ESTRUCTURAS LAMINARES

Elementos finitos Semiloof

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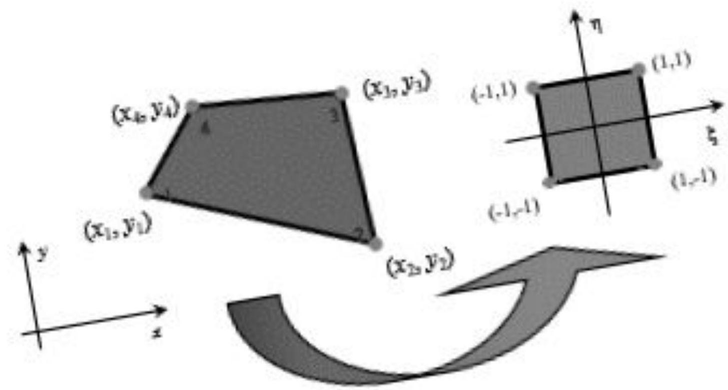
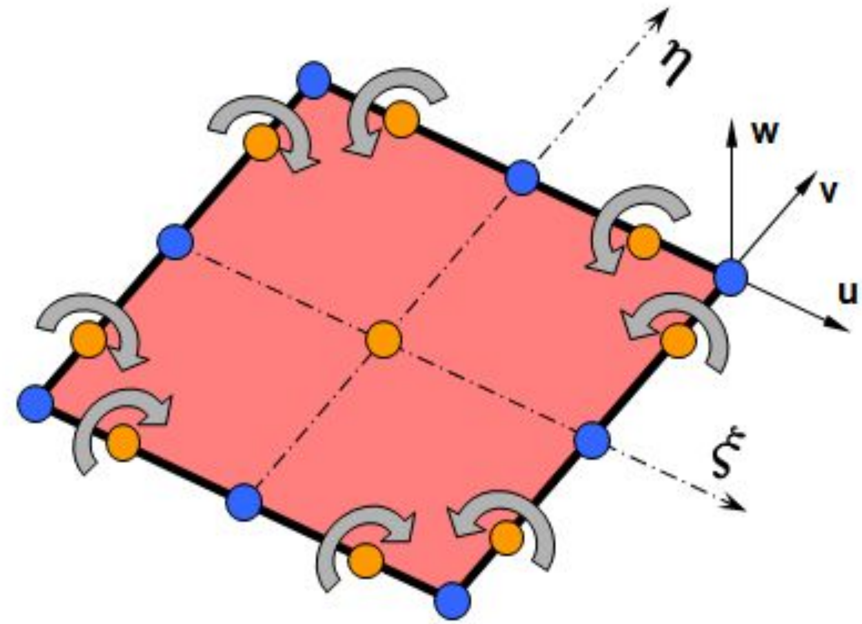


$$\begin{bmatrix}\frac{\partial N_i}{\partial x}\\ \frac{\partial N_i}{\partial y}\end{bmatrix} = \frac{1}{|\mathbf{J}^e|}\begin{bmatrix}\frac{\partial y}{\partial \eta} & -\frac{\partial y}{\partial \xi}\\ -\frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi}\end{bmatrix}\begin{bmatrix}\frac{\partial N_i}{\partial \xi}\\ \frac{\partial N_i}{\partial \eta}\end{bmatrix}$$

$$\mathbf{K}_{ij}^e = \frac{t}{|\mathbf{J}^e|}\int_{-1}^{1}\int_{-1}^{1}\mathbf{B}_i^T(\xi,\eta):\mathbf{D}:\mathbf{B}_j(\xi,\eta)\,d\xi\, d\eta$$

Historia Elementos Semiloof

- **Estudios en la cátedra: Ing. Elbio Villafañe.**
- **Creador: Bruce Irons 1966-1974.**
- **Campo de aplicación:**
  Cáscaras delgadas (thin shells)
- **Motivación: Disconformidad con elementos shell de ese momento.**
- **Nombre del elemento: Homenaje a su amigo Henk Loof, Investigador elementos shell.**



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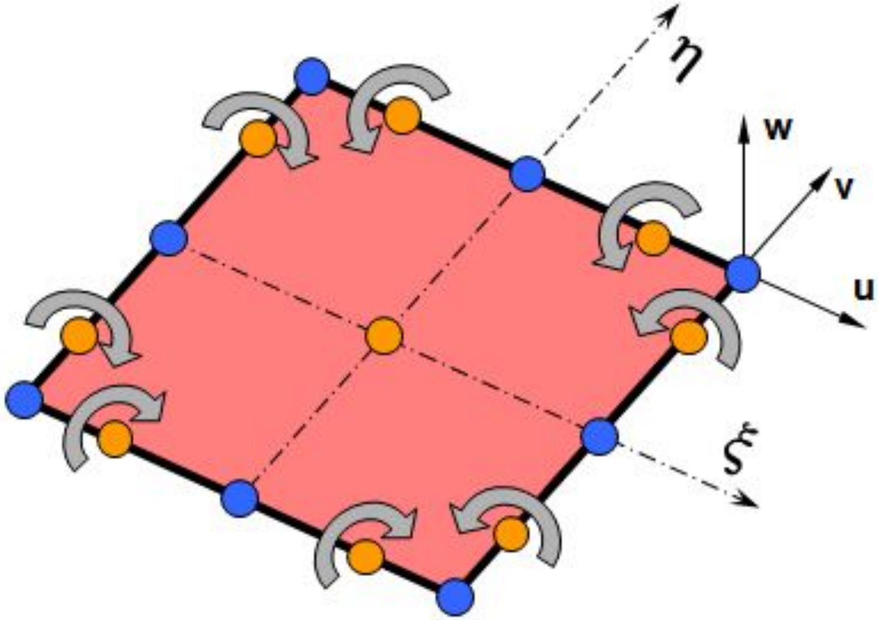
Características



- **Isoparamétricos**
- **32 grados de libertad**
- **Ocho nodos serendípticos (Desplazamientos u, v, w)**
- **Nueve nodos Loof (Rotaciones locales en lados)**
- **Dos grupos de funciones de forma**
- **Es no conforme**
- **Utiliza la teoría de placas de Kirchhoff**
- **Utiliza integración reducida 2x2**

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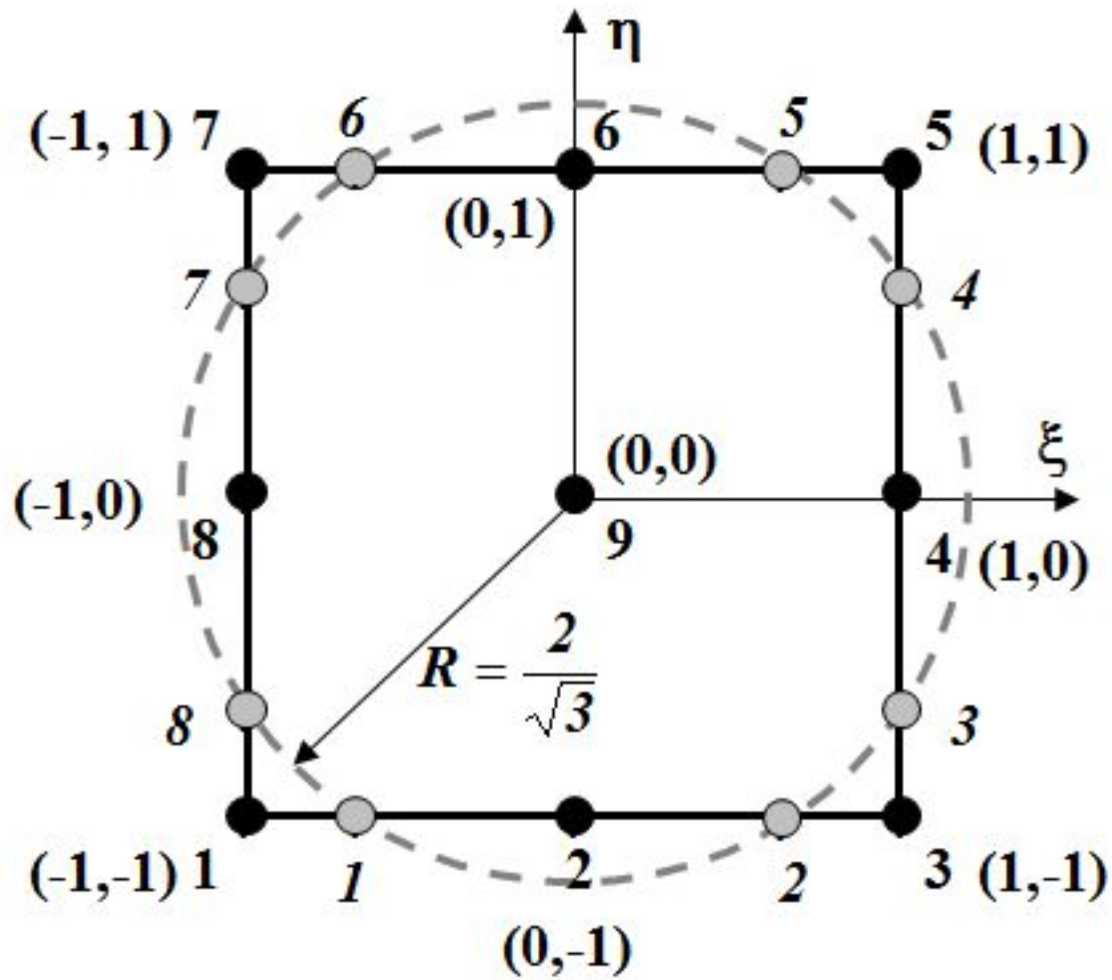
Funciones de Forma



Funciones de Forma (Nodos Serendípitos)

Triángulo de Pascal

$$1$$

$$\xi \quad \eta$$

$$\xi^2 \quad \xi\eta \quad \eta^2$$

$$\xi^3 \quad \xi^2\eta \quad \xi\eta^2 \quad \eta^3$$

$$\xi^4 \quad \xi^3\eta \quad \xi^2\eta^2 \quad \xi\eta^3 \quad \eta^4$$

Forma del Polinomio

$$N_i = a_{i1}1 + a_{i2}\xi + a_{i3}\eta + a_{i4}\xi^2 + a_{i5}\xi\eta + a_{i6}\eta^2 + a_{i7}\xi^2\eta + a_{i8}\xi\eta^2$$

Funciones de Forma (Nodos Serendípitos)

$$
\begin{bmatrix}
1 & \xi & \eta & \xi^2 & \xi\eta & \eta^2 & \xi^2\eta & \xi\eta^2 \\
1 & \xi & \eta & \xi^2 & \xi\eta & \eta^2 & \xi^2\eta & \xi\eta^2 \\
1 & \xi & \eta & \xi^2 & \xi\eta & \eta^2 & \xi^2\eta & \xi\eta^2 \\
1 & \xi & \eta & \xi^2 & \xi\eta & \eta^2 & \xi^2\eta & \xi\eta^2 \\
1 & \xi & \eta & \xi^2 & \xi\eta & \eta^2 & \xi^2\eta & \xi\eta^2 \\
1 & \xi & \eta & \xi^2 & \xi\eta & \eta^2 & \xi^2\eta & \xi\eta^2 \\
1 & \xi & \eta & \xi^2 & \xi\eta & \eta^2 & \xi^2\eta & \xi\eta^2 \\
1 & \xi & \eta & \xi^2 & \xi\eta & \eta^2 & \xi^2\eta & \xi\eta^2
\end{bmatrix}
\cdot
\begin{bmatrix}
a_{11} & a_{21} & a_{31} & a_{41} & a_{51} & a_{61} & a_{71} & a_{81} \\
a_{12} & a_{22} & a_{32} & a_{42} & a_{52} & a_{62} & a_{72} & a_{82} \\
a_{13} & a_{23} & a_{33} & a_{43} & a_{53} & a_{63} & a_{73} & a_{83} \\
a_{14} & a_{24} & a_{34} & a_{44} & a_{54} & a_{64} & a_{74} & a_{84} \\
a_{15} & a_{25} & a_{35} & a_{45} & a_{55} & a_{65} & a_{75} & a_{85} \\
a_{16} & a_{26} & a_{36} & a_{46} & a_{56} & a_{66} & a_{76} & a_{86} \\
a_{17} & a_{27} & a_{37} & a_{47} & a_{57} & a_{67} & a_{77} & a_{87} \\
a_{18} & a_{28} & a_{38} & a_{48} & a_{58} & a_{68} & a_{78} & a_{88}
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$T \cdot C = I$$

$$C = T^{-1} \cdot I$$

$$C = T^{-1}$$

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Funciones de Forma (Nodos Serendípitos)

1	-1	-1	1	1	1	-1	-1
1	0	-1	0	0	1	0	0
1	1	-1	1	-1	1	1	-1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1
1	0	1	0	0	1	0	0
1	-1	1	1	-1	1	-1	1
1	-1	0	1	0	0	0	0

→

-0.25	0.5	-0.25	0.5	-0.25	0.5	-0.25	0.5
0	0	0	0.5	0	0	0	-0.5
0	-0.5	0	0	0	0.5	0	0
0.25	-0.5	0.25	0	0.25	-0.5	0.25	0
0.25	0	-0.25	0	0.25	0	-0.25	0
0.25	0	0.25	-0.5	0.25	0	0.25	-0.5
-0.25	0	0.25	-0.5	0.25	0	-0.25	0.5
-0.25	0.5	-0.25	0	0.25	-0.5	0.25	0

F.F. Para Nodos Serendípitos

$$NS1 = \frac{1}{4}\left(-1 + \xi^2 + \xi\eta + \eta^2 - \xi^2\eta - \xi\eta^2\right)$$

$$NS2 = \frac{1}{2}\left(1 - \eta - \xi^2 + \xi^2\eta\right)$$

$$NS3 = \frac{1}{4}\left(-1 + \xi^2 - \xi\eta + \eta^2 + \xi^2\eta - \xi\eta^2\right)$$

$$NS4 = \frac{1}{2}\left(1 + \xi - \eta^2 - \xi\eta^2\right)$$

$$NS5 = \frac{1}{4}\left(-1 + \xi^2 + \xi\eta + \eta^2 + \xi^2\eta + \xi\eta^2\right)$$

$$NS6 = \frac{1}{2}\left(1 + \eta - \xi^2 - \xi^2\eta\right)$$

$$NS7 = \frac{1}{4}\left(-1 + \xi^2 - \xi\eta + \eta^2 - \xi^2\eta + \xi\eta^2\right)$$

$$NS8 = \frac{1}{2}\left(1 - \xi - \eta^2 + \xi\eta^2\right)$$

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Funciones de Forma (Nodos Serendípitos)

1	ξ	η	ξ^2	$\xi\eta$	η^2	$\xi^2\eta$	$\xi\eta^2$
1	ξ	η	ξ^2	$\xi\eta$	η^2	$\xi^2\eta$	$\xi\eta^2$
1	ξ	η	ξ^2	$\xi\eta$	η^2	$\xi^2\eta$	$\xi\eta^2$
1	ξ	η	ξ^2	$\xi\eta$	η^2	$\xi^2\eta$	$\xi\eta^2$
1	ξ	η	ξ^2	$\xi\eta$	η^2	$\xi^2\eta$	$\xi\eta^2$
1	ξ	η	ξ^2	$\xi\eta$	η^2	$\xi^2\eta$	$\xi\eta^2$
1	ξ	η	ξ^2	$\xi\eta$	η^2	$\xi^2\eta$	$\xi\eta^2$
1	ξ	η	ξ^2	$\xi\eta$	η^2	$\xi^2\eta$	$\xi\eta^2$

•

-0.25	0.5	-0.25	0.5	-0.25	0.5	-0.25	0.5
0	0	0	0.5	0	0	0	-0.5
0	-0.5	0	0	0	0.5	0	0
0.25	-0.5	0.25	0	0.25	-0.5	0.25	0
0.25	0	-0.25	0	0.25	0	-0.25	0
0.25	0	0.25	-0.5	0.25	0	0.25	-0.5
-0.25	0	0.25	-0.5	0.25	0	-0.25	0.5
-0.25	0.5	-0.25	0	0.25	-0.5	0.25	0

F.F. Para Nodos Serendípitos

$$NS1 = \frac{1}{4}\left(-1 + \xi^2 + \xi\eta + \eta^2 - \xi^2\eta - \xi\eta^2\right)$$

$$NS2 = \frac{1}{2}\left(1 - \eta - \xi^2 + \xi^2\eta\right)$$

$$NS3 = \frac{1}{4}\left(-1 + \xi^2 - \xi\eta + \eta^2 + \xi^2\eta - \xi\eta^2\right)$$

$$NS4 = \frac{1}{2}\left(1 + \xi - \eta^2 - \xi\eta^2\right)$$

$$NS5 = \frac{1}{4}\left(-1 + \xi^2 + \xi\eta + \eta^2 + \xi^2\eta + \xi\eta^2\right)$$

$$NS6 = \frac{1}{2}\left(1 + \eta - \xi^2 - \xi^2\eta\right)$$

$$NS7 = \frac{1}{4}\left(-1 + \xi^2 - \xi\eta + \eta^2 - \xi^2\eta + \xi\eta^2\right)$$

$$NS8 = \frac{1}{2}\left(1 - \xi - \eta^2 + \xi\eta^2\right)$$

Funciones de Forma (Nodos Serendípitos)

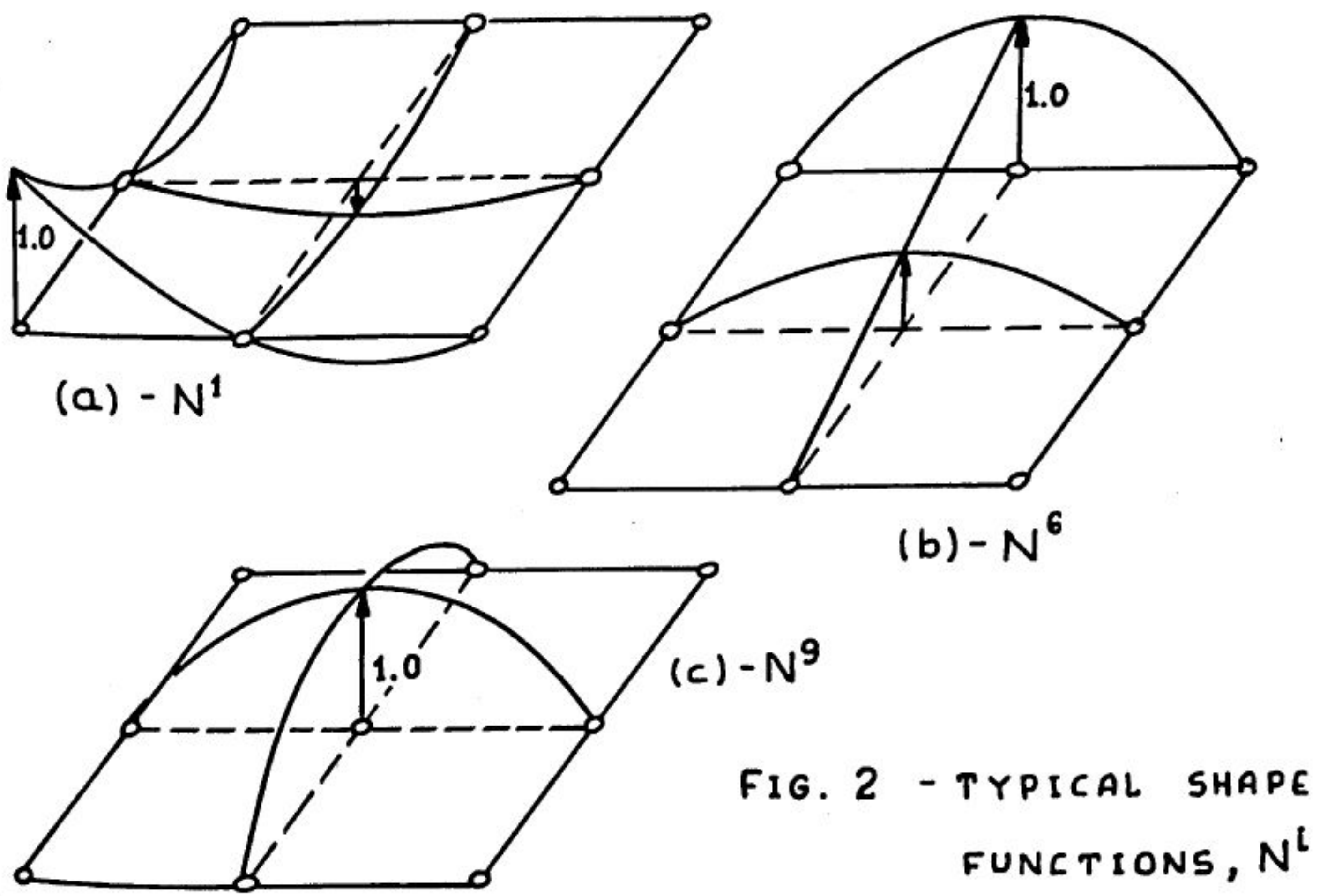


FIG. 2 - TYPICAL SHAPE FUNCTIONS, N^i

Analysis of plates and arbitrary shells by the use of the semiloof element. R. Martins, 1976

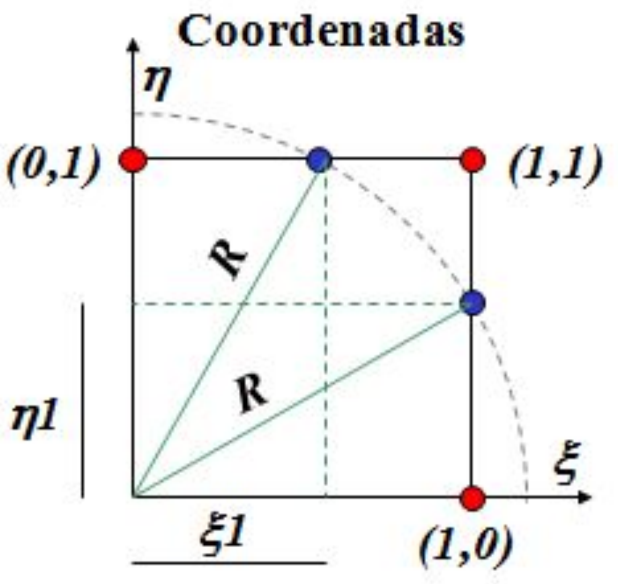
Funciones de Forma (Nodos Loof)

F.F. Para Nodos Loof

- **En principio ocho ("8") nodos**
- **Necesidad de al menos un polinomio con 8 términos**

Forma del Polinomio

$$N_i = a_{i1}1 + a_{i2}\xi + a_{i3}\eta + a_{i4}\xi^2 + a_{i5}\xi\eta + a_{i6}\eta^2 + a_{i7}\xi^2\eta + a_{i8}\xi\eta^2$$



$$R = \frac{2}{\sqrt{3}} = \sqrt{1^2 + \xi_1^2} \Rightarrow \xi_1 = \frac{1}{\sqrt{3}}$$

$$R = \frac{2}{\sqrt{3}} = \sqrt{1^2 + \eta_1^2} \Rightarrow \eta_1 = \frac{1}{\sqrt{3}}$$

Funciones de Forma (Nodos Loof)

1	-0.577	-1	0.333	0.577	1	-0.577	-0.333
1	0.577	-1	0.333	-0.577	1	0.577	-0.333
1	1	-0.577	1	-0.577	0.333	0.333	-0.577
1	1	0.577	1	0.577	0.333	0.333	0.577
1	0.577	1	0.333	0.577	1	0.577	0.333
1	-0.577	1	0.333	-0.577	1	-0.577	0.333
1	-1	0.577	1	-0.577	0.333	-0.333	0.577
1	-1	-0.577	1	0.577	0.333	-0.333	-0.577

#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!
#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!
#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!
#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!
#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!
#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!
#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!
#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!	#¡NUM!

Funciones de Forma (Nodos Loof)

Triángulo de Pascal

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & \xi & & \eta & & & \\
 & & \xi^2 & & \xi\eta & & \eta^2 & & \\
 & \xi^3 & & \xi^2\eta & & \xi\eta^2 & & \eta^3 & \\
\xi^4 & & \xi^3\eta & & \xi^2\eta^2 & & \xi\eta^3 & & \eta^4
\end{array}
$$

Forma del Polinomio

$$N_i = a_{i1}1 + a_{i2}\xi + a_{i3}\eta + a_{i4}\xi^2 + a_{i5}\xi\eta + a_{i6}\eta^2 + a_{i7}\xi^2\eta + a_{i8}\xi\eta^2 + a_{i9}(\xi^3\eta - \xi\eta^3)$$

Funciones de Forma (Nodos Loof)

1	-0.577	-1	0.333	0.577	1	-0.577	-0.333	-0.385
1	0.577	-1	0.333	-0.577	1	0.577	-0.333	0.385
1	1	-0.577	1	-0.577	0.333	0.333	-0.577	-0.385
1	1	0.577	1	0.577	0.333	0.333	0.577	0.385
1	0.577	1	0.333	0.577	1	0.577	0.333	-0.385
1	-0.577	1	0.333	-0.577	1	-0.577	0.333	0.385
1	-1	0.577	1	-0.577	0.333	-0.333	0.577	-0.385
1	-1	-0.577	1	0.577	0.333	-0.333	-0.577	0.385
1	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	1
0.217	-0.217	0.375	0.375	-0.217	0.217	-0.375	-0.375	0
-0.375	-0.375	0.217	-0.217	0.375	0.375	-0.217	0.217	0
-0.094	-0.094	0.281	0.281	-0.094	-0.094	0.281	0.281	-0.750
0.217	-0.217	-0.217	0.217	0.217	-0.217	-0.217	0.217	0
0.281	0.281	-0.094	-0.094	0.281	0.281	-0.094	-0.094	-0.750
-0.650	0.650	-0.375	-0.375	0.650	-0.650	0.375	0.375	0
0.375	0.375	-0.650	0.650	-0.375	-0.375	0.650	-0.650	0
-0.325	0.325	-0.325	0.325	-0.325	0.325	-0.325	0.325	0

Funciones de Forma (Nodos Loof)

$$NL1 = \frac{1}{8}\left(\sqrt{3}\xi - 3\eta - \frac{3}{4}\xi^2 + \sqrt{3}\xi\eta + \frac{9}{4}\eta^2 - 3\sqrt{3}\xi\eta^2 + 3\xi^2\eta - \frac{3\sqrt{3}}{2}\left(\xi^3\eta - \xi\mu^3\right)\right)$$

$$NL2 = \frac{1}{8}\left(-\sqrt{3}\xi - 3\eta - \frac{3}{4}\xi^2 - \sqrt{3}\xi\eta + \frac{9}{4}\eta^2 + 3\sqrt{3}\xi\eta^2 + 3\xi^2\eta + \frac{3\sqrt{3}}{2}\left(\xi^3\eta - \xi\mu^3\right)\right)$$

$$NL3 = \frac{1}{8}\left(3\xi + \sqrt{3}\eta + \frac{9}{4}\xi^2 - \sqrt{3}\xi\eta - \frac{3}{4}\eta^2 - 3\xi\xi^2 - 3\sqrt{3}\xi^2\eta - \frac{3\sqrt{3}}{2}\left(\xi^3\eta - \xi\mu^3\right)\right)$$

$$NL4 = \frac{1}{8}\left(3\xi - \sqrt{3}\eta + \frac{9}{4}\xi^2 + \sqrt{3}\xi\eta - \frac{3}{4}\eta^2 - 3\xi\xi^2 + 3\sqrt{3}\xi^2\eta + \frac{3\sqrt{3}}{2}\left(\xi^3\eta - \xi\mu^3\right)\right)$$

$$NL5 = \frac{1}{8}\left(-\sqrt{3}\xi + 3\eta - \frac{3}{4}\xi^2 + \sqrt{3}\xi\eta + \frac{9}{4}\eta^2 + 3\sqrt{3}\xi\eta^2 - 3\xi^2\eta - \frac{3\sqrt{3}}{2}\left(\xi^3\eta - \xi\mu^3\right)\right)$$

$$NL6 = \frac{1}{8}\left(\sqrt{3}\xi + 3\eta - \frac{3}{4}\xi^2 - \sqrt{3}\xi\eta + \frac{9}{4}\eta^2 - 3\sqrt{3}\xi\eta^2 - 3\xi^2\eta + \frac{3\sqrt{3}}{2}\left(\xi^3\eta - \xi\mu^3\right)\right)$$

$$NL7 = \frac{1}{8}\left(-3\xi - \sqrt{3}\eta + \frac{9}{4}\xi^2 - \sqrt{3}\xi\eta - \frac{3}{4}\eta^2 + 3\xi\xi^2 + 3\sqrt{3}\xi^2\eta - \frac{3\sqrt{3}}{2}\left(\xi^3\eta - \xi\mu^3\right)\right)$$

$$NL8 = \frac{1}{8}\left(-3\xi + \sqrt{3}\eta + \frac{9}{4}\xi^2 + \sqrt{3}\xi\eta - \frac{3}{4}\eta^2 + 3\xi\xi^2 - 3\sqrt{3}\xi^2\eta + \frac{3\sqrt{3}}{2}\left(\xi^3\eta - \xi\mu^3\right)\right)$$

$$NL9 = 1 - \frac{3}{4}\xi^2 - \frac{3}{4}\eta^2$$

Funciones de Forma (Nodos Loof)

$$NL1 = \frac{1}{8}\left(\sqrt{3}\xi - 3\eta - \frac{3}{4}\xi^{2} + \sqrt{3}\xi\eta + \frac{9}{4}\eta^{2} - 3\sqrt{3}\xi\eta^{2} + 3\xi^{2}\eta - \frac{3\sqrt{3}}{2}\left(\xi^{3}\eta - \xi\mu^{3}\right)\right)$$

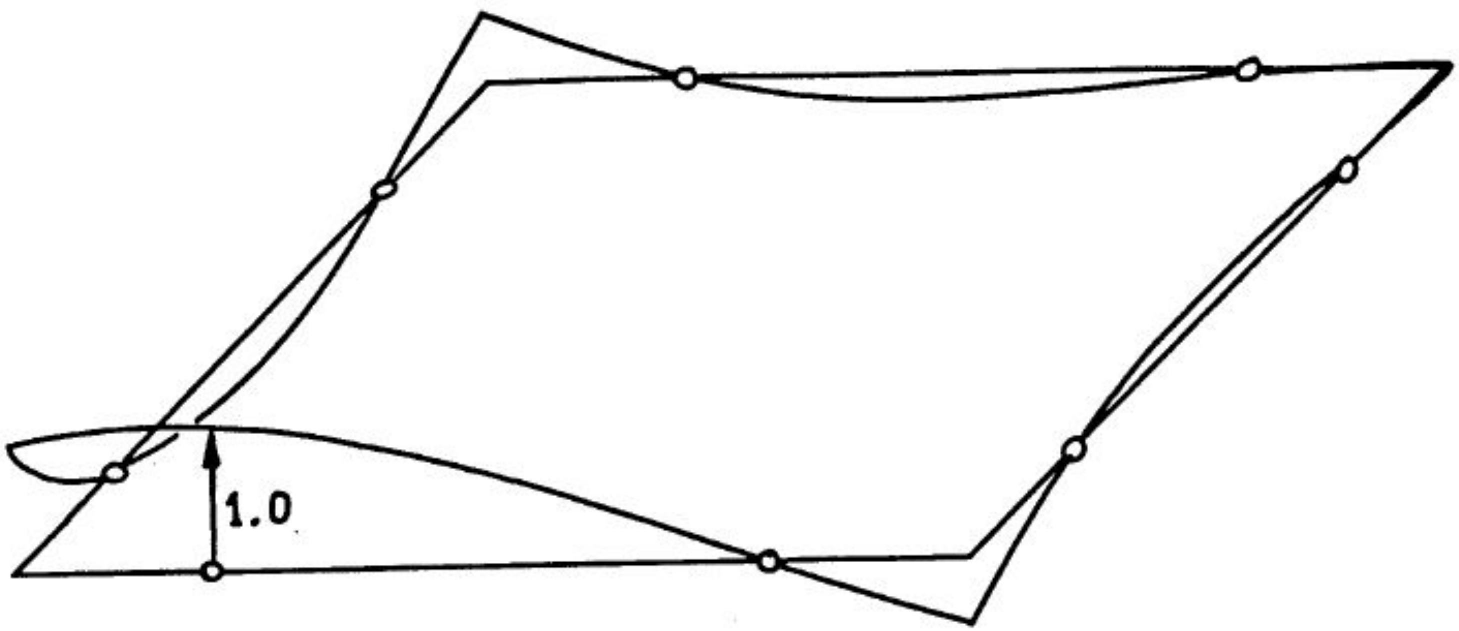
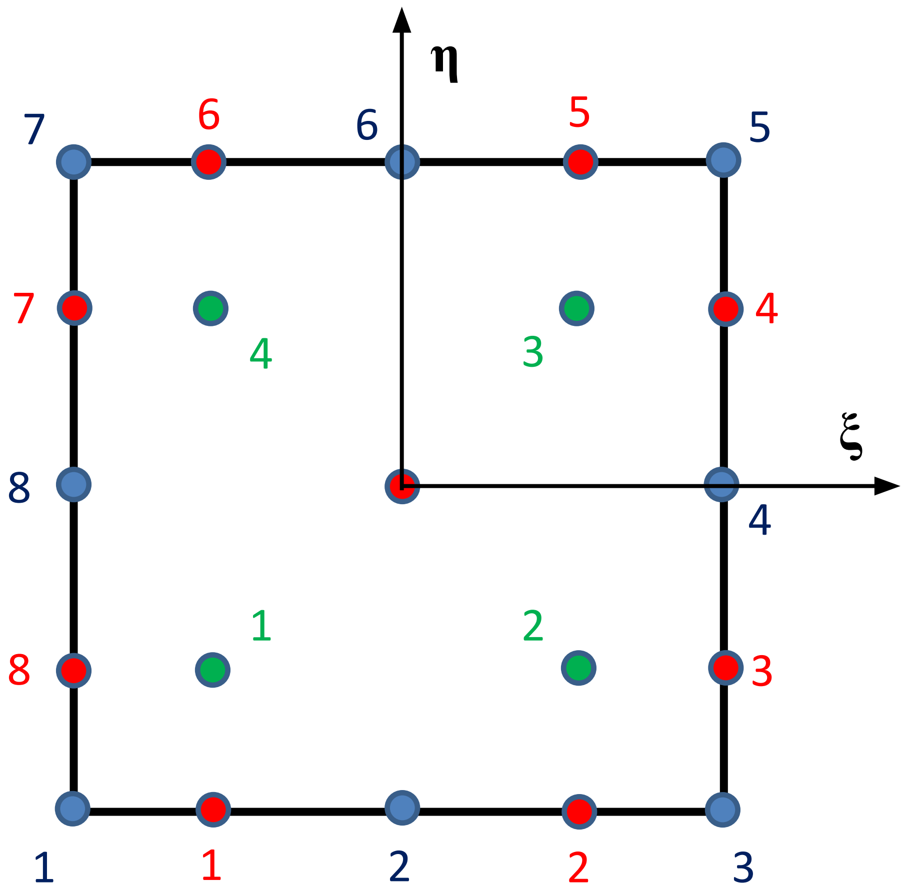


FIG. 5 - SHAPE FUNCTION FOR LOOF NODE 1

Analysis of plates and arbitrary shells by the use of the semiloof element. R. Martins, 1976

Integración numérica

Esquema 2x2 – Integración reducida



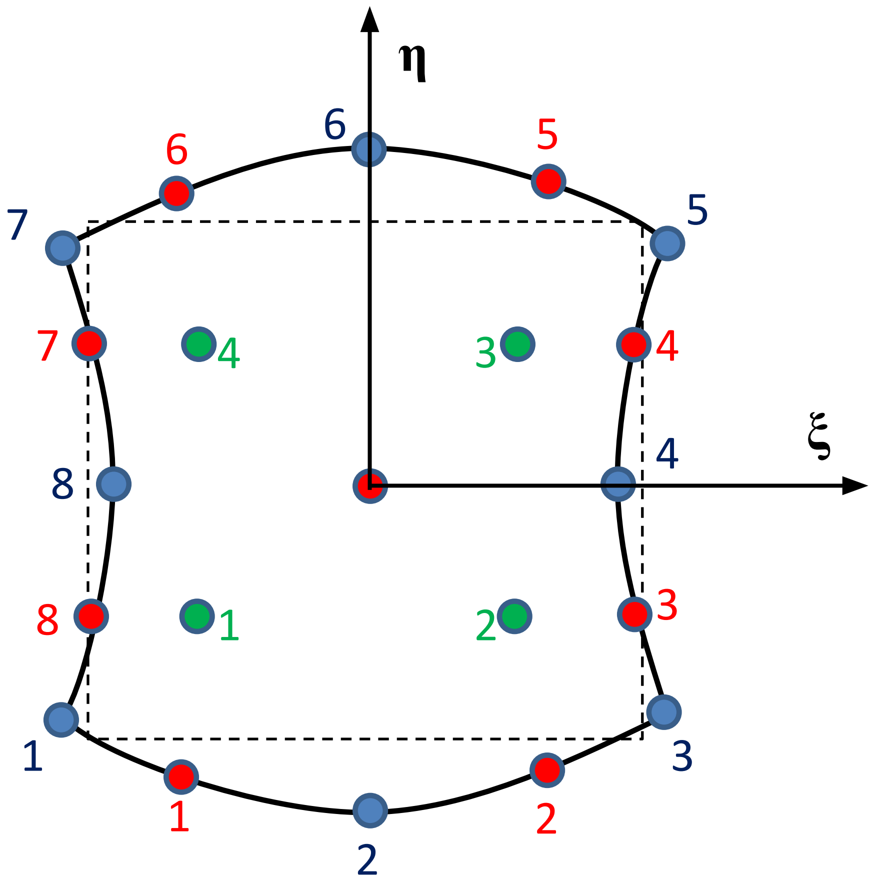
n	$\pm\xi_i$	W_i
1	0.0	2.0
2	0.5773502692	1.0
3	0.774596697 0.0	0.5555555556 0.8888888889
4	0.8611363116 0.3399810436	0.3478548451 0.6521451549
5	0.9061798459 0.5384693101 0.0	0.2369268851 0.4786286705 0.5688888889
6	0.9324695142 0.6612093865 0.2386191861	0.1713244924 0.3607615730 0.4679139346
7	0.9491079123 0.7415311856 0.4058451514 0.0	0.1294849662 0.2797053915 0.3818300505 0.4179591837
8	0.9602898565 0.7966664774 0.5255324099 0.1834346425	0.1012285363 0.2223810345 0.3137066459 0.3626837834

Integración completa para polinomios de grado igual o superior a 2n-1

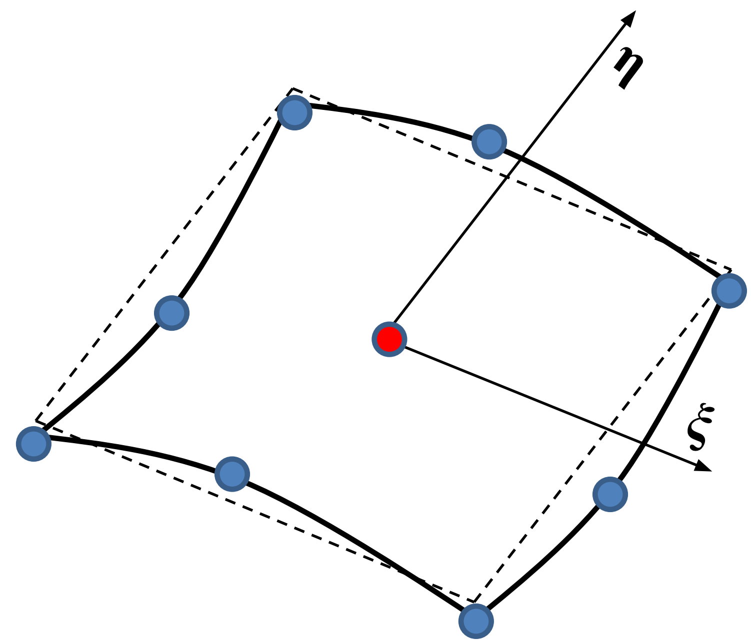
Integración numérica

Modos espurios (Energía de deformación nula)

Reloj de arena (membranal)



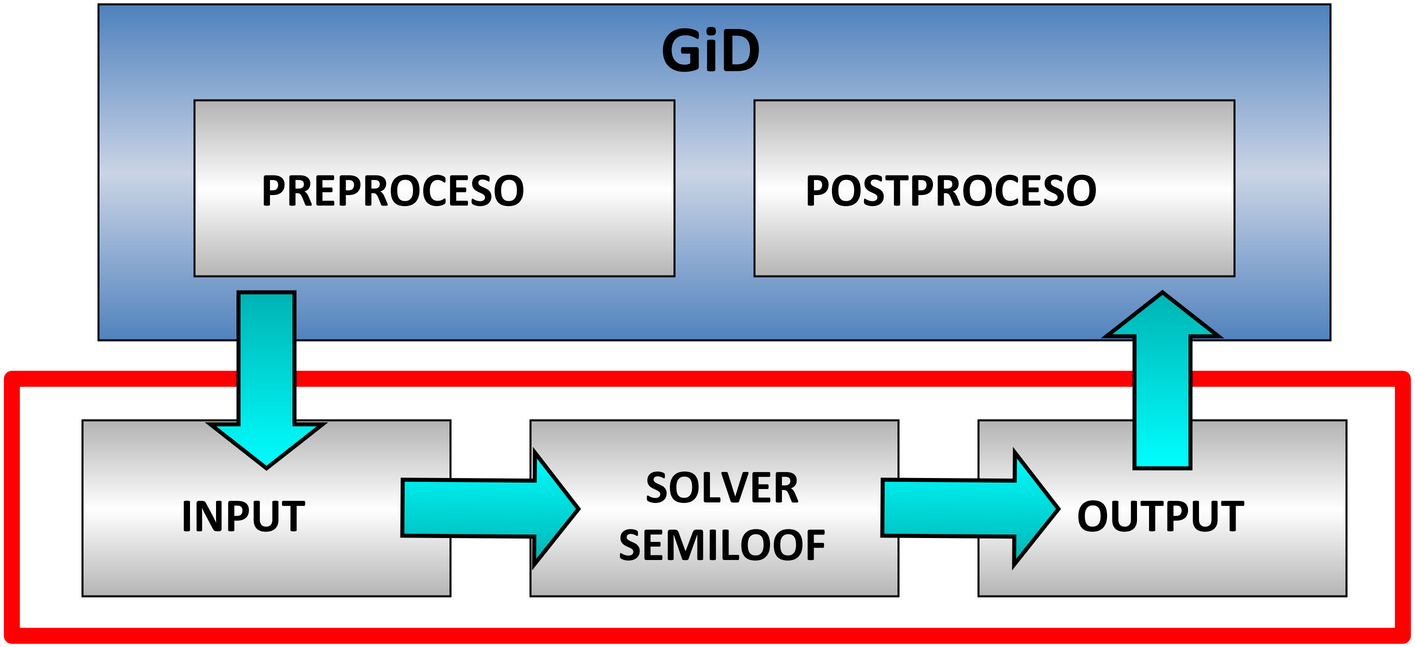
Reloj de arena (flexional)



Implementación en GiD

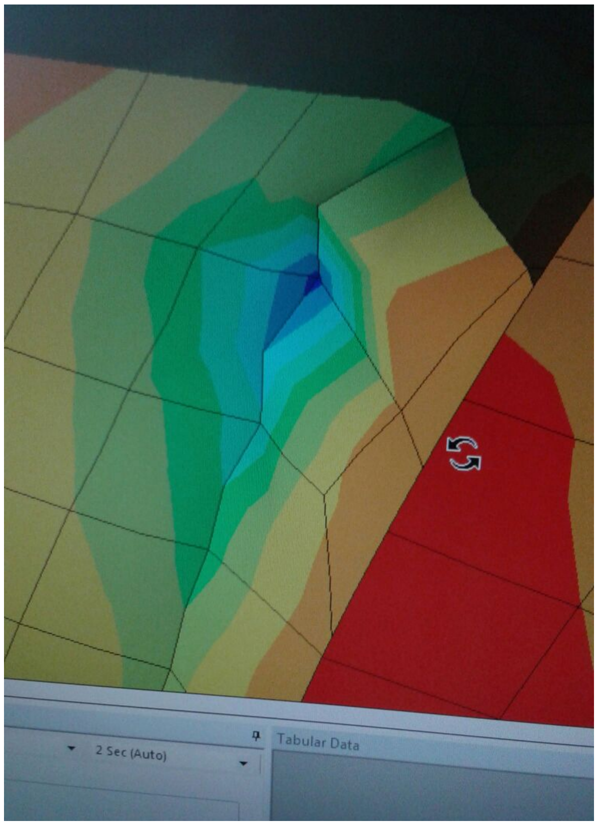
Solver: Master Tyrant (Irons)

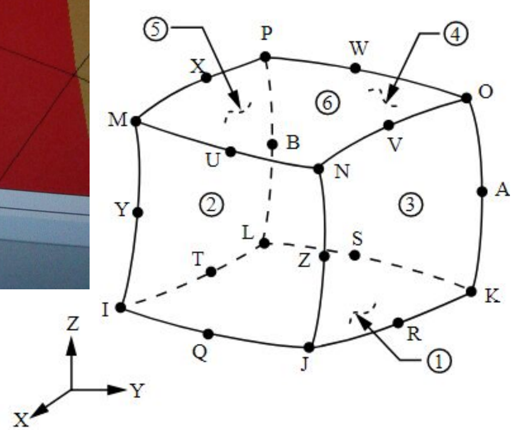
GiD: software de pre y post proceso (CIMNE)



Integración Reducida

Hourglass mode en Solid186 de ANSYS (hexaedro de 20 nodos)





n	$\pm\xi_i$	W_i
1	0.0	2.0
2	0.5773502692	1.0
3	0.774596697 0.0	0.5555555556 0.8888888889
4	0.8611363116 0.3399810436	0.3478548451 0.6521451549
5	0.9061798459 0.5384693101 0.0	0.2369268851 0.4786286705 0.5688888889
6	0.9324695142 0.6612093865 0.2386191861	0.1713244924 0.3607615730 0.4679139346
7	0.9491079123 0.7415311856 0.4058451514 0.0	0.1294849662 0.2797053915 0.3818300505 0.4179591837
8	0.9602898565 0.7966664774 0.5255324099 0.1834346425	0.1012285363 0.2223810345 0.3137066459 0.3626837834