TP5

37 a) $4y'' + y' = 0$

E.C.

$4r^2 + r = 0$

$r_1 = 0 \quad r_2 = -1/4$

$y_1 = e^{0x} \quad y_2 = e^{-1/4 x}$

$y_g(x) = C_1 \cdot 1 + C_2 e^{-1/4 x}$

b) $y'' + 8y' + 16y = 0$

E.C. $r^2 + 8r + 16 = 0$

$r_{1,2} = -4$

$y_1 = e^{-4x} \quad y_2 = x e^{-4x}$

$y_g(x) = C_1 e^{-4x} + C_2 x e^{-4x}$

c) $y''' + 3y'' - 4y' - 12y = 0$

E.C.

$r^3 + 3r^2 - 4r - 12 = 0$

$r^2(r+3) - 4(r+3) = 0$

$(r^2 - 4)(r+3) = 0$

$r_1 = 2 \quad r_2 = -2 \quad r_3 = -3$

$y_g(x) = C_1 e^{2x} + C_2 e^{-2x} + C_3 e^{-3x}$

d) $2y'' + 2y' + y = 0$

E.C. $2r^2 + 2r + 1 = 0$

$r_{1,2} = -\frac{1}{2} \pm \frac{1}{2} i$

$y_g = e^{-1/2 x}\left(C_1 \cos(\tfrac{1}{2}x) + C_2 \sin(\tfrac{1}{2}x)\right)$

41 ⓐ $4y'' + 3y' + 2y = 6$ $\qquad y_g = y_c + y_p$

EC $\quad 4r^2 + 3r + 2 = 0$

$$r_{1,2} = -\frac{3}{8} \pm \frac{\sqrt{23}}{8} i$$

$$y_c(x) = e^{-3/8x}\left(C_1 \cos\left(\frac{\sqrt{23}}{8}x\right) + C_2 \sin\left(\frac{\sqrt{23}}{8}x\right)\right)$$

$y_p(x) = A$

$y' = 0$

$y'' = 0$

$4.0 + 3.0 + 2.A = 6$

$\boxed{A = 3}$

$$y_g(x) = C_1 e^{-\frac{3}{8}x} \cos\left(\frac{\sqrt{23}}{8}x\right) + C_2 e^{-3/8x} \sin\left(\frac{\sqrt{23}}{8}x\right) + 3$$

41c) $y''' - 2y'' - 4y' + 8y = \underbrace{6xe^{2x}}$

$y''' - 2y'' - 4y' + 8y = 0$

$m^3 - 2m^2 - 4m + 8 = 0$

$m^2(m-2) - 4(m-2) = 0$

$(m-2)(m^2-4) = 0 \rightarrow (m-2)(m-2)(m+2) = 0$

$m_1 = 2 \quad m_2 = -2$

$y_c = c_1 e^{2x} + c_2 x e^{2x} + c_3 e^{-2x}$

$y_p = (Ax^3 + Bx^2)e^{2x}$

$y_p' = 2e^{2x}(Ax^3 + Bx^2) + e^{2x}(3Ax^2 + 2Bx)$

$= e^{2x}(2Ax^3 + (2B+3A)x^2 + 2Bx)$

$y_p'' = 2e^{2x}(2Ax^3 + (2B+3A)x^2 + 2Bx) +$

$e^{2x}(6Ax^2 + (4B+6A)x + 2B)$

$= e^{2x}(4Ax^3 + (4B+12A)x^2 + (8B+6A)x + 2B)$

$y_p''' = 2e^{2x}(4Ax^3 + (4B+12A)x^2 + (8B+6A)x + 2B)$

$+ e^{2x}(12Ax^2 + (8B+24A)x + (8B+6A))$

$= e^{2x}(8Ax^3 + (36A+8B)x^2 + (24B+36A)x +$

$(12B+6A))$

92.b $y'' + y = \tan x$

$$y'' + y = 0 \to y_c = C_1 \cos x + C_2 \sin x$$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \tan x & \cos x \end{vmatrix} = -\tan x \sin x$$

$$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix} = \cos x \tan x = \sin x$$

$$u_1' = \frac{\Delta_1}{\Delta} = -\tan x \sin x$$

$$u_1 = -\int \tan x \sin x \, dx = -\int \frac{\sin^2 x}{\cos x} dx =$$

$$-\int \frac{(1-\cos^2 x)}{\cos x} dx = -\int (\sec x - \cos x) dx$$

$t(x) = \sec x + \tan x$

$$u_2' = \frac{\Delta_2}{\Delta} = \sin x \to u_2 = -\cos x + C$$

$$y = y_c + y_p = C_1 \cos x + C_2 \sin x + \cos x \, u_1 \overset{?}{+} \sin x (-\cos x)$$

$(-\cos x + C)$

42.2 $y'' + y = \sec x$

$$y'' + y = 0 \to m^2 + 1 = 0 \to m^2 = -1$$

$$m = i \quad m = -i \to 0 + 1i, \; 0 - 1i$$

$$y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$$

$$\boxed{y_c = c_1 \cos x + c_2 \sin x}$$

$$\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = \sec x \end{cases}$$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix} = -\tan x$$

$$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix} = \cos x \sec x$$

$$u_1' = -\tan x \to u_1 = -\int \tan x \, dx = \ln|\cos x|$$

$$u_2' = \cos x \sec x \to u_2 = \int \cos x \sec x \, dx = \int dx = x$$

$$y = c_1 \cos x + c_2 \sin x + \underbrace{\cos x}_{y_1} \underbrace{\ln|\cos x|}_{u_1} + \underbrace{\sin x}_{y_2} \cdot \underbrace{x}_{u_2}$$

$$y''' - 2y'' - 4y' + 8y = 6xe^{2x}$$

$$e^{2x}\Big[8Ax^3 + (36A+8B)x^2 + (24B+36A)x + (12B+6A) - 2\big(4Ax^3 + (4B+12A)x^2 + (8B+6A)x + 2B\big) - 4\big(2Ax^3 + (2B+3A)x^2 + 2Bx\big) + 8\big(Ax^3 + Bx^2\big)\Big] = 6xe^{2x}$$

$$e^{2x}\Big[0x^3 + (\underbrace{0.A + 0B}_{0})x^2 + (0B + 24A)x + (8B + 6A)\Big] = 6xe^{2x}$$

$$24Ax + (8B + 6A) = 6x$$

$$24A = 6 \rightarrow \boxed{A = 1/4}$$

$$8B + 6A = 0$$

$$B = \left(-\tfrac{3}{2}\right)\cdot\tfrac{1}{8} = \boxed{-\tfrac{3}{16} = B}$$

$$Y_p = (Ax^3 + Bx^2)e^{2x}$$

$$Y_p = \left(\tfrac{1}{4}x^3 - \tfrac{3}{16}x^2\right)e^{2x}$$

$$\boxed{Y = Y_c + Y_p}$$