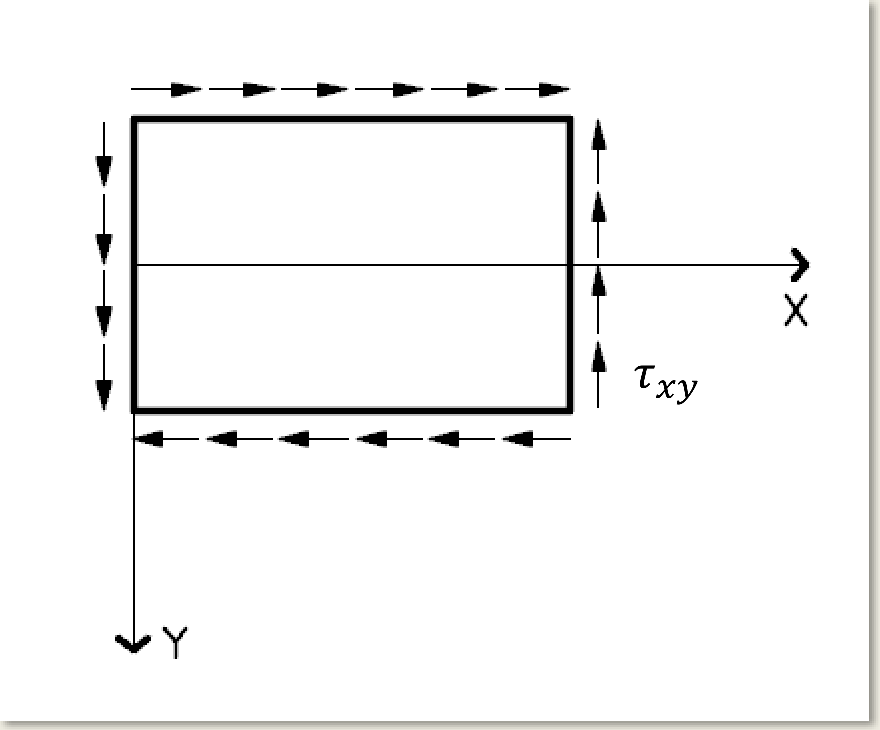
$$a_2 = c_2 = 0$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = 0$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = 0$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = -b_2$$



FUNCIÓN DE TENSIÓN

Polinomio 3° Grado

$$\boldsymbol{\emptyset_3 = a_3\, x^3 + b_3\, x^2\, y + c_3\, x\, y^2 + d_3\, y^3}$$

E.C.

$$\nabla^4 \emptyset_3 = 0$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = 2\, c_3\, x + 6\, d_3\, y^2$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = 6\, a_3\, x^2 + 2\, b_3\, y$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = -2\, b_3\, x - 2\, c_3\, y$$

FUNCIÓN DE TENSIÓN

Polinomio 3° Grado

$$\emptyset_3 = a_3\, x^3 + b_3\, x^2\, y + c_3\, x\, y^2 + d_3\, y^3$$

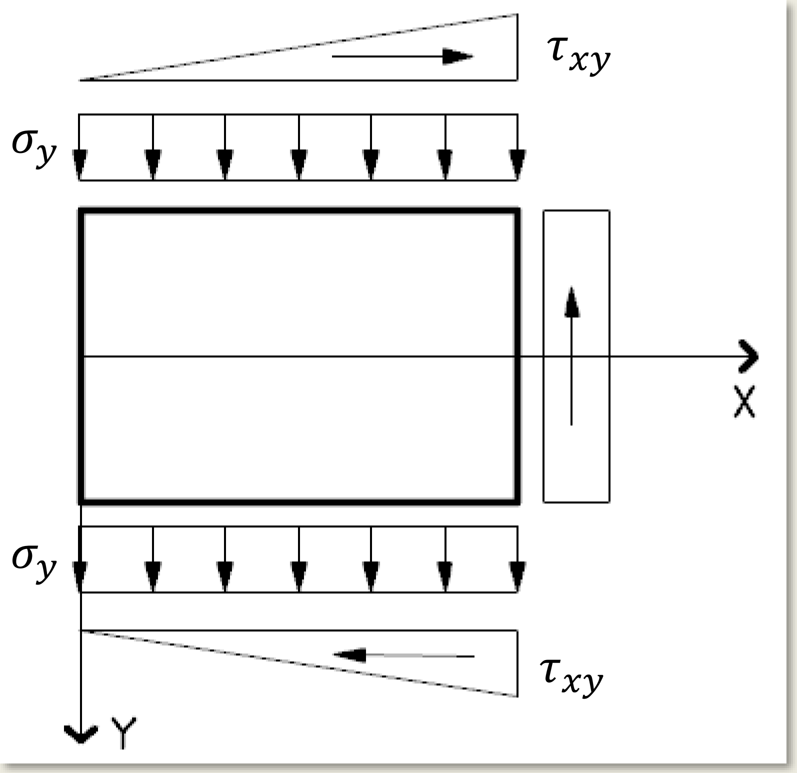
$$a_3 = c_3 = d_3 = 0 \quad b_3 \neq 0$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = 0$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = 2\, b_3\, y$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = -2\, b_3\, x$$



FUNCIÓN DE TENSIÓN

Polinomio 3° Grado

$$\emptyset_3 = a_3\,x^3 + b_3\,x^2\,y + c_3\,x\,y^2 + d_3\,y^3$$

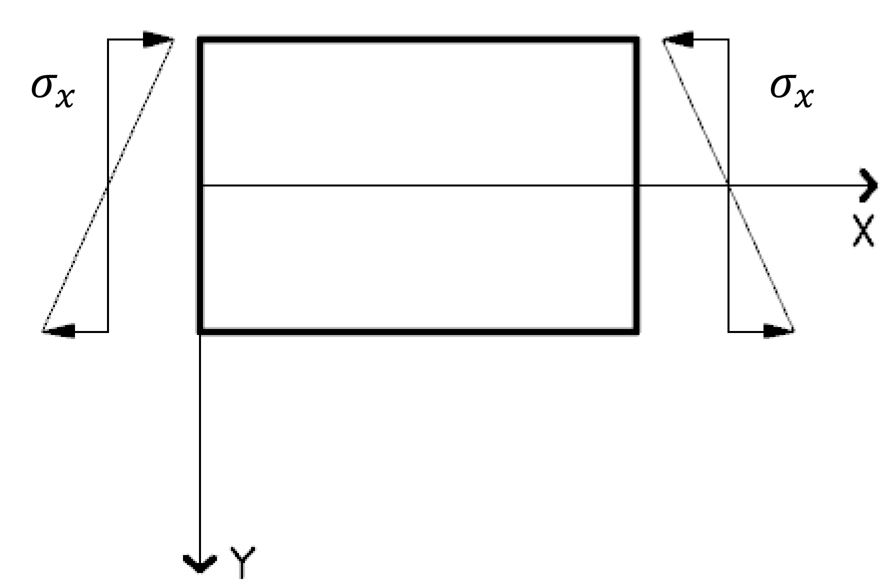
$$a_3 = b_3 = c_3 = 0 \quad d_3 \neq 0$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = 6\,d_3\,y$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = 0$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = 0$$



FUNCIÓN DE TENSIÓN

Polinomio 3° Grado

$$\emptyset_3 = a_3\, x^3 + b_3\, x^2\, y + c_3\, x\, y^2 + d_3\, y^3$$

$$a_3 = b_3 = c_3 = 0 \quad d_3 \neq 0$$

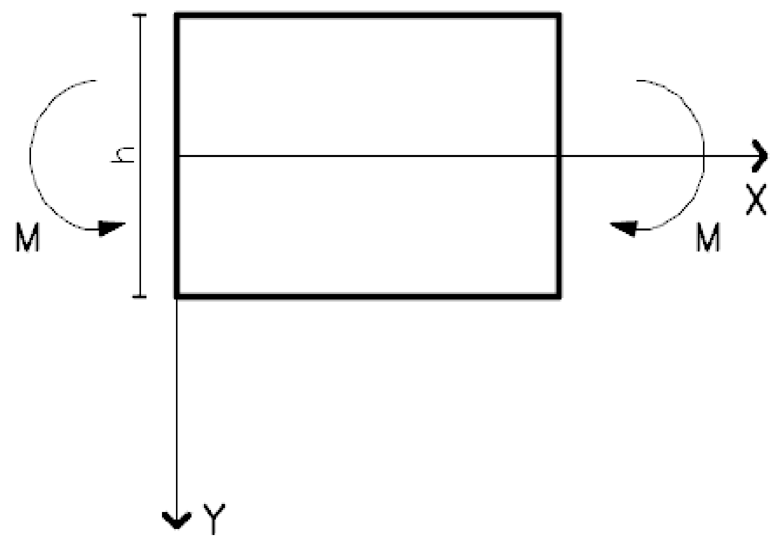
$$\sigma_x = 6\, d_3\, y$$

$$\sum M = 0 = \int_{-h/2}^{h/2} \sigma_x\, y\, dF + M$$

$$M = -\int_{-h/2}^{h/2} \sigma_x\, y\, dF = -6\, d_3 \int_{-h/2}^{h/2} 1\, y^2\, dy$$

$$M = -6\, d_3\, I_z$$

$$d_3 = -\frac{M}{6\, I_z}$$



FUNCIÓN DE TENSIÓN

Polinomio 3° Grado

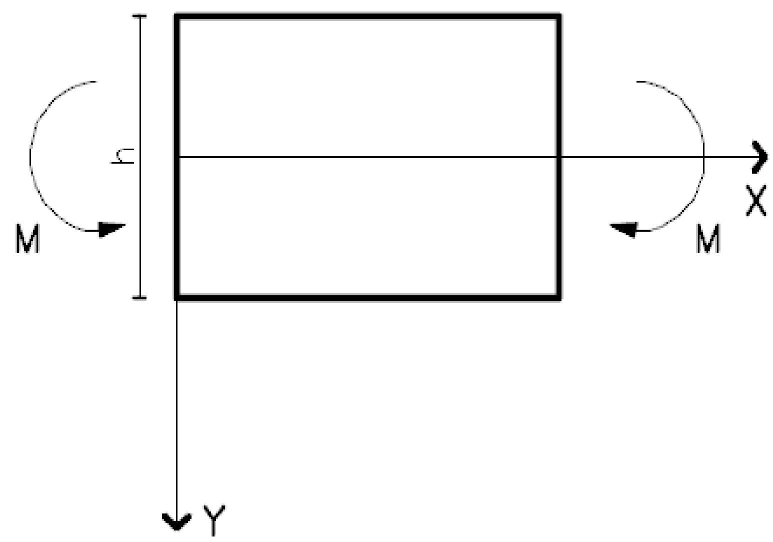
$$\emptyset_3 = a_3\, x^3 + b_3\, x^2\, y + c_3\, x\, y^2 + d_3\, y^3$$

$$\sigma_x = -\frac{M}{I_z} y$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\, \sigma_y = -\frac{M}{E\, I_z} y$$

$$\varepsilon_y = \cdots$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\tau_{xy}}{G} = \cdots$$



FUNCIÓN DE TENSIÓN

Polinomio 4° Grado

$$\emptyset_4 = a_4\, x^4 + b_4\, x^3\, y + c_4\, x^2\, y^2 + d_4\, x\, y^3 + e_4\, y^4$$

E.C.

$\nabla^4 \emptyset_4 = 0$?

$$\frac{\partial^4 \emptyset}{\partial x^4} = 24\, a_4$$

$$\frac{2\, \partial^4 \emptyset}{\partial x^2 \partial y^2} = 4\, c_4$$

$$\frac{\partial^4 \emptyset}{\partial y^4} = 24\, e_4$$

FUNCIÓN DE TENSIÓN

Polinomio 4° Grado

$$\boldsymbol{\emptyset_4 = a_4\, x^4 + b_4\, x^3\, y + c_4\, x^2\, y^2 + d_4\, x\, y^3 + e_4\, y^4}$$

E.C.

$$\nabla^4 \emptyset_4 = 24\, a_4 + 4\, c_4 + 24\, e_4 \neq 0$$

FUNCIÓN DE TENSIÓN

Polinomio 4° Grado. Caso $d_4 \neq 0$

$$\emptyset_4 = d_4\, x\, y^3$$

E.C.

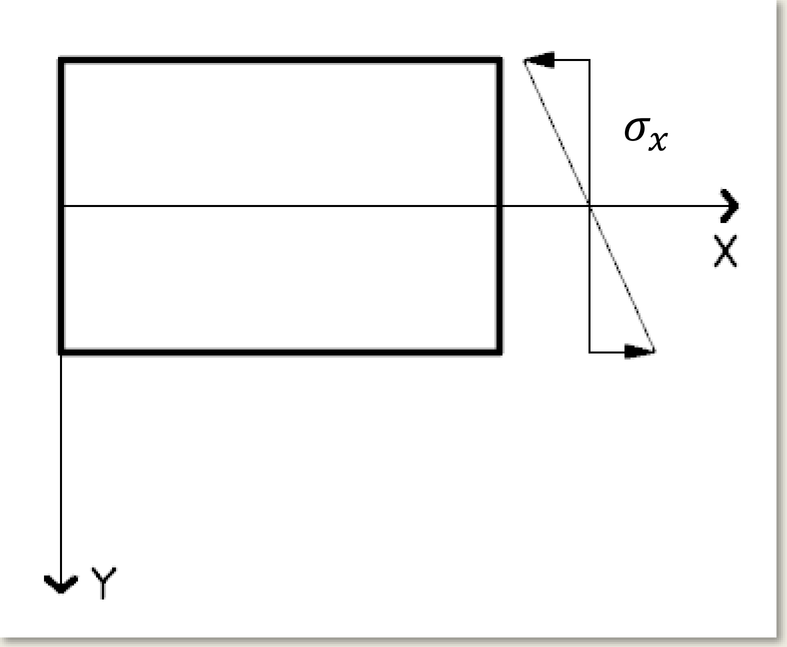
$$\nabla^4 \emptyset_4 = 0$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = 6\, d_4\, xy$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = 0$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = -3\, d_4\, y^2$$



FUNCIÓN DE TENSIÓN

Polinomio 4° Grado. Caso $d_4 \neq 0$

$$\emptyset_4 = d_4\, x\, y^3$$

E.C.

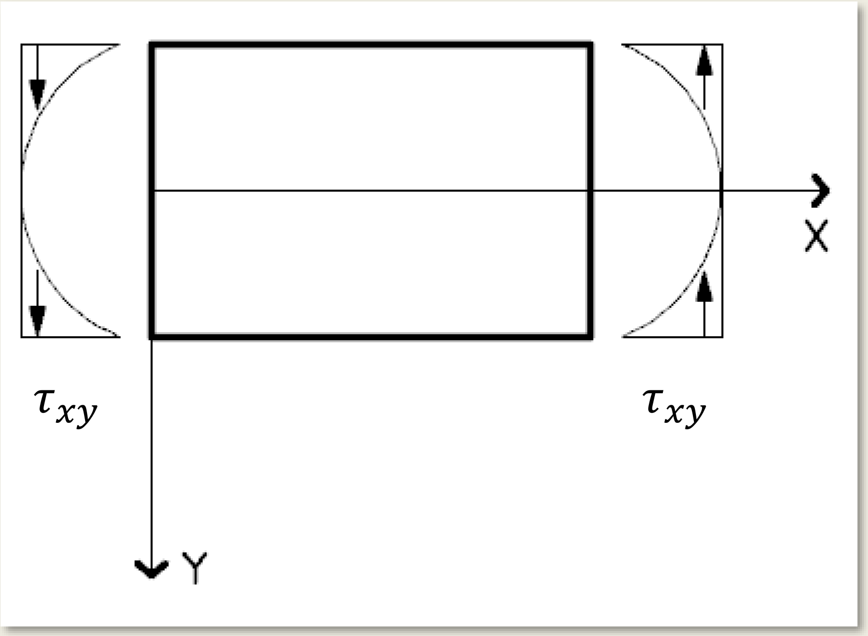
$$\nabla^4 \emptyset_4 = 0$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = 6\, d_4\, xy$$

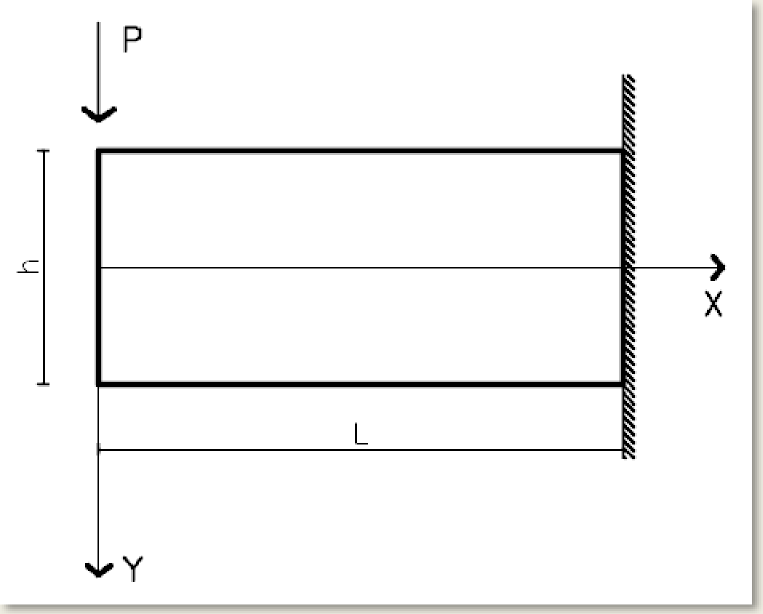
$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = 0$$

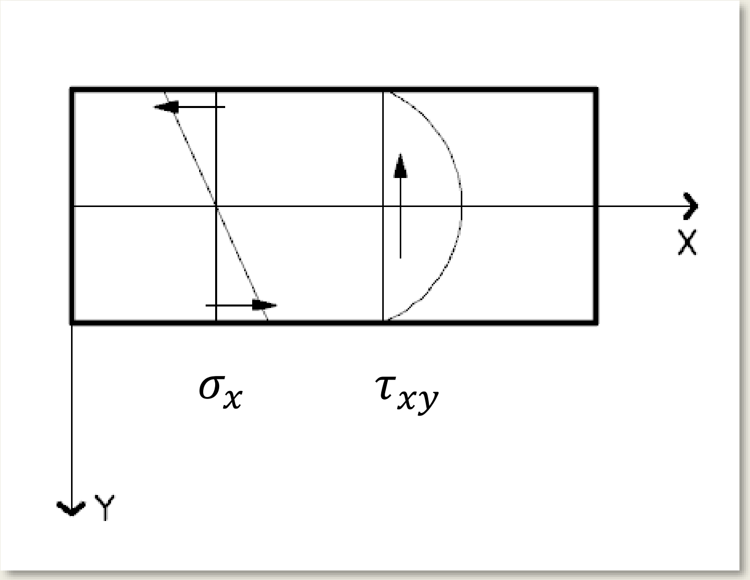
$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = -3\, d_4\, y^2$$



FUNCIÓN DE TENSIÓN

Aplicación Viga en Voladizo





FUNCIÓN DE TENSIÓN

Aplicación Viga en Voladizo

$$\emptyset = d_4\, x\, y^3 \; - b_2\, x\, y$$

E.C.

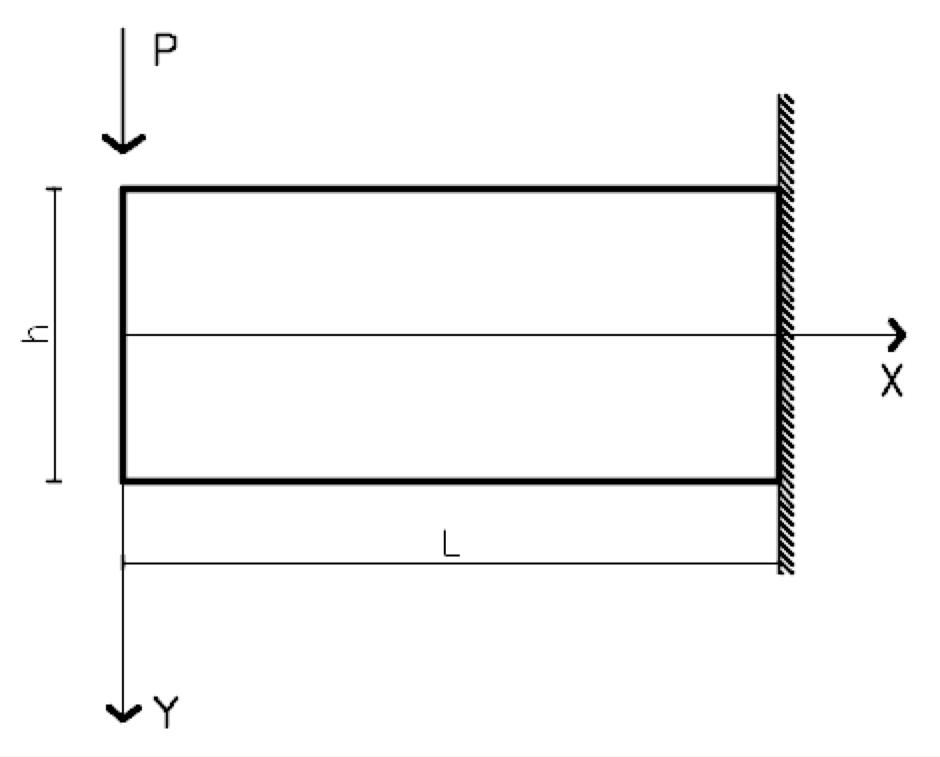
$$\nabla^4 \emptyset_4 = ?$$

Tensiones

$$\sigma_x = \frac{\partial^2 \emptyset}{\partial y^2} = ?$$

$$\sigma_y = \frac{\partial^2 \emptyset}{\partial x^2} = ?$$

$$\tau_{xy} = -\frac{\partial^2 \emptyset}{\partial x \partial y} = ?$$



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Fin