# TEORÍA ELASTICIDAD

# TP#4 DIFERENCIAS FINITAS

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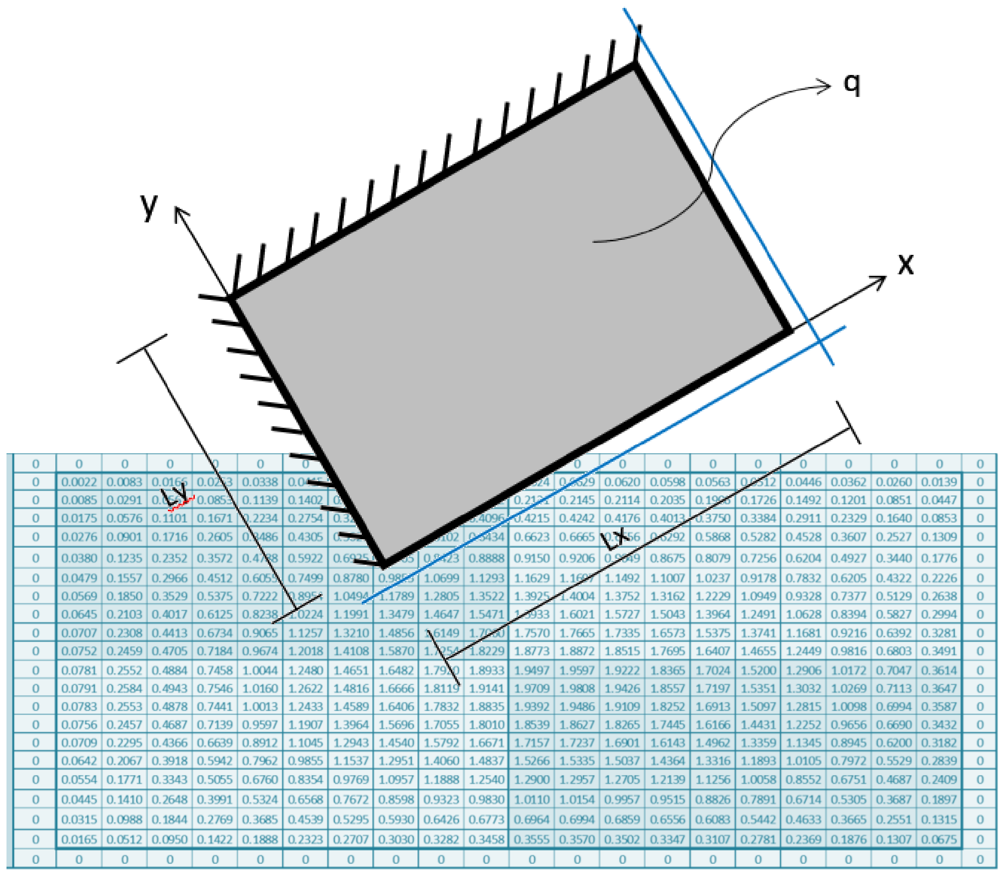
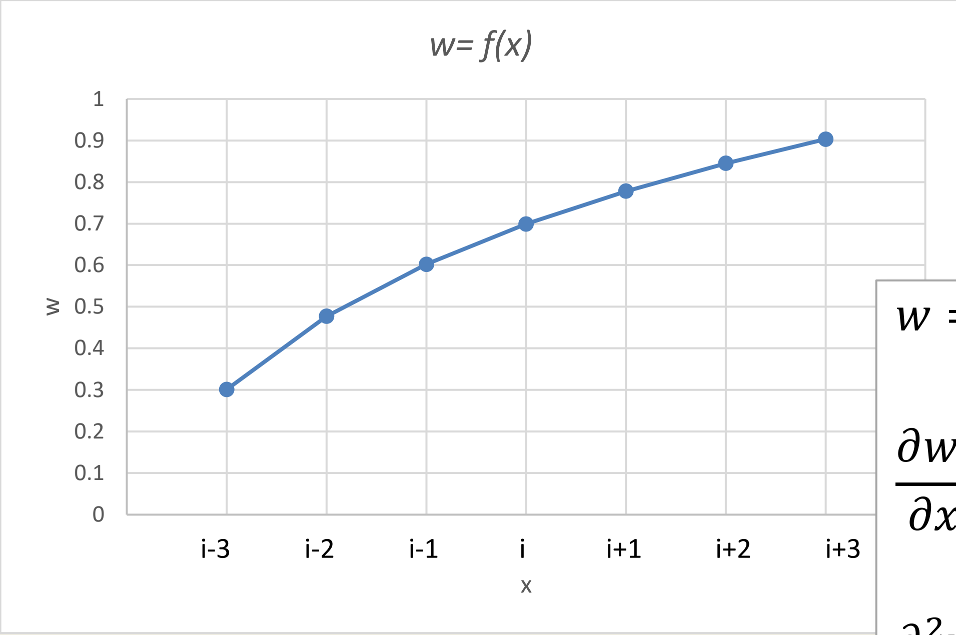
## ANALISIS ESTRUCTURAL II



# DIFERENCIAS FINITAS

## Aproximación de derivadas




$$w = f(x)$$

$$\frac{\partial w_i}{\partial x} \cong -\frac{1\, w_{i-1} - 1\, w_{i+1}}{2\, \Delta x}$$

$$\frac{\partial^2 w_i}{\partial x^2} \cong \frac{1\, w_{i-1} - 2\, w_i + 1\, w_{i+1}}{\Delta x^2}$$

$$\frac{\partial^3 w_i}{\partial x^3} \cong \frac{-1\, w_{i-2} + 2\, w_{i-1} - 2\, w_{i+1} + w_{i+2}}{2\, \Delta x^3}$$

$$\frac{\partial^4 w_i}{\partial x^4} \cong \frac{w_{i-2} - 4\, w_{i-1} + 6\, w_i - 4\, w_{i+1} + w_{i+2}}{\Delta x^4}$$

# DIFERENCIAS FINITAS

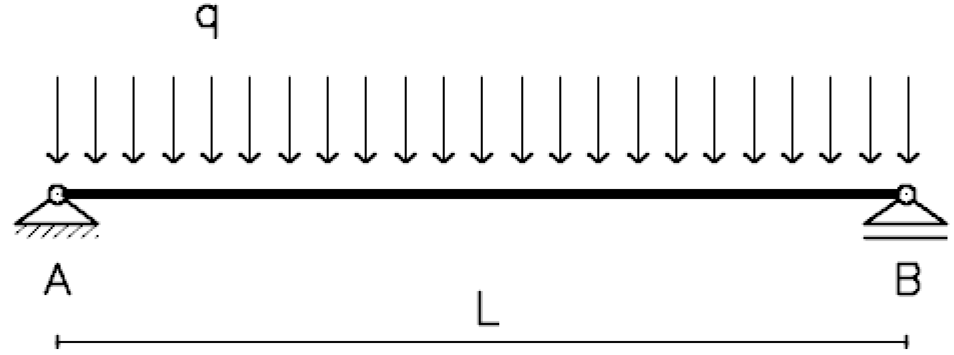
## Viga Simplemente Apoyada con carga q

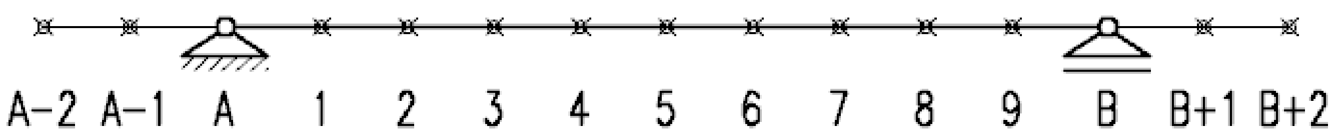
$$w = f(x)$$

$$M = -EI\,\frac{\partial^2 w}{\partial x^2}$$

$$V = -EI\,\frac{\partial^3 w}{\partial x^3}$$

$$q = EI\,\frac{\partial^4 w}{\partial x^4}$$




$$\frac{\partial^4 w_i}{\partial x^4} \cong \frac{w_{i-2} - 4\,w_{i-1} + 6\,w_i - 4\,w_{i+1} + w_{i+2}}{\Delta x^4}$$

$$\frac{q}{EI}\Delta L^4 = \frac{\partial^4 w_i}{\partial x^4} \cong w_{i-2} - 4\,w_{i-1} + 6\,w_i - 4\,w_{i+1} + w_{i+2}$$

$$[A][w] = \left[\frac{q}{EI}\Delta L^4\right] \qquad [w] = [A]^{-1}\left[\frac{q}{EI}\Delta L^4\right]$$

# DIFERENCIAS FINITAS

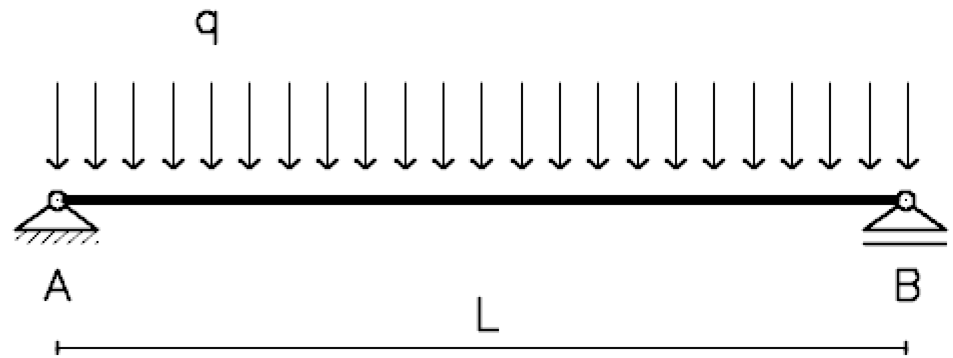
## Viga Simplemente Apoyada con carga q

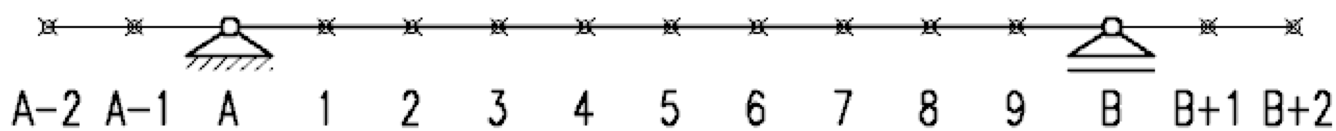
En el Apoyo A

$$w_A = 0$$

$$M_A = 0$$

$$M_A = -EI\,\frac{\partial^2 w_A}{\partial x^2}$$




$$M_A = 0 \cong -EI\,\frac{1\,w_{A-1} - 2\,w_A + 1\,w_1}{\Delta x^2}$$

$$w_{A-1} = -w_1$$

$$\frac{q}{EI}\Delta L^4 = \frac{\partial^4 w_1}{\partial x^4} \cong -w_1 - 4\,w_A + 6\,w_1 - 4\,w_2 + w_3$$

$$\frac{q}{EI}\Delta L^4 = 5\,w_1 - 4\,w_2 + w_3 \quad \rightarrow \quad [5 \quad -4 \quad 1 \quad \cdots]$$

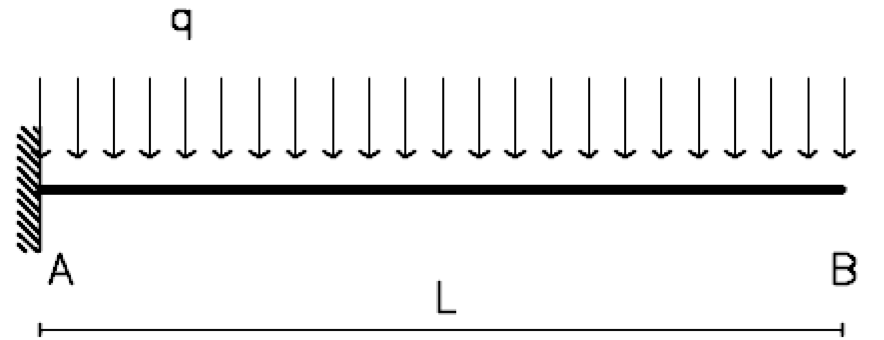
# DIFERENCIAS FINITAS

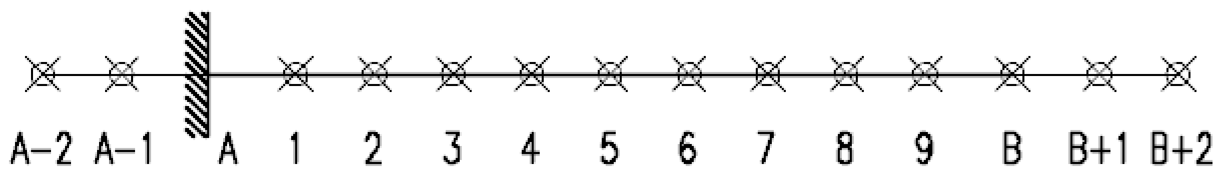
## Viga Empotrada con carga q

En el Apoyo A

$\theta_A = 0$

$$\theta_A = \frac{\partial w_A}{\partial x}$$




$$\theta_A = 0 \cong \frac{1\, w_1 \; - \; 1\, w_{A-1}}{2\, \Delta x}$$

$$w_{A-1} = w_1$$

# DIFERENCIAS FINITAS

## Viga Empotrada con carga q

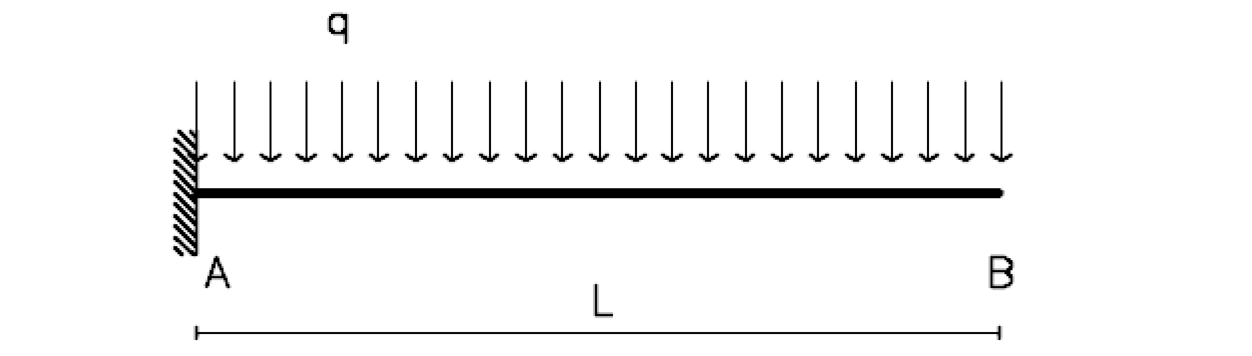
En el Extremo B

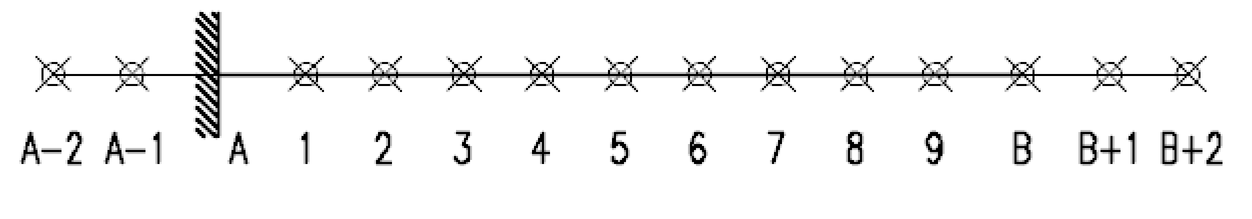
$$V_B = 0$$

$$M_B = 0$$

$$V_B = -EI\,\frac{\partial^3 w_B}{\partial x^3}$$

$$M_B = -EI\,\frac{\partial^2 w_B}{\partial x^2}$$




$$V_B = 0 \cong -EI\,\frac{-1\,w_8 + 2\,w_9 - 2\,w_{B+1} + 1\,w_{B+2}}{2\,\Delta x^3}$$

$$w_{B+2} = 1\,w_8 - 2\,w_9 + 2w_{B+1}$$

$$M_B = 0 \cong -EI\,\frac{1\,w_9 - 2\,w_B + 1\,w_{B+1}}{\Delta x^2}$$

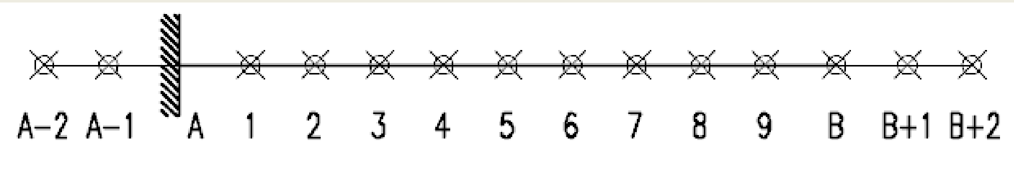
$$\boxed{w_{B+1} = -1\,w_9 + 2\,w_B}$$

$$w_{B+2} = 1\,w_8 - 2\,w_9 - 2\,w_9 + 4\,w_B$$

$$\boxed{w_{B+2} = 1\,w_8 - 4\,w_9 + 4\,w_B}$$

# DIFERENCIAS FINITAS

## Viga Empotrada con carga q




$$\frac{q}{EI}\Delta L^4 = \frac{\partial^4 w_i}{\partial x^4} \cong w_{i-2} - 4\,w_{i-1} + 6\,w_i - 4\,w_{i+1} + w_{i+2}$$

$$[A][w] = \frac{q}{EI}\Delta L^4 \qquad [w] = [A]^{-1}\frac{q}{EI}\Delta L^4$$

$$\frac{\partial^4 w_1}{\partial x^4} \cong \frac{w_{A-1} - 4\,w_A + 6\,w_1 - 4\,w_2 + w_3}{\Delta L^4} = \frac{w_1 + 6\,w_1 - 4\,w_2 + 1\,w_3}{\Delta L^4} = \frac{7\,w_1 - 4\,w_2 + 1\,w_3}{\Delta L^4} \rightarrow [7 \quad -4 \quad 1 \quad \cdots]$$

$$\frac{\partial^4 w_2}{\partial x^4} \cong \frac{1\,w_A - 4\,w_1 + 6\,w_2 - 4\,w_3 + w_4}{\Delta L^4} = \frac{-4\,w_1 + 6\,w_2 - 4\,w_3 + 1\,w_4}{\Delta L^4} \rightarrow [-4 \quad 6 \quad -4 \quad 1 \quad \cdots]$$

$$\frac{\partial^4 w_3}{\partial x^4} \cong \frac{1\,w_1 - 4\,w_2 + 6\,w_3 - 4\,w_4 + 1\,w_5}{\Delta L^4} \rightarrow [0 \quad 1 - 4 \quad 6 \quad -4 \quad 1 \quad \cdots]$$

$$\frac{\partial^4 w_9}{\partial x^4} \cong \frac{1\,w_7 - 4\,w_8 + 6\,w_9 - 4\,w_B - 1\,w_9 + 2\,w_B}{\Delta L^4} = \frac{1\,w_7 - 4\,w_8 + 5\,w_9 - 2\,w_B}{\Delta L^4} \rightarrow [\cdots \quad 1 \quad -4 \quad 5 \quad -2]$$

$$\frac{\partial^4 w_B}{\partial x^4} \cong \frac{1\,w_8 - 4\,w_9 + 6\,w_B - 4\,w_{B+1} + 1\,w_{B+2}}{\Delta L^4} = \frac{1\,w_8 - 4\,w_9 + 6\,w_B + 4\,w_9 - 8\,w_B + 1\,w_8 - 4\,w_9 + 4\,w_B}{\Delta L^4}$$

$$\frac{\partial^4 w_B}{\partial x^4} \cong \frac{2\,w_8 - 4\,w_9 + 2\,w_B}{\Delta L^4} \rightarrow [\cdots \quad 2 \quad -4 \quad 2]$$

# DIFERENCIAS FINITAS

## Losas

La ecuación diferencial se debe satisfacer en todos los puntos

$$\nabla^2\nabla^2 w = \frac{q}{B}$$

$$B = \frac{E\ I}{(1-\nu^2)} \qquad I = 1\,\frac{t^3}{12}$$

$$\nabla^2\nabla^2 w_i = \left(\frac{\partial^4 w}{\partial x^4}\right)_i + 2\left(\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)_i + \left(\frac{\partial^4 w}{\partial y^4}\right)_i$$

$$\nabla^2\nabla^2 w_i \cong \frac{1}{\Delta^4}[1w_{i-2}^{-x-} - 8w_{i-1}^{-x-} + 20w_i - 8w_{i+1}^{-x-} + 1w_{i+2}^{-x-} +$$

$$+ 1w_{i-2}^{-y-} - 8w_{i-1}^{-y-} - 8w_{i+1}^{-y-} + 1w_{i+2}^{-y-} +$$

$$+ 2\,w_{i-1\ \ i-1}^{-x-\ \ -y-} + \ 2\,w_{i+1\ \ i-1}^{-x-\ \ -y-} + \ 2\,w_{i+1\ \ i+1}^{-x-\ \ -y-} + \ 2\,w_{i-1\ \ i+1}^{-x-\ \ -y-}]$$

$$\Delta = \ \Delta_x = \Delta_y$$

# DIFERENCIAS FINITAS

## Losas. Corrimientos

La ecuación diferencial se debe satisfacer en todos los puntos

$$\nabla^2\nabla^2 w = \frac{q}{B}$$

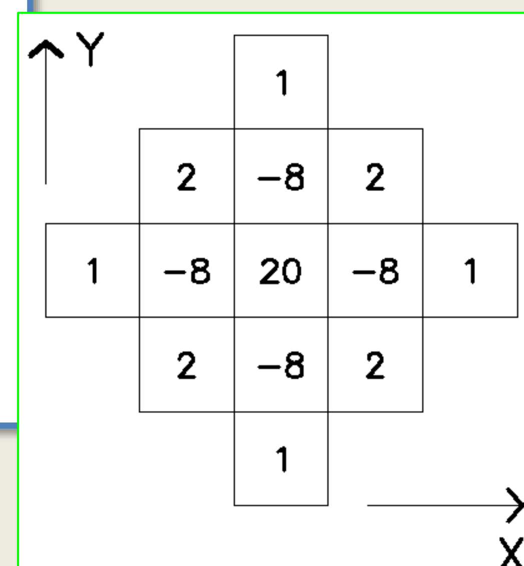
$$\nabla^2\nabla^2 w_i = \left(\frac{\partial^4 w}{\partial x^4}\right)_i + 2\left(\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)_i + \left(\frac{\partial^4 w}{\partial y^4}\right)_i$$

$$\nabla^2\nabla^2 w_i \cong \frac{1}{\Delta^4}\left[1w_{i-2}^{-x-} - 8w_{i-1}^{-x-} + 20w_i - 8w_{i+1}^{-x-} + 1w_{i+2}^{-x-} + \right.$$

$$+ 1w_{i-2}^{-y-} - 8w_{i-1}^{-y-} - 8w_{i+1}^{-y-} + 1w_{i+2}^{-y-} +$$

$$\left. + 2\, w_{i-1\;\; i-1}^{-x-\;\; -y-} + 2\, w_{i+1\;\; i-1}^{-x-\;\; -y-} + 2\, w_{i+1\;\; i+1}^{-x-\;\; -y-} + 2\, w_{i-1\;\; i+1}^{-x-\;\; -y-}\right]$$

$$\Delta = \Delta_x = \Delta_y$$



# DIFERENCIAS FINITAS

## Losas. Esfuerzos generalizados de flexión (momentos)

$$m_x = -B\left(\frac{\partial^2 w}{\partial x^2} + \nu\,\frac{\partial^2 w}{\partial y^2}\right)$$

$$m_y = -B\left(\frac{\partial^2 w}{\partial y^2} + \nu\,\frac{\partial^2 w}{\partial x^2}\right)$$

$$m_{xy} = -(1-\nu)\,B\,\frac{\partial^2 w}{\partial x \partial y}$$

$$m_x = -\,\frac{B}{\Delta^2}\left[1\,w_{i-1}^{-x-} - 2\,w_i + 1\,w_{i+1}^{-x-} + \nu\left(1\,w_{i-1}^{-y-} - 2\,w_i + 1\,w_{i+1}^{-y-}\right)\right]$$

$$m_y = -\,\frac{B}{\Delta^2}\left[1\,w_{i-1}^{-y-} - 2\,w_i + 1\,w_{i+1}^{-y-} + \nu\,\left(1\,w_{i-1}^{-x-} - 2\,w_i + 1\,w_{i+1}^{-x-}\right)\right]$$

$$m_{xy} = -\,\frac{(1-\nu)\,B}{2\Delta^2}\left[1\,w_{i+1}^{-x-} - 1\,w_{i-1}^{-x-} + 1\,w_{i+1}^{-y-} - 1\,w_{i-1}^{-y-}\right]$$

# DIFERENCIAS FINITAS

## Losas. Tensiones

$$\sigma_x = m_x \frac{z}{I}$$

$$\sigma_y = m_y \frac{z}{I}$$

$$\tau_{xy} = m_{xy} \frac{z}{I}$$

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# Fin