TEORÍA ELASTICIDAD

ACTIVIDAD 5 TORSIÓN

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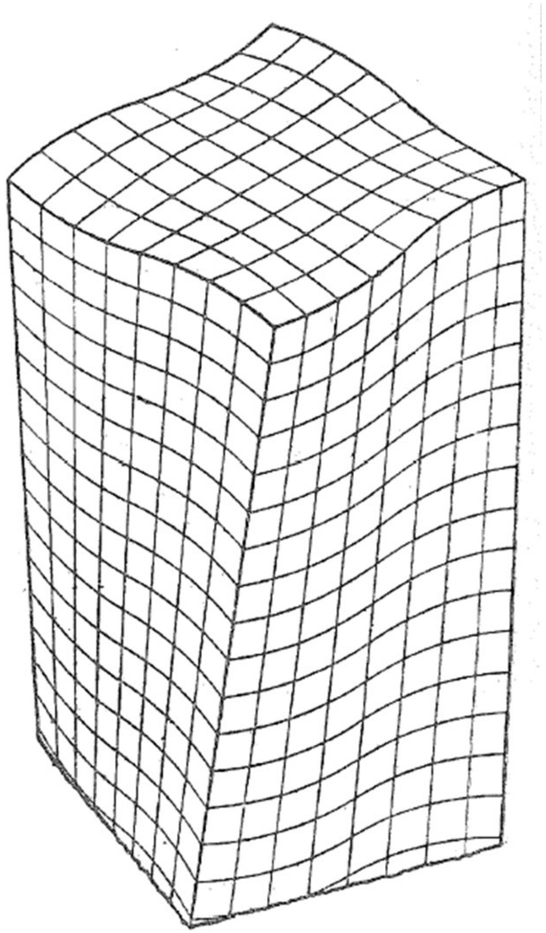
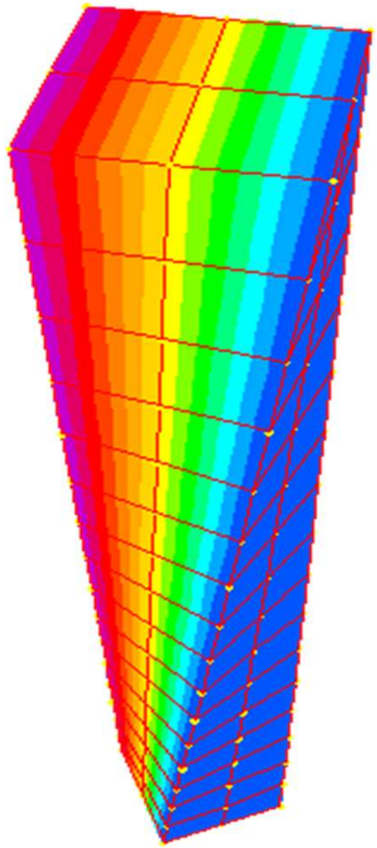
UNCUYO
UNIVERSIDAD NACIONAL DE CUYO
FACULTAD DE INGENIERÍA

ANALISIS ESTRUCTURAL II





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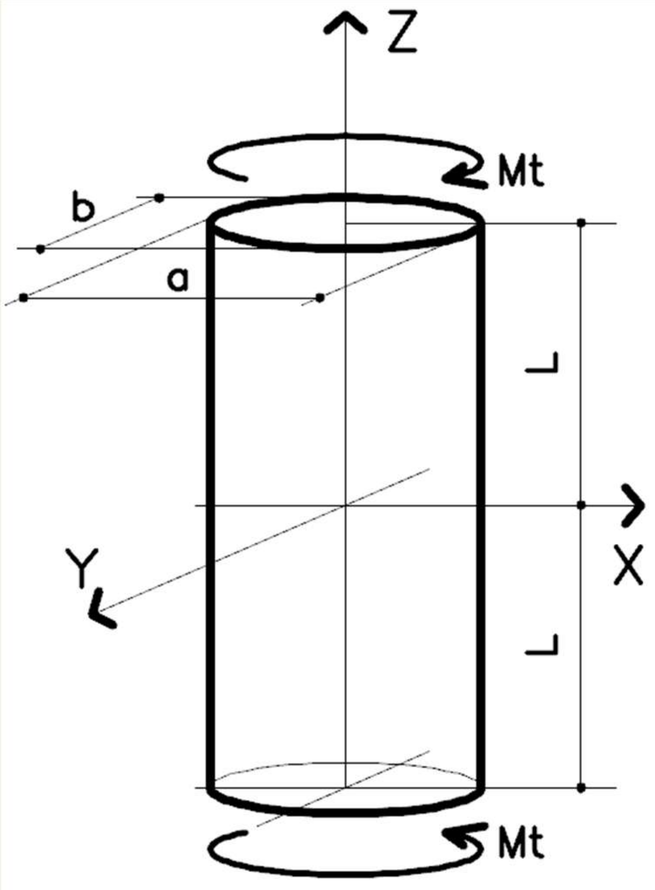
Barra Sección Elíptica



De la observación del comportamiento

a. Las secciones giran alrededor del eje de simetría un ángulo variable, 0 en el plano medio y ω en las secciones extremas.

b. Las secciones no cambian de forma, ni de tamaño.

c. La generatrices conservan su longitud pero se trasladan verticalmente.

S/a. $\theta = {}^{\omega}/_{L} \qquad u = -\theta\, z\, y \qquad v = \theta\, z\, x$

S/b. $\varepsilon_x = \varepsilon_y = 0$

S/c. $\varepsilon_z = 0$

$w \neq 0 = \theta\, f(x, y)$

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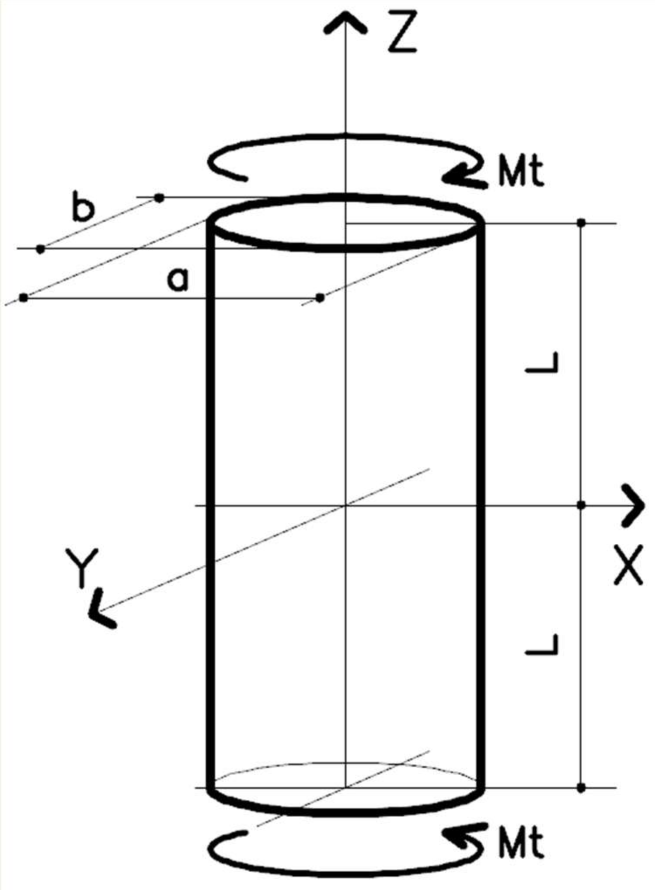

Solución de Saint-Venant

Si existe un función $\emptyset(x, y)$ tal que:

$$-\frac{\partial \emptyset}{\partial x} = \tau_{yz} \qquad \frac{\partial \emptyset}{\partial y} = \tau_{xz}$$

será solución del problema si satisface EEI, EEC y EC

EEI (3°Ec)

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0$$

EEC (3°Ec)

$$\tau_{xz} \cos\alpha + \tau_{yz} \cos\beta = 0$$

$$\frac{\partial \emptyset}{\partial y} \cos\alpha - \frac{\partial \emptyset}{\partial x} \cos\beta = 0$$

$$\frac{\partial \emptyset}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial \emptyset}{\partial x}\frac{\partial x}{\partial s} = 0$$

$$\frac{\partial \emptyset}{\partial s} = 0 \qquad \emptyset = ctte$$

en el Contorno

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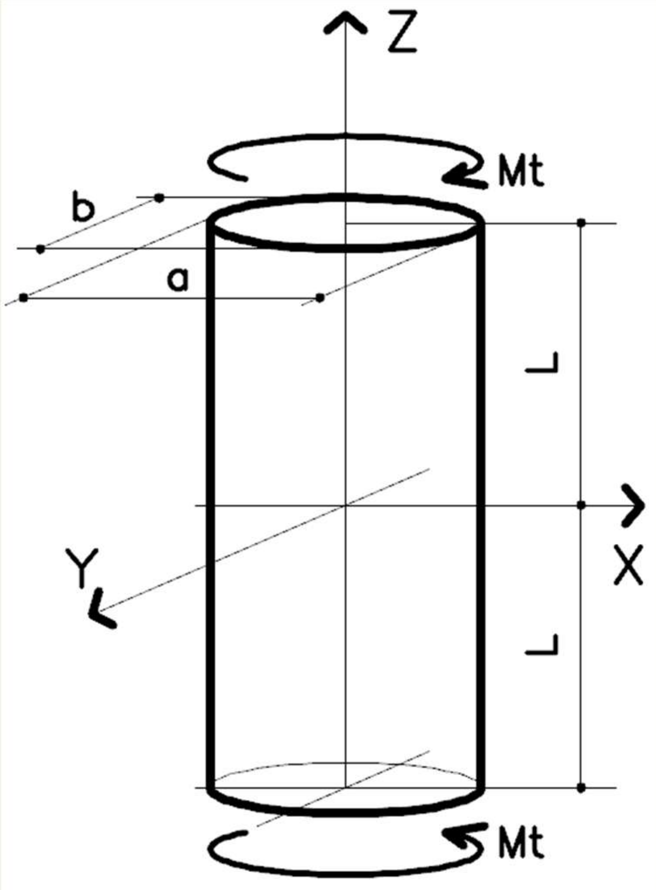


Solución de Saint-Venant

EC (2 Ult)

$$\frac{\partial}{\partial y}\nabla^2 \emptyset(x,y) = 0$$

$$-\frac{\partial}{\partial x}\nabla^2 \emptyset(x,y) = 0$$

$$\nabla^2 \emptyset = ctte$$

Resumen

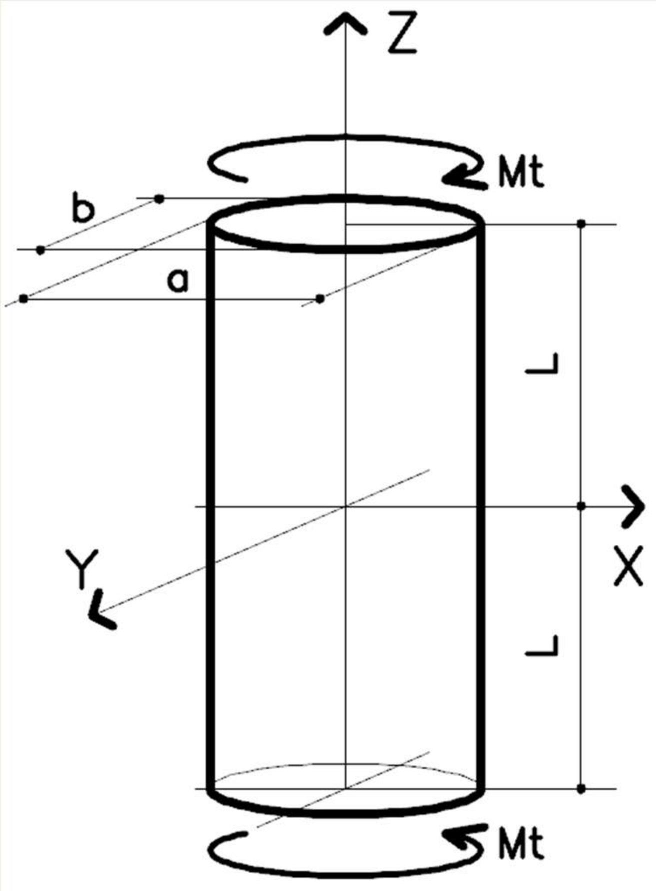
$$-\frac{\partial \emptyset}{\partial x} = \tau_{yz} \qquad \frac{\partial \emptyset}{\partial y} = \tau_{xz}$$

$$\frac{\partial \tau_{xz}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0$$

$\emptyset = ctte$ en el Contorno

$$\nabla^2 \emptyset = ctte$$

Barra Sección Elíptica



Solución de Saint-Venant

$$\nabla^2 \emptyset = ctte$$

$$\nabla^2 \emptyset = \frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = -\frac{\partial \tau_{yz}}{\partial x} + \frac{\partial \tau_{xz}}{\partial y} = G\left[-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y}\right]$$

reemplazando

$$u = -\theta\, z\, y \qquad v = \theta\, z\, x \qquad w = \theta\, f(x, y)$$

resulta

$$\nabla^2 \emptyset = -2G\frac{\omega}{L} = -2G\theta$$

Función ∅

$$\emptyset = k\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right) \qquad \text{Satisface} \qquad \frac{\partial \emptyset}{\partial S} = 0$$

reemplazando

$$2k\left(\frac{1}{a^2} + \frac{1}{b^2}\right) = -2G\theta \qquad \rightarrow \qquad k = -G\theta\left(\frac{a^2 b^2}{a^2 + b^2}\right)$$

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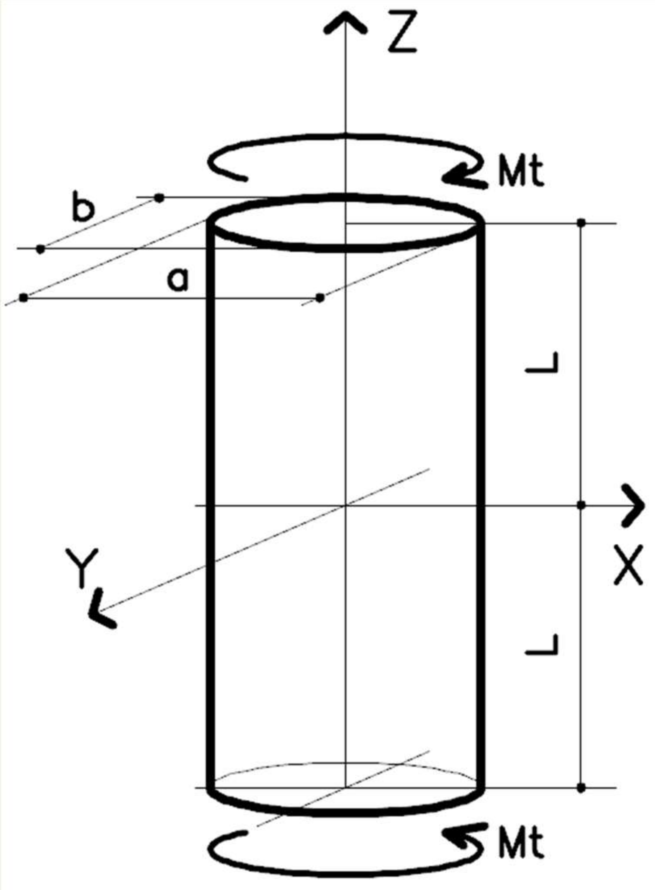


Solución de Saint-Venant

Función Ø

$$\emptyset = -G\theta\left(\frac{a^2b^2}{a^2+b^2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)$$

Equilibrio Mt

$$Mt - \iint_{\Omega} \tau_{xz}\, y\, dxdy + \iint_{\Omega} \tau_{yz}\, x\, dxdy$$

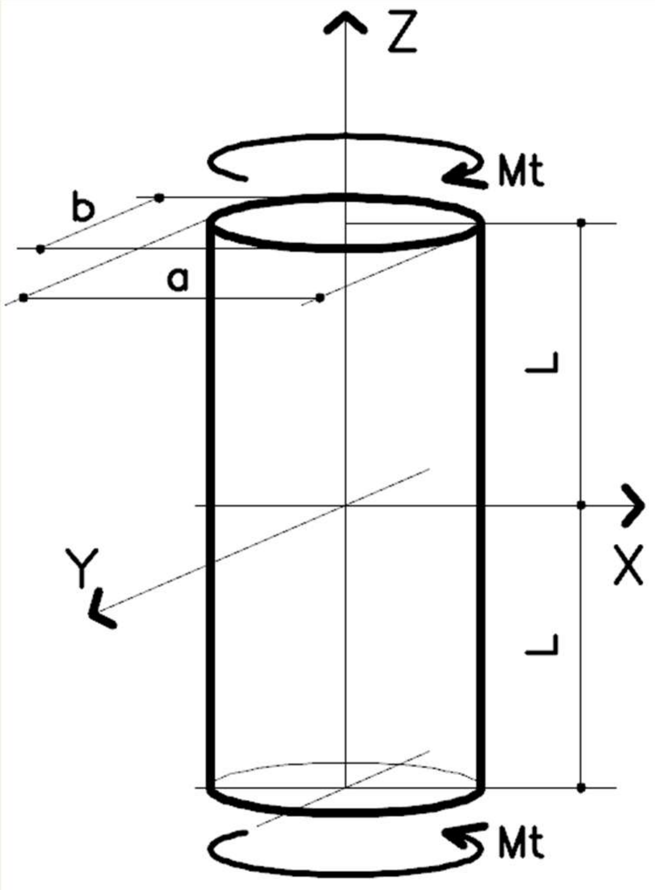
reemplazando $\tau = f(\emptyset)$

$$Mt = 2\iint_{\Omega} \emptyset\, dxdy$$

$$Mt = 2\iint_{\Omega} -G\theta\left(\frac{a^2b^2}{a^2+b^2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)dxdy$$

$$Mt = \pi G\theta\, f(a,b) \quad \rightarrow \quad \theta = \ \dots$$

Barra Sección Elíptica



Solución de Saint-Venant

Cálculo de w

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$$\frac{\partial w}{\partial x} = \frac{\tau_{xz}}{G} - \frac{\partial u}{\partial z}$$

$$\frac{\partial w}{\partial x} = \frac{1}{G}\frac{\partial \emptyset}{\partial y} + \frac{\omega}{L} y$$

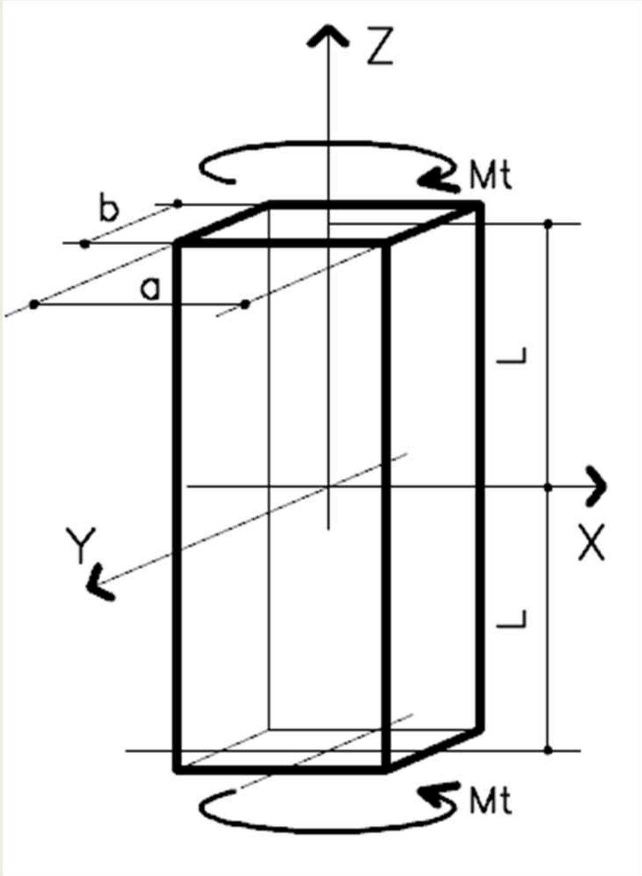
$$w(x) = \int \left(\frac{1}{G}\frac{\partial \emptyset}{\partial y} + \frac{\omega}{L} y\right) dx \quad \rightarrow \quad w(x) = (\dots)xy + F1(y)$$

$$w(y) = \int \left(\frac{1}{G}\frac{\partial \emptyset}{\partial x} - \frac{\omega}{L} x\right) dx \quad \rightarrow \quad w(y) = (\dots)xy + F2(x)$$

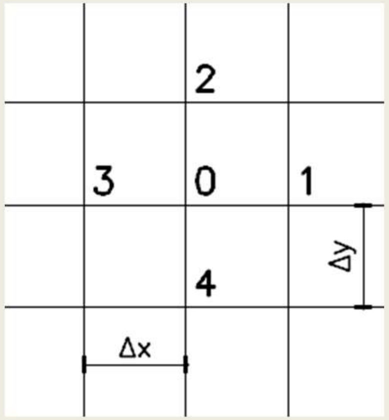
$$w(x) = w(y) \quad \rightarrow \quad F1(y) = ? \qquad F2(x) = ?$$

TORSIÓN

Barra Sección Rectangular

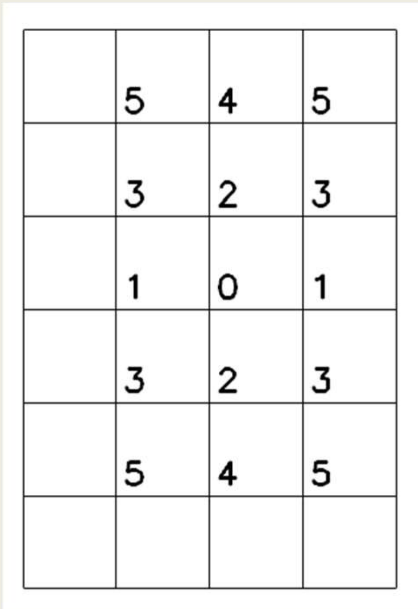


Diferencias Finitas



Cálculo de Ø

$$(\nabla^2 \emptyset)_0 = \frac{\emptyset_1 + \emptyset_2 + \emptyset_3 + \emptyset_4 - 4\emptyset_0}{\Delta^2} = -2G\theta$$



$$\emptyset_1 + \emptyset_1 + \emptyset_2 + \emptyset_2 - 4\emptyset_0 = -2G\theta\Delta^2$$

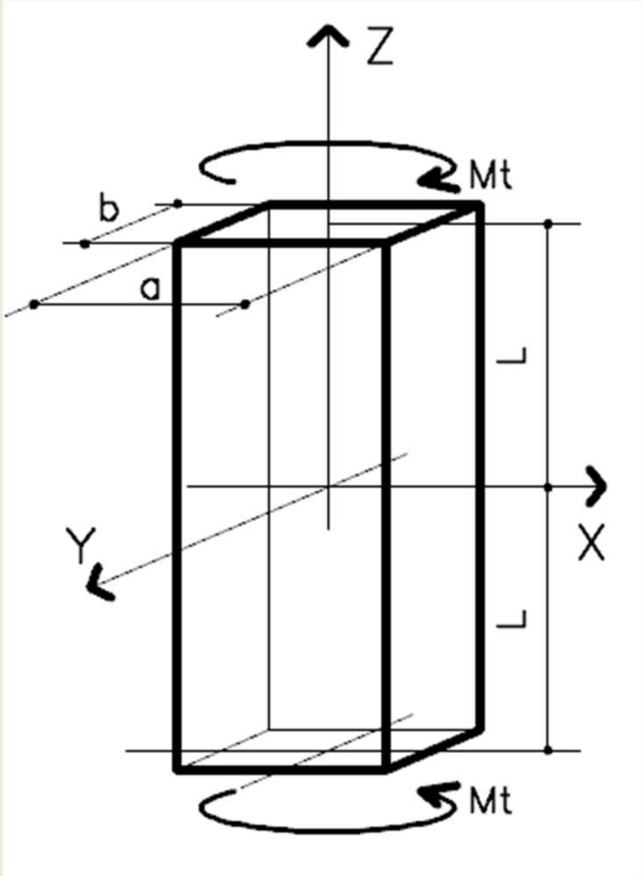
$$2\,\emptyset_1 + 2\,\emptyset_2 - 4\emptyset_0 = -2G\theta\Delta^2$$

en n puntos

$$[A][\emptyset] = -2G\theta\Delta^2[1]$$

$$[\emptyset] = -2G\theta\Delta^2[A]^{-1}[1]$$

Barra Sección Rectangular



Diferencias Finitas

Cálculo de θ

$$Mt = 2 \iint_{\Omega} \emptyset \, dxdy$$

$$Mt = 2 \sum \emptyset \, \Delta x \Delta y$$

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Fin