

$$g(x, y) = \frac{x-y}{xy+2} \quad P_0(1, -1) \quad \vec{u} = (12, 5)$$

$$D_{\vec{u}} f(P_0) = \nabla f(P_0) \cdot \vec{u}$$

$$\|\vec{u}\| = \sqrt{12^2 + 5^2} = 13$$

$$v = \frac{\vec{u}}{\|\vec{u}\|} = (12/13, 5/13)$$

$$g_x(x, y) = \frac{(xy+2) - (x-y)y}{(xy+2)^2}$$

$$g_x(1, -1) = \frac{1 - 2 \cdot (-1)}{1} = 3$$

$$g_y(x, y) = \frac{(-1)(xy+2) - (x-y)x}{(xy+2)^2}$$

$$g_y(1, -1) = \frac{-1 - 2}{1} = -3$$

$$\nabla f(1, -1) = (3, -3)$$

$$D_{\vec{u}} f(1, -1) = (3, -3) \cdot \left(\frac{12}{13}, \frac{5}{13}\right)$$

$$= \frac{36}{13} - \frac{15}{13} = \boxed{\frac{21}{13}}$$