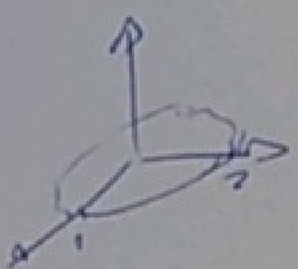
15 ⓐ $\quad F = (x^2, 2x, z^2) \quad C: 4x^2+y^2=4$

sobre plano xy
antihorario

$$\oint_C F\cdot T\,ds = \iint_S \nabla\times F\cdot n\,d\sigma.$$

$S: \quad z=0 \quad$ en $\quad 4x^2+y^2 \leq 4$



$$r(u,v) = (u\cos v,\ 2u\sin v,\ 0) \quad \begin{cases} 0\leq u\leq 1 \\ 0\leq v\leq 2\pi \end{cases}$$

$$r_u = (\cos v,\ 2\sin v,\ 0)$$

$$r_v = (-u\sin v,\ 2u\cos v,\ 0).$$

$$r_u\times r_v = (0,0,2u)$$

$$\nabla\times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2x & z^2 \end{vmatrix} = (0,0,2)$$

$$\oint_C F\cdot T\,ds = \int_0^{2\pi}\int_0^1 (0,0,2)\cdot(0,0,2u)\,du\,dv$$

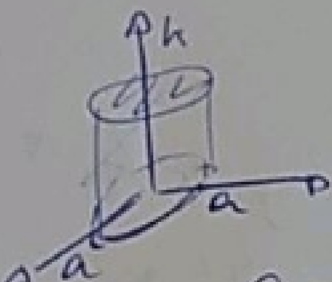
$$= \int_0^{2\pi}\int_0^1 4u\,du\,dv = \int_0^{2\pi} \frac{4u^2}{2}\Big|_0^1 dv =$$

$$= \int_0^{2\pi} 2\,dv = 2v\Big|_0^{2\pi} = \boxed{4\pi}$$

TP4 B.

18 $S: \; x^2+y^2=a^2 \quad 0\leq z\leq h$ con tapa $z=h$

$F=(-y, x, x^2)$ Flujo $\nabla \times F$.



$$\oint_C F\cdot T\,ds = \iint_S \nabla\times F\cdot \bar{n}\,d\sigma$$

$C \quad r(t)=(a\cos t, a\sin t, 0) \quad 0\leq t\leq 2\pi$.

$$\int_C F\cdot T\,ds = \int_0^{2\pi} (-a\sin t, a\cos t, a^2\cos^2 t)\cdot(-a\sin t, a\cos t, 0)\,dt$$

$$= \int_0^{2\pi} (a^2\sin^2 t + a^2\cos^2 t + 0)\,dt = \int_0^{2\pi} a^2\,dt = a^2\cdot t\Big|_0^{2\pi}$$

$$= \boxed{a^2 2\pi}.$$

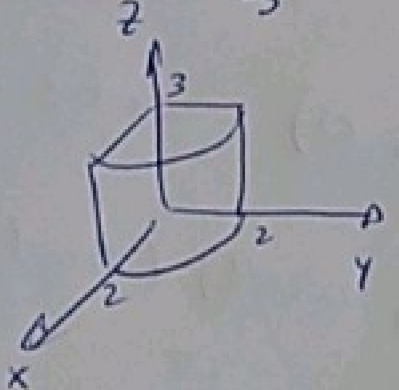
$$\nabla\times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & x^2 \end{vmatrix} = (0, -2x, 2)$$

$$\iint_S (0,-2x,2)\cdot\bar{n}\,d\sigma = \boxed{2a^2\pi}.$$

(26) (b) $F = (6x^2 + 2xy \,;\, 2y + x^2 z \,;\, 4x^2 y^3)$

$D = \quad x^2+y^2 = 4 \quad z = 3 \quad$ 1º octante.

$$\iint_S F \cdot \bar{n}\, d\sigma = \iiint_D \operatorname{div} F \, dV$$



$$\operatorname{div} F = 12x + 2y + 2 + 0.$$

$$\iiint_D (12x+2y+2)\, dV = \int_0^{\pi/4}\int_0^{2}\int_0^{3} (12 r\cos\theta + 2 r\sin\theta + 2)\; r\,dz\,dr\,d\theta,$$

$$= \int_0^{\pi/4}\int_0^{2}\int_0^{3} (12 r^2\cos\theta + 2r^2\sin\theta + 2r)\, dz\, dr\, d\theta =$$

$$= \int_0^{\pi/4}\int_0^{2} (12r^2\cos\theta + 2r^2\sin\theta + 2r)\cdot z\Big|_0^3 dr\, d\theta =$$

$$= \int_0^{\pi/4}\int_0^{2} (36r^2\cos\theta + 6r^2\sin\theta + 6r)\, dr\, d\theta =$$

$$= \int_0^{\pi/4} \frac{36r^3}{3}\cos\theta + \frac{6r^3}{3}\sin\theta + \frac{6r^2}{2}\Big|_0^2 d\theta =$$

$$= \int_0^{\pi/4} (96\cos\theta + 16\sin\theta + 12)\, d\theta =$$

$$= 96\cdot\sin\theta - 16\cos\theta + 12\theta\Big|_0^{2\pi} =$$

$$= (96\cdot 0 - 16 + 24\pi) - (0 - 16 + 0) =$$

$$= \boxed{24\pi}.$$

32 $F = (x, y, z)$ S, D hipót de T.G.

Volume de $D = \iiint_D dV$

$$\iint_S F \cdot n \, d\sigma = \iiint_D \operatorname{div} F \, dV =$$

$$\operatorname{div} F = 1 + 1 + 1 = \boxed{3}$$

$$\iint_S F \cdot n \, d\sigma = \iiint_D 3 \, dV.$$

$$\Rightarrow \iiint_D dV = \frac{1}{3} \iint_S F \cdot n \, dV.$$

$S: \; z=0 \qquad (x,y): 4x^2+y^2 \leq 4$

$$\iint_S (\nabla \times F) \cdot n \, d\sigma = \iint_R (\nabla \times F)(S(r,\theta)) \cdot (S_r \times S_\theta) \, dA$$

$S(r,\theta) = (r\cos\theta, 2r\sin\theta, 0)$

$0 \leq r \leq 1 \qquad 0 \leq \theta \leq 2\pi$

$S_r = (\cos\theta, 2\sin\theta, 0)$

$S_\theta = (-r\sin\theta, 2r\cos\theta, 0)$

$S_r \times S_\theta = (0, 0, 2r)$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & 2x & z^2 \end{vmatrix} = (0, 0, 2)$$

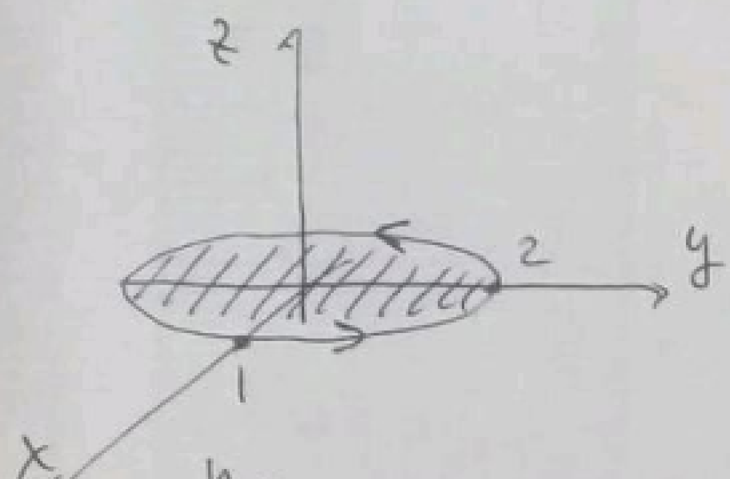
$(\nabla \times F)(S(r,\theta)) = (0, 0, 2)$

$$\iint_S (\nabla \times F) \cdot n \, d\sigma = \int_0^{2\pi} \int_0^1 4r \, dr \, d\theta. = B$$

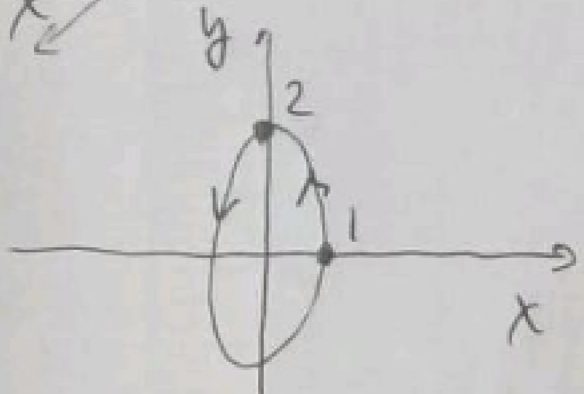
Falta verificar que $A = B$!

15.2 $F=(x^2, 2x, z^2)$

$C: 4x^2+y^2=4 \quad z=0$ antihorario



$$\oint_C F\cdot dr = \iint_S (\nabla\times F)\cdot n\, d\sigma$$



$$r(t)=(\cos t, 2\sin t, 0)$$

$$0\le t\le 2\pi$$

$$r'(t)=(-\sin t, 2\cos t, 0)$$

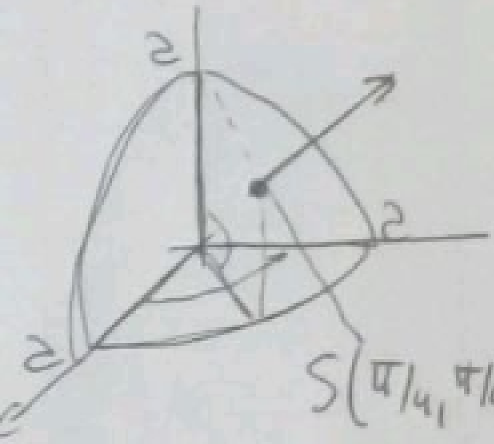
$$\oint_C F\cdot dr = \int_a^b F(r(t))\cdot r'(t)\,dt =$$

$$\int_0^{2\pi}(\cos^2 t, 2\cos t, 0)\cdot(-\sin t, 2\cos t, 0)\,dt =$$

$$\int_0^{2\pi}(-\cos^2 t\sin t + 4\cos^2 t)\,dt = A$$

$x^2+y^2+z^2=2^2 \quad (z>0)$ 1° octante
n alejando origen.

$F=(0,0,z)$



$S(\varphi,\theta)=(2\operatorname{sen}\varphi\cos\theta,\ 2\operatorname{sen}\varphi\operatorname{sen}\theta,\ 2\cos\varphi)$

$S(\pi/4,\pi/4)$ $\underbrace{0\le\varphi\le\pi/2 \quad 0\le\theta\le\pi/2}_{R}$

$$\iint_S F\cdot n\,d\sigma=\iint_R F(S(\varphi,\theta))\cdot\underbrace{(S_\varphi\times S_\theta)}_{n}\,dA$$

$S_\varphi=(2\cos\varphi\cos\theta,\ 2\cos\varphi\operatorname{sen}\theta,\ -2\operatorname{sen}\varphi)$

$S_\theta=(-2\operatorname{sen}\varphi\operatorname{sen}\theta,\ 2\operatorname{sen}\varphi\cos\theta,\ 0)$

$S_\varphi\times S_\theta=(2^2\operatorname{sen}^2\varphi\cos\theta,\ 2^2\operatorname{sen}^2\varphi\operatorname{sen}\theta,\ 2^2\cos\varphi\operatorname{sen}\varphi)$

$P(\pi/4,\pi/4)$

$F(S(\varphi,\theta))\cdot S_\varphi\times S_\theta=(0,0,2\cos\varphi)\cdot(S_\varphi\times S_\theta)$

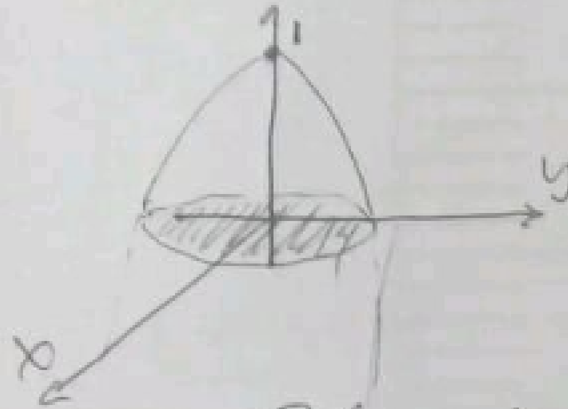
$=2^3\cos^2\varphi\operatorname{sen}\varphi$

$$\iint_S F\cdot n\,d\sigma=\int_0^{\pi/2}\int_0^{\pi/2}2^3\cos^2\varphi\operatorname{sen}\varphi\,d\varphi\,d\theta.$$

$$=\frac{2^3}{6}\pi$$

(11.c) $f(x,y,z) = x^2\sqrt{5-4z}$

$S: \ z = 1 - x^2 - y^2 \qquad z \geq 0$



$$\iint_S f \, d\sigma = \iint_R f(S(u,v)) \, \|S_u \times S_v\| \, dA$$

$$S(r,\theta) = (r\cos\theta, r\sin\theta, 1-r^2)$$

$$\boxed{0 \leq r \leq 1 \qquad 0 \leq \theta \leq 2\pi} \ R$$

$$S_r(r,\theta) = (\cos\theta, \sin\theta, -2r)$$

$$S_\theta(r,\theta) = (-r\sin\theta, r\cos\theta, 0)$$

$$S_r \times S_\theta = (2r^2\cos\theta, 2r^2\sin\theta, r)$$

$$\|S_r \times S_\theta\| = \sqrt{4r^4 + r^2} = r\sqrt{4r^2+1}$$

$$\iint_S f \, d\sigma = \iint_R \underbrace{r^2\cos^2\theta\sqrt{5-4(1-r^2)}}_{f(S(r,\theta))} \cdot \underbrace{r\sqrt{4r^2+1}}_{\|S_r \times S_\theta\|} \, dA$$

$$= \int_0^{2\pi}\int_0^1 r^3\cos^2\theta\,(4r^2+1)\,dr\,d\theta.$$

$$= \frac{17}{15}\pi$$