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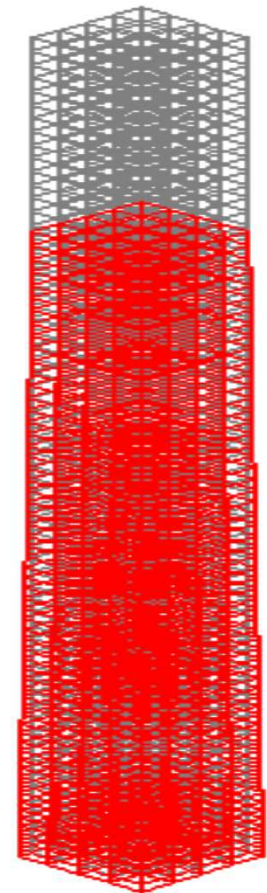
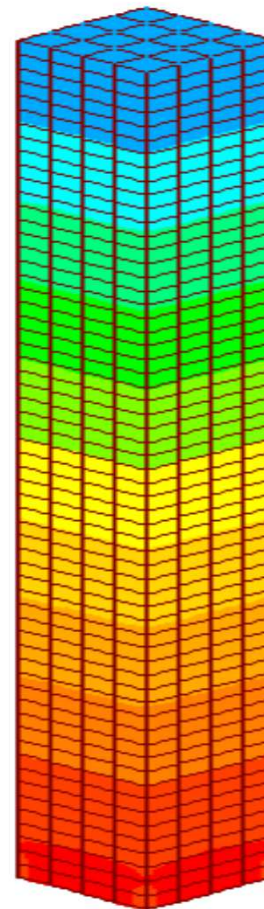
FACULTAD  
DE INGENIERÍA

# TEORÍA ELASTICIDAD. MEF

## A2 CORRIMIENTOS

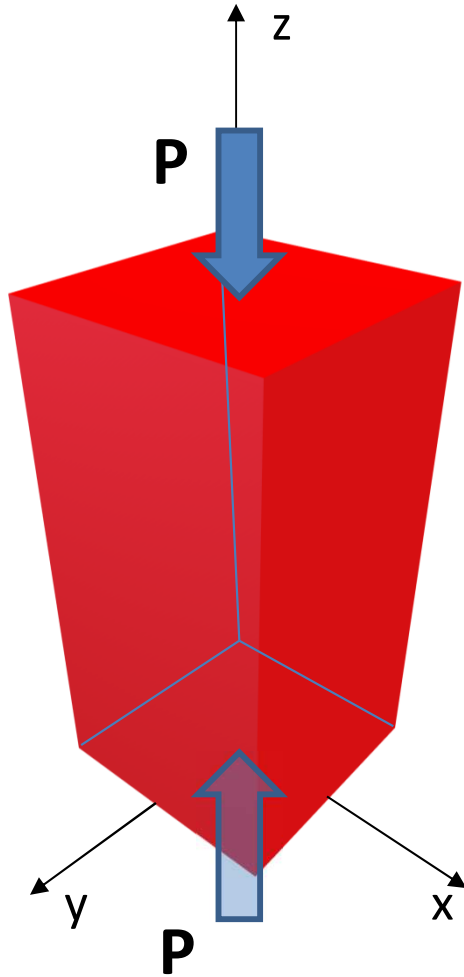
Mg. Ing. DANIEL E. LÓPEZ

## ANALISIS ESTRUCTURAL II



# CALCULO DE CORRIMIENTOS. A2.1

## Hipótesis de Tensiones Prisma en Compresión mas Peso Propio



Campo tensiones

$$\sigma_x = 0$$

$$\sigma_y = 0$$

$$\sigma_z = \frac{P}{A} + \gamma(h - z)$$

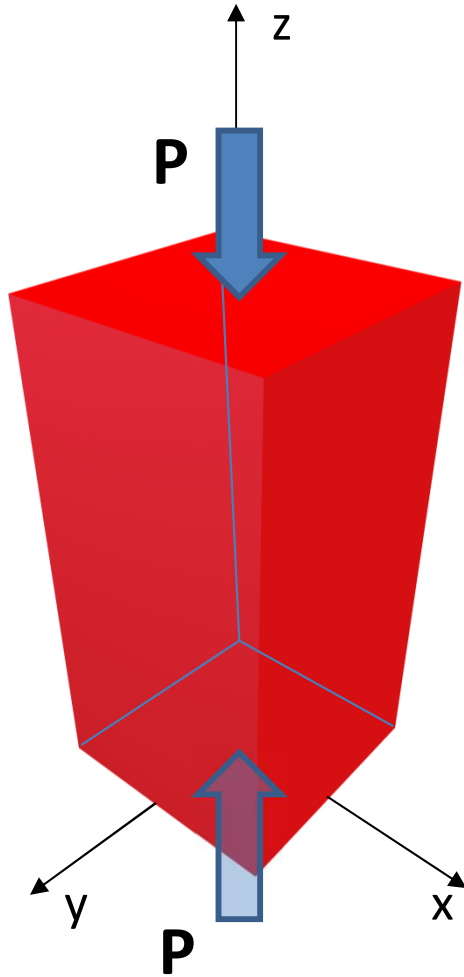
$$\tau_{xy} = 0$$

$$\tau_{yz} = 0$$

$$\tau_{zx} = 0$$

# CALCULO DE CORRIMIENTOS. A2.1

## Tensiones y Deformaciones



### Campo tensiones

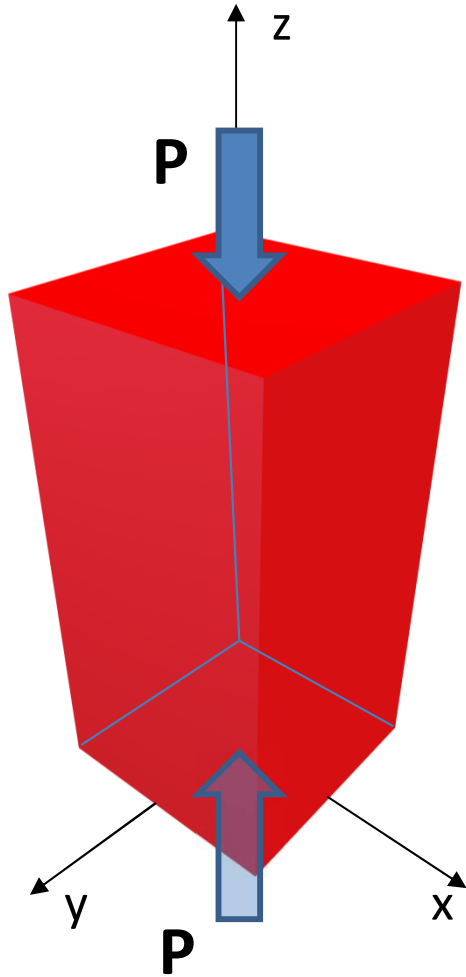
$$\begin{aligned}\sigma_x &= 0 \\ \sigma_y &= 0 \\ \sigma_z &= \frac{P}{A} + \gamma(h - z) \\ \tau_{xy} &= 0 \\ \tau_{yz} &= 0 \\ \tau_{zx} &= 0\end{aligned}$$

### Campo deformaciones

$$\begin{aligned}\varepsilon_x &= \frac{\partial u}{\partial x} = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) \\ \varepsilon_y &= \frac{\partial v}{\partial y} = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) \\ \varepsilon_z &= \frac{\partial w}{\partial z} = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\tau_{xy}}{G} \\ \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\tau_{yz}}{G} = 0 \\ \gamma_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \frac{\tau_{zx}}{G} = 0\end{aligned}$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\varepsilon_z = \frac{\partial w}{\partial z} = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$\sigma_z = \frac{P}{A} + \gamma(h - z), \text{ llamando } P_0 = \frac{P}{A}$$

$$\frac{\partial w}{\partial z} = \frac{P_0}{E} + \frac{\gamma}{E}h - \frac{\gamma}{E}z$$

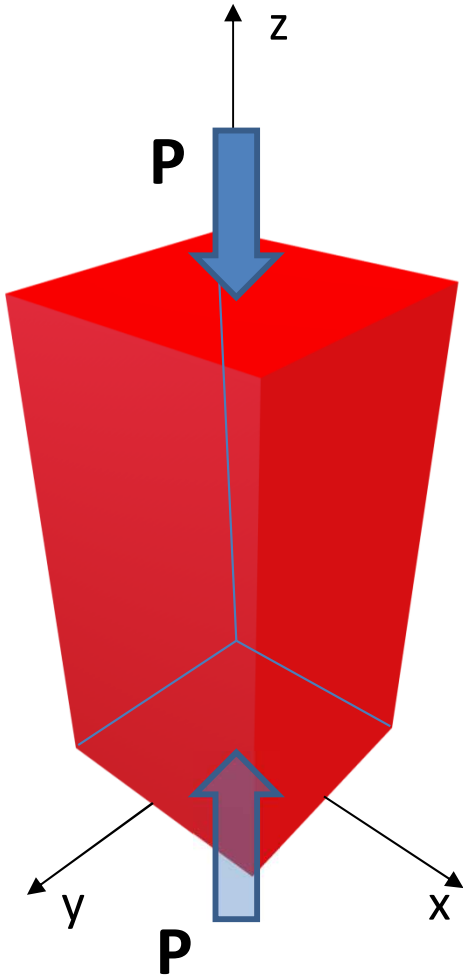
$$\partial w = \left( \frac{P_0}{E} + \frac{\gamma}{E}h - \frac{\gamma}{E}z \right) \partial z$$

$$w = \int \left( \frac{P_0}{E} + \frac{\gamma}{E}h - \frac{\gamma}{E}z \right) dz$$

$$w = \frac{P_0}{E}z + \frac{\gamma}{E}hz - \frac{\gamma}{2E}z^2 + F_1(x, y)$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \frac{\tau_{zx}}{G} = 0$$

$$\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0 \quad \therefore \quad \frac{\partial u}{\partial z} = -\frac{\partial w}{\partial x}$$

$$\partial u = -\frac{\partial w}{\partial x} dz$$

$$u = \int -\frac{\partial w}{\partial x} dz$$

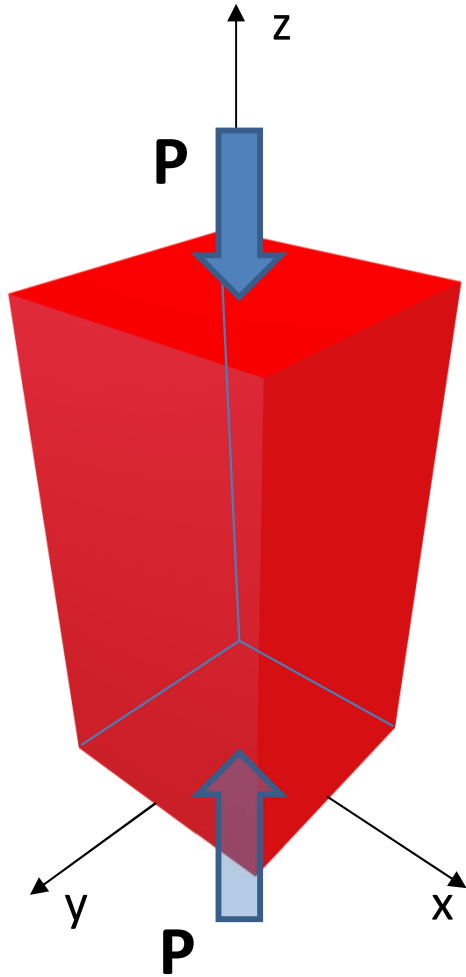
$$w = \frac{P_0}{E} z + \frac{\gamma}{E} h z - \frac{\gamma}{2 E} z^2 + F_1(x, y)$$

$$u = \int -\frac{\partial}{\partial x} F_1(x, y) dz$$

$$u = -\frac{\partial F_1(x, y)}{\partial x} z + F_2(x, y)$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\tau_{yz}}{G} = 0$$

$$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \quad \therefore \frac{\partial v}{\partial z} = -\frac{\partial w}{\partial y}$$

$$dv = -\frac{\partial w}{\partial y} dz$$

$$v = \int -\frac{\partial w}{\partial y} dz$$

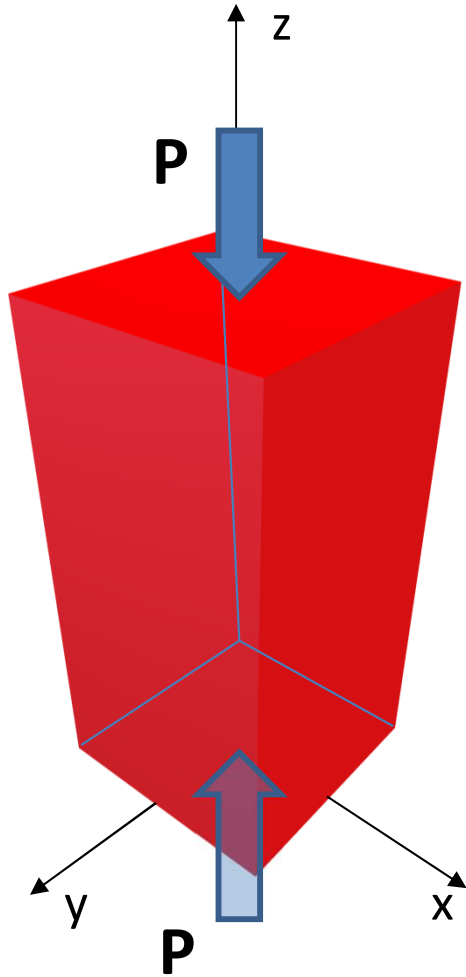
$$w = \frac{P_0}{E} z + \frac{\gamma}{E} h z - \frac{\gamma}{2 E} z^2 + F_1(x, y)$$

$$v = \int -\frac{\partial}{\partial y} F_1(x, y) dz$$

$$v = -\frac{\partial F_1(x, y)}{\partial y} z + F_3(x, y)$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\frac{\partial u}{\partial x} = -\frac{\nu}{E}(P_0 + \gamma(h - z))$$

$$\frac{\partial u}{\partial x} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h + \frac{\nu}{E}\gamma z$$

$$u = -\frac{\partial F_1(x, y)}{\partial x} z + F_2(x, y)$$

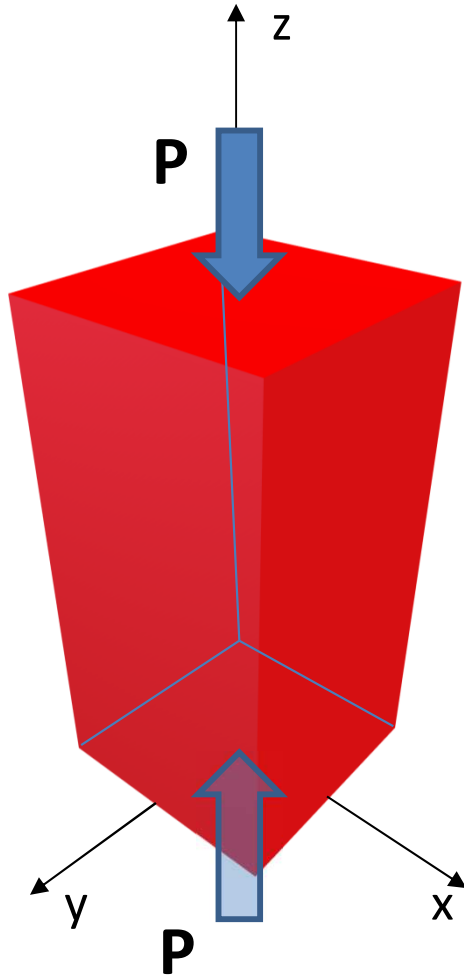
$$\frac{\partial u}{\partial x} = -\frac{\partial^2 F_1(x, y)}{\partial x^2} z + \frac{\partial F_2(x, y)}{\partial x}$$

$$\frac{\partial^2 F_1(x, y)}{\partial x^2} = -\frac{\nu}{E}\gamma$$

$$\frac{\partial F_2(x, y)}{\partial x} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



De Hip. Tensiones y Ley de Hooke

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$\frac{\partial v}{\partial y} = -\frac{\nu}{E}(P_0 + \gamma(h - z))$$

$$\frac{\partial v}{\partial y} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h + \frac{\nu}{E}\gamma z$$

$$v = -\frac{\partial F_1(x, y)}{\partial y} z + F_3(x, y)$$

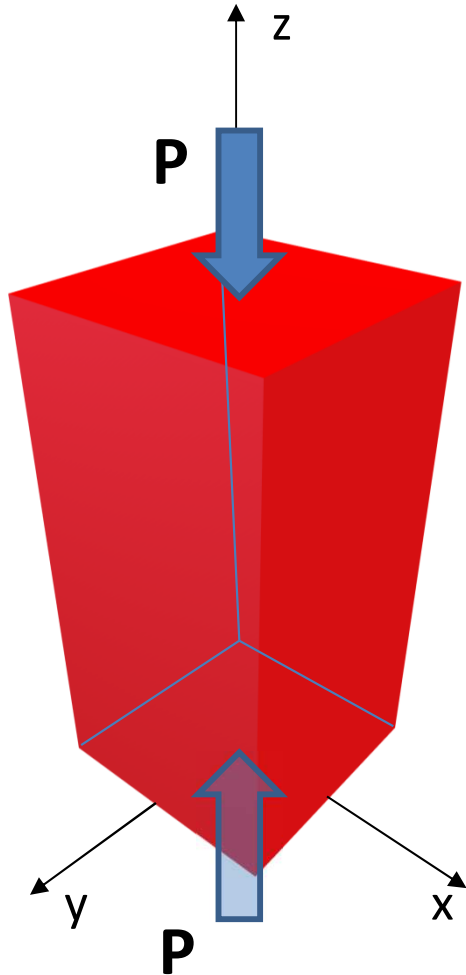
$$\frac{\partial v}{\partial y} = -\frac{\partial^2 F_1(x, y)}{\partial y^2} z + \frac{\partial F_3(x, y)}{\partial y}$$

$$\frac{\partial^2 F_1(x, y)}{\partial y^2} = -\frac{\nu}{E}\gamma$$

$$\frac{\partial F_3(x, y)}{\partial y} = -\frac{\nu}{E}P_0 - \frac{\nu}{E}\gamma h$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



Condición

De Relaciones Anteriores

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\tau_{xy}}{G} = 0$$

$$u = -\frac{\partial F_1(x, y)}{\partial x} z + F_2(x, y)$$

$$v = -\frac{\partial F_1(x, y)}{\partial y} z + F_3(x, y)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial^2 F_1(x, y)}{\partial x \partial y} z + \frac{\partial F_2(x, y)}{\partial y}$$

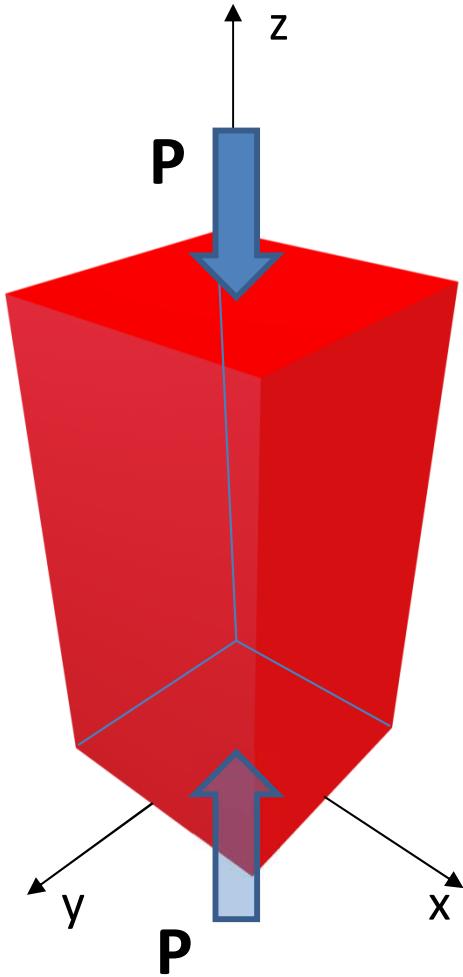
$$\frac{\partial v}{\partial x} = -\frac{\partial^2 F_1(x, y)}{\partial x \partial y} z + \frac{\partial F_3(x, y)}{\partial x}$$

$$-2 \frac{\partial^2 F_1(x, y)}{\partial x \partial y} z + \frac{\partial F_2(x, y)}{\partial y} + \frac{\partial F_3(x, y)}{\partial x} = 0$$

$$\frac{\partial F_2(x, y)}{\partial y} = -\frac{\partial F_3(x, y)}{\partial x}$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



### De Relaciones Anteriores

$$\frac{\partial F_2(x, y)}{\partial x} = -\frac{\nu}{E} P_0 - \frac{\nu}{E} \gamma h$$

$$\frac{\partial F_3(x, y)}{\partial y} = -\frac{\nu}{E} P_0 - \frac{\nu}{E} \gamma h$$

$$\frac{\partial F_2(x, y)}{\partial y} = -\frac{\partial F_3(x, y)}{\partial x}$$

### Integrando

$$F_2(x, y) = -\frac{\nu}{E} P_0 x - \frac{\nu}{E} \gamma h x + F_4(y)$$

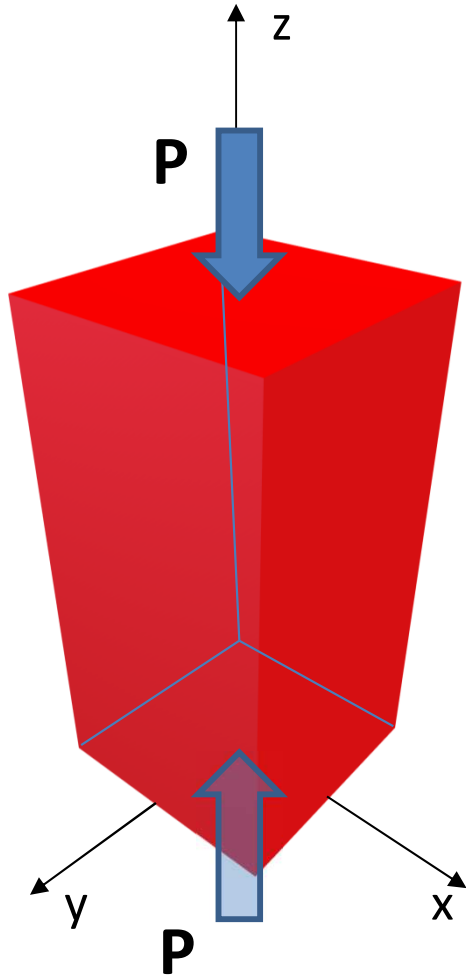
$$F_3(x, y) = -\frac{\nu}{E} P_0 y - \frac{\nu}{E} \gamma h y + F_5(x)$$

$$F_4(y) = Ay + B$$

$$F_5(x) = -Ax + C$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



Reemplazando

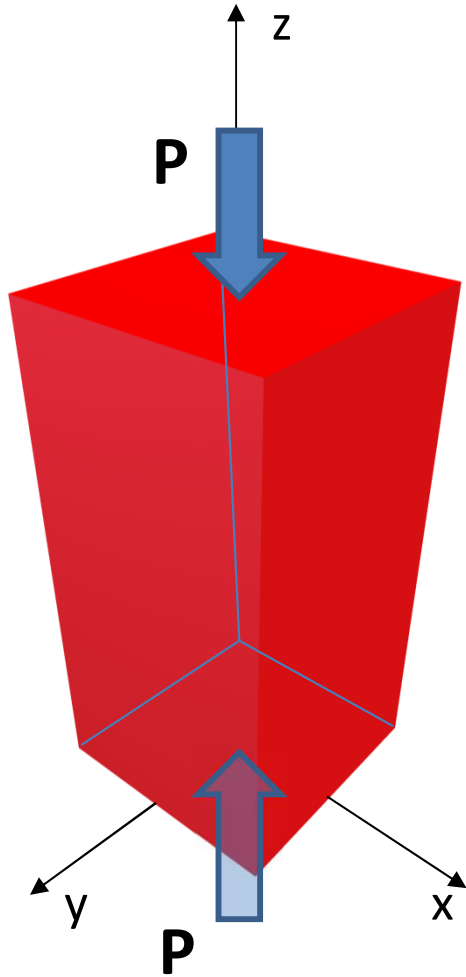
$$F_2(x, y) = -\frac{\nu}{E} P_0 x - \frac{\nu}{E} \gamma h x + Ay + B$$
$$F_3(x, y) = \frac{\nu}{E} P_0 y - \frac{\nu}{E} \gamma h y - Ax + C$$

Integrando  $F_1$

$$\frac{\partial^2 F_1(x, y)}{\partial x^2} = -\frac{\nu}{E} \gamma \quad , \quad \frac{\partial^2 F_1(x, y)}{\partial y^2} = -\frac{\nu}{E} \gamma$$
$$\frac{\partial F_1(x, y)}{\partial x} = -\frac{\nu}{E} \gamma x + F_6(y) \quad , \quad F_1^* = -\frac{\nu}{E} \gamma \frac{x^2}{2} + F_6(y) x + F_7(y)$$
$$\frac{\partial F_1(x, y)}{\partial y} = -\frac{\nu}{E} \gamma y + F_8(x) \quad , \quad F_1^{**} = -\frac{\nu}{E} \gamma \frac{y^2}{2} + F_8(x) y + F_9(x)$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



Sumando  $F_1^* + F_1^{**}$

$$F_1^* + F_1^{**} = -\frac{\nu}{E} \gamma \frac{x^2}{2} + F_6(y) x + F_7(y) - \frac{\nu}{E} \gamma \frac{y^2}{2} + F_8(x) y + F_9(x)$$

$$F_1^* + F_1^{**} = -\frac{\nu}{E} \gamma \left( \frac{x^2}{2} + \frac{y^2}{2} \right) + F_6(y) x + F_7(y) + F_8(x) y + F_9(x)$$

La condición

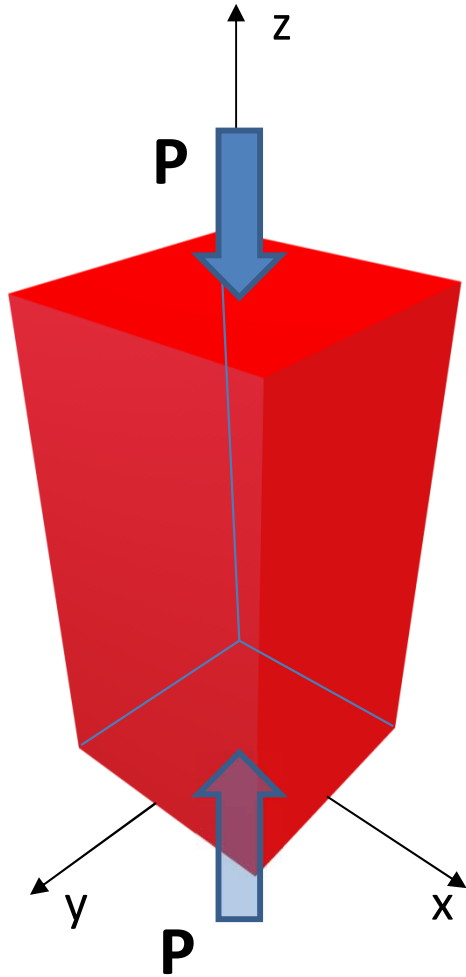
$$\frac{\partial^2 F_1(x, y)}{\partial x \partial y} = 0$$

$$\frac{\partial^2 F_1^*}{\partial x \partial y} = \frac{\partial F_6(y)}{\partial y} \Rightarrow F_6(y) = \text{ctte} = H$$

$$\frac{\partial^2 F_1^{**}}{\partial x \partial y} = \frac{\partial F_8(x)}{\partial x} \Rightarrow F_8(x) = \text{ctte} = J$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



Reemplazando en  $F_1$

$$F_1 = -\frac{\nu}{E}\gamma\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + Hx + Jy + F_7(y) + F_9(x)$$

Derivando

$$\frac{\partial F_1(x, y)}{\partial x} = \frac{\partial F_1^*}{\partial x}$$

$$-\frac{\nu}{E}\gamma x + H + \frac{\partial F_9(x)}{\partial x} = -\frac{\nu}{E}\gamma x + H \Rightarrow \frac{\partial F_9(x)}{\partial x} = 0 \Rightarrow F_9(x) = ctte = K$$

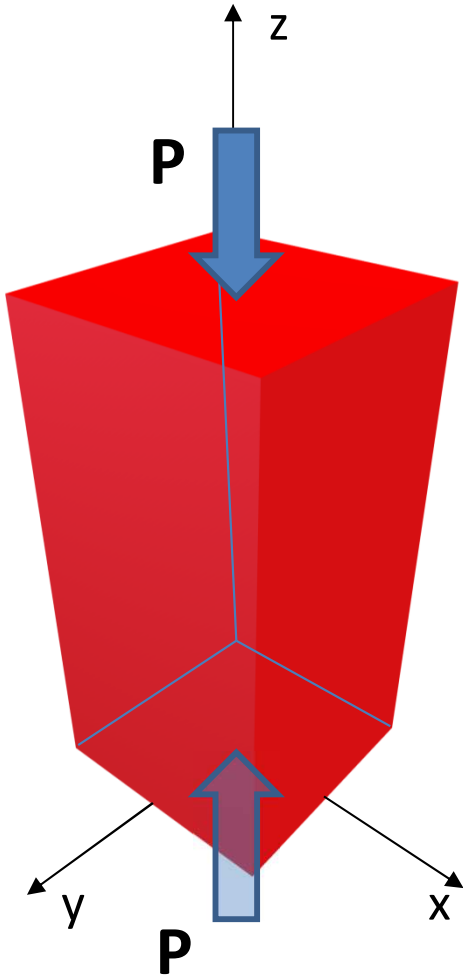
$$\frac{\partial F_1(x, y)}{\partial y} = \frac{\partial F_1^{**}}{\partial y}$$

$$-\frac{\nu}{E}\gamma y + J + \frac{\partial F_7(y)}{\partial y} = -\frac{\nu}{E}\gamma y + J \Rightarrow \frac{\partial F_7(y)}{\partial y} = 0 \Rightarrow F_7(y) = ctte = L$$

$$M = K + L$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



Reemplazando en  $F_1$

$$F_1 = -\frac{\nu}{E} \gamma \left( \frac{x^2}{2} + \frac{y^2}{2} \right) + Hx + Jy + M$$

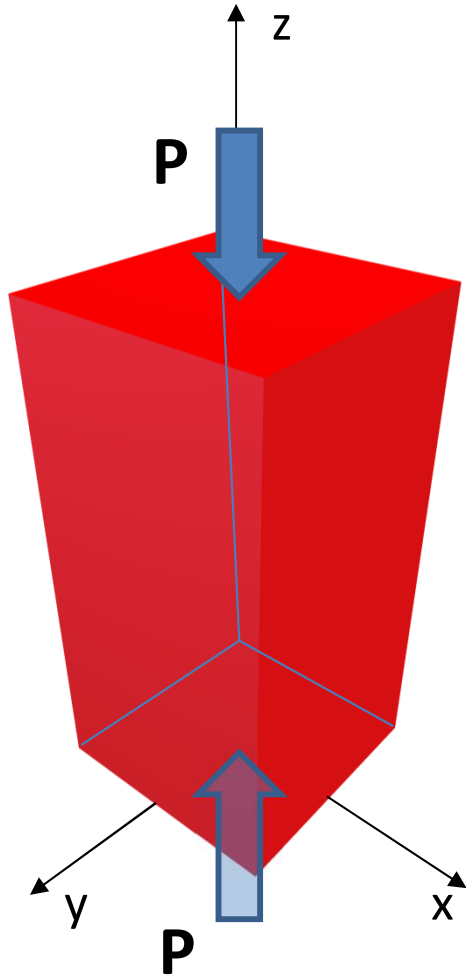
Reemplazando en la expresiones de  $w, u, v$

$$w = \frac{P_0}{E} z + \frac{\gamma}{E} h z - \frac{\gamma}{2E} z^2 + F_1(x, y)$$

$$w = \frac{1}{E} \left( P_0 z + \gamma h z - \gamma \frac{z^2}{2} \right) - \frac{\nu}{2E} \gamma (x^2 + y^2) + Hx + Jy + M$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



Reemplazando en  $F_1$

$$F_1 = -\frac{\nu}{E}\gamma\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + Hx + Jy + M$$

Reemplazando en la expresiones de  $w, u, v$

$$u = -\frac{\partial F_1(x, y)}{\partial x} z + F_2(x, y)$$

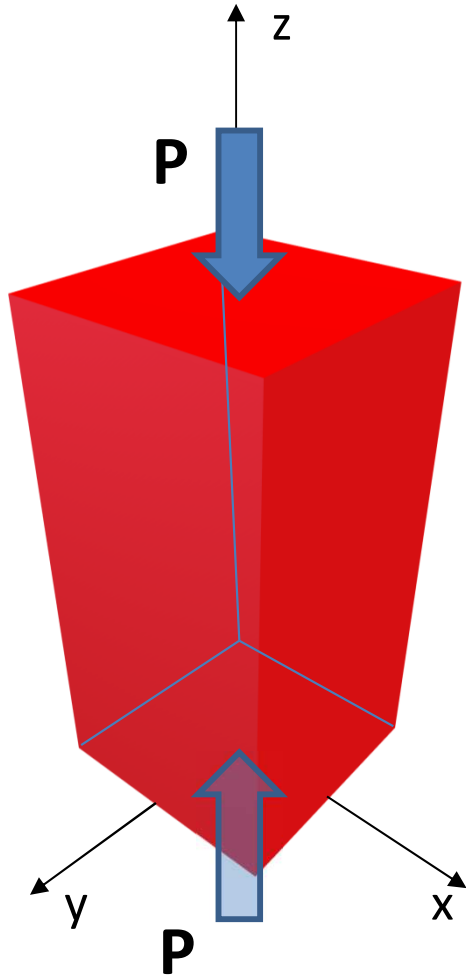
$$u = \frac{\nu}{E}\gamma x z - H z - \frac{\nu}{E}P_0 x - \frac{\nu}{E}\gamma h x + A y + B$$

$$v = -\frac{\partial F_1(x, y)}{\partial y} z + F_3(x, y)$$

$$v = \frac{\nu}{E}\gamma y z - J z - \frac{\nu}{E}P_0 y - \frac{\nu}{E}\gamma h y - A x + C$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



### Expresiones Generales de $w, u, v$

$$w = \frac{1}{E} \left( P_0 z + \gamma h z - \gamma \frac{z^2}{2} \right) - \frac{\nu}{2E} \gamma (x^2 + y^2) + Hx + Jy + M$$

$$u = \frac{\nu}{E} \gamma x z - H z - \frac{\nu}{E} P_0 x - \frac{\nu}{E} \gamma h x + A y + B$$

$$v = \frac{\nu}{E} \gamma y z - J z - \frac{\nu}{E} P_0 y - \frac{\nu}{E} \gamma h y - A x + C$$

$$\text{Cond. Contorno } x = y = z = 0 \Rightarrow u = v = w = 0$$

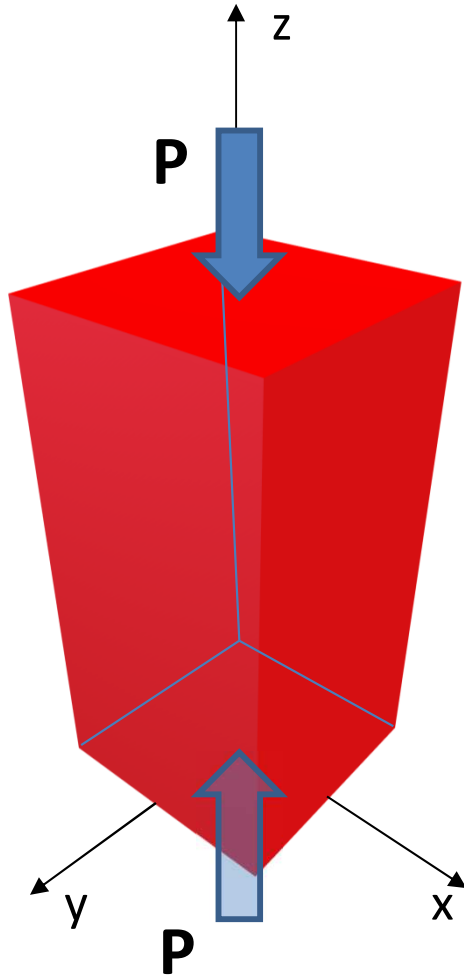
$$w = 0 \Rightarrow M = 0$$

$$u = 0 \Rightarrow B = 0$$

$$v = 0 \Rightarrow C = 0$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



### Expresiones Generales de $w, u, v$

$$w = \frac{1}{E} \left( P_0 z + \gamma h z - \gamma \frac{z^2}{2} \right) - \frac{\nu}{2E} \gamma (x^2 + y^2) + Hx + Jy + M$$

$$u = \frac{\nu}{E} \gamma x z - H z - \frac{\nu}{E} P_0 x - \frac{\nu}{E} \gamma h x + A y + B$$

$$v = \frac{\nu}{E} \gamma y z - J z - \frac{\nu}{E} P_0 y - \frac{\nu}{E} \gamma h y - A x + C$$

$$\text{C. Cont. } x = y = z = 0 \Rightarrow g_{xy} = g_{yz} = g_{zx} = 0$$

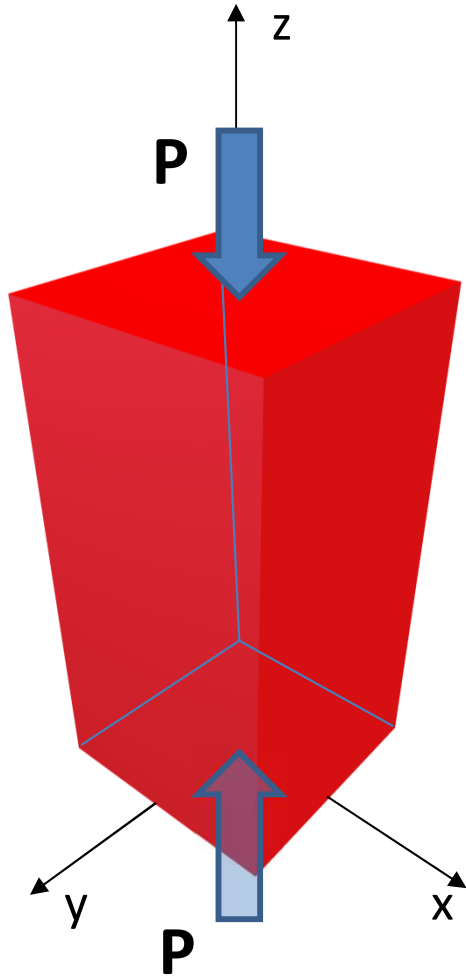
$$g_{xy} = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0 \Rightarrow \frac{1}{2} (-A - A) = 0 \Rightarrow A = 0$$

$$g_{yz} = \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) = 0 \Rightarrow \frac{1}{2} (+J + J) = 0 \Rightarrow J = 0$$

$$g_{zx} = \frac{1}{2} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = 0 \Rightarrow \frac{1}{2} (-H - H) = 0 \Rightarrow H = 0$$

# CALCULO DE CORRIMIENTOS. A2.1

## Desarrollo



Expresiones Particulares de  $w, u, v$

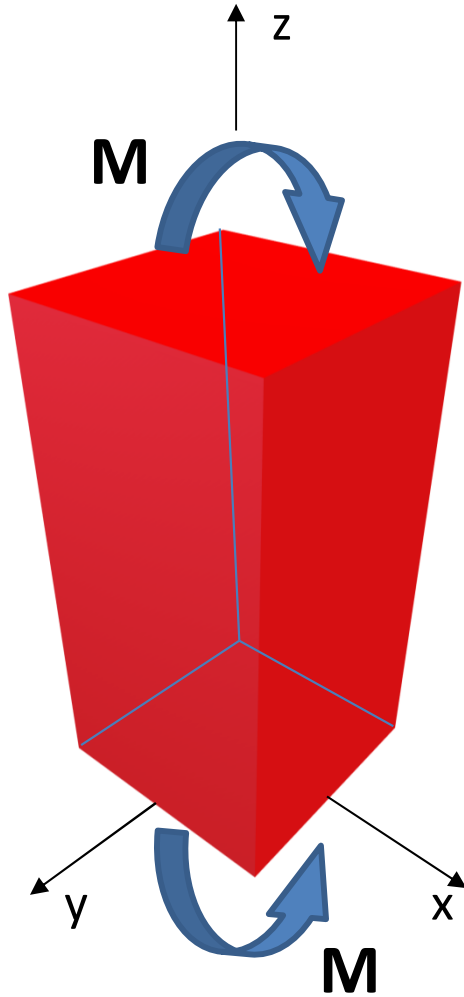
$$u = -\frac{\nu}{E} P_0 x + \frac{\nu}{E} \gamma x z - \frac{\nu}{E} \gamma h x$$

$$v = -\frac{\nu}{E} P_0 y + \frac{\nu}{E} \gamma y z - \frac{\nu}{E} \gamma h y$$

$$w = \frac{1}{E} P_0 z + \frac{1}{E} \gamma h z - \frac{1}{E} \gamma \frac{z^2}{2} - \frac{\nu}{2E} \gamma (x^2 + y^2)$$

# CALCULO DE CORRIMIENTOS. A2.1

## Hipótesis de Tensiones Prisma en Flexión Simple



Campo tensiones

$$\sigma_x = 0$$

$$\sigma_y = 0$$

$$\sigma_z = -\frac{M}{J} x$$

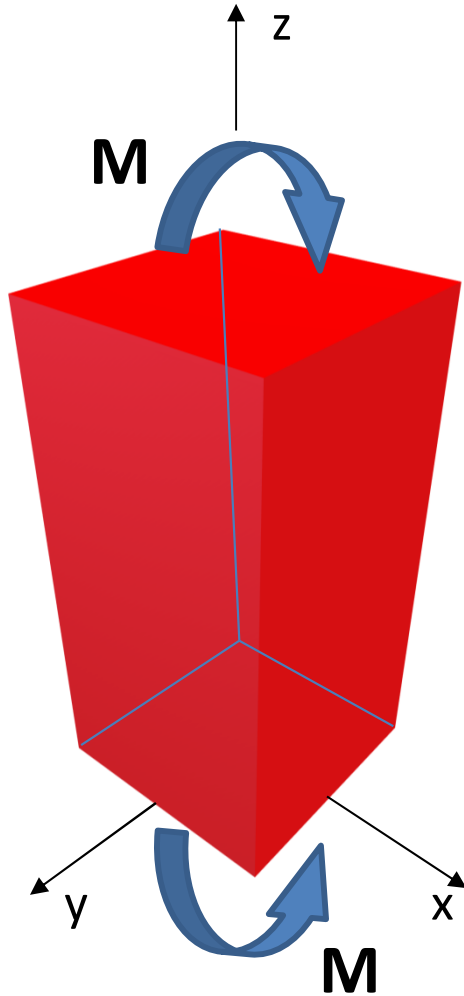
$$\tau_{xy} = 0$$

$$\tau_{yz} = 0$$

$$\tau_{zx} = 0$$

# CALCULO DE CORRIMIENTOS. A2.2

## Tensiones y Deformaciones



### C. tensiones

$$\sigma_x = 0$$

$$\sigma_y = 0$$

$$\sigma_z = -\frac{M}{J} x$$

$$\tau_{xy} = 0$$

$$\tau_{yz} = 0$$

$$\tau_{zx} = 0$$

### Campo deformaciones

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E} (\sigma_y + \sigma_z) = \frac{\nu}{EJ} Mx$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E} (\sigma_x + \sigma_z) = \frac{\nu}{EJ} Mx$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E} (\sigma_x + \sigma_y) = -\frac{1}{EJ} Mx$$

$$\frac{\partial u}{\partial x} = \frac{\nu}{EJ} Mx \Rightarrow u = \frac{\nu}{2EJ} Mx^2 + F_1(y, z)$$

$$\frac{\partial v}{\partial y} = \frac{\nu}{EJ} Mx \Rightarrow v = \frac{\nu}{2EJ} Mxy + F_2(x, z)$$

$$\frac{\partial w}{\partial z} = -\frac{1}{EJ} Mx \Rightarrow w = -\frac{1}{EJ} Mxz + F_3(x, y)$$



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## A2 CORRIMIENTOS

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## ANALISIS ESTRUCTURAL II

# Fin