4 THE JOINTED ROCK MODEL (ANISOTROPY)

Materials may have different properties in different directions. As a result, they may respond differently when subjected to particular conditions in one direction or another. This aspect of material behaviour is called anisotropy. When modelling anisotropy, distinction can be made between elastic anisotropy and plastic anisotropy. Elastic anisotropy refers to the use of different elastic stiffness properties in different directions. Plastic anisotropy may involve the use of different strength properties in different directions, as considered in the Jointed Rock model. Another form of plastic anisotropy is kinematic hardening. The latter is not considered in PLAXIS program.

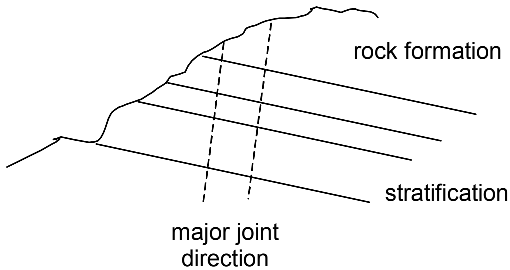

Figure 4.1 Visualization of concept behind the Jointed Rock model

The Jointed Rock model is an anisotropic elastic perfectly-plastic model, especially meant to simulate the behaviour of stratified and jointed rock layers. In this model it is assumed that there is intact rock with an eventual stratification direction and major joint directions. The intact rock is considered to behave as a transversly anisotropic elastic material, quantified by five parameters and a direction. The anisotropy may result from stratification or from other phenomena. In the major joint directions it is assumed that shear stresses are limited according to Coulomb's criterion. Upon reaching the maximum shear stress in such a direction, plastic sliding will occur. A maximum of three sliding directions ('planes') can be defined, of which the first plane is assumed to coincide with the direction of elastic anisotropy. Each plane may have different shear strength properties. In addition to plastic shearing, the tensile stresses perpendicular to the three planes are limited according to a predefined tensile strength (tension cut-off).

The application of the Jointed Rock model is justified when families of joints or joint sets are present. These joint sets have to be parallel, not filled with fault gouge, and their spacing has to be small compared to the characteristic dimension of the structure.

Some basic characteristics of the Jointed Rock model are:

- Anisotropic elastic behaviour for intact rock — Parameters E_1, E_2, ν_1, ν_2, G_2
- Shear failure according to Coulomb in three directions, i — Parameters c_i, φ_i and ψ_i
- Limited tensile strength in three directions, i — Parameters $\sigma_{t,i}$

4.1 ANISOTROPIC ELASTIC MATERIAL STIFFNESS MATRIX

The elastic material behaviour in the Jointed Rock model is described by an elastic material stiffness matrix, $\underline{\underline{D}}^*$. In contrast to Hooke's law, the D^*-matrix as used in the Jointed Rock model is transversely anisotropic. Different stiffnesses can be used normal to and in a predefined direction ('plane 1'). This direction may correspond to the stratification direction or to any other direction with significantly different elastic stiffness properties.

Consider, for example, a horizontal stratification, where the stiffness in horizontal direction, E_1, is different from the stiffness in vertical direction, E_2. In this case the 'Plane 1' direction is parallel to the *x-z*-plane and the following constitutive relations exist (See: Zienkiewicz & Taylor: The Finite Element Method, 4th Ed.):

$$\dot{\varepsilon}_{xx} = \frac{\dot{\sigma}_{xx}}{E_1} - \frac{\nu_2\dot{\sigma}_{yy}}{E_2} - \frac{\nu_1\dot{\sigma}_{zz}}{E_1} \tag{4.1a}$$

$$\dot{\varepsilon}_{yy} = -\frac{\nu_2\dot{\sigma}_{xx}}{E_2} + \frac{\dot{\sigma}_{yy}}{E_2} - \frac{\nu_2\dot{\sigma}_{zz}}{E_2} \tag{4.1b}$$

$$\dot{\varepsilon}_{zz} = -\frac{\nu_1\dot{\sigma}_{xx}}{E_1} - \frac{\nu_2\dot{\sigma}_{yy}}{E_2} + \frac{\dot{\sigma}_{zz}}{E_1} \tag{4.1c}$$

$$\dot{\gamma}_{xy} = \frac{\dot{\sigma}_{xy}}{G_2} \tag{4.1d}$$

$$\dot{\gamma}_{yz} = \frac{\dot{\sigma}_{yz}}{G_2} \tag{4.1e}$$

$$\dot{\gamma}_{zx} = \frac{2(1+\nu_1)\dot{\sigma}_{zx}}{E_1} \tag{4.1f}$$

The inverse of the anisotropic elastic material stiffness matrix, $(\underline{\underline{D}}^*)^{-1}$, follows from the above relations. This matrix is symmetric. The regular material stiffness matrix $\underline{\underline{D}}^*$ can only be obtained by numerical inversion.

In general, the stratification plane will not be parallel to the global *x-z*-plane, but the above relations will generally hold for a local (*n,s,t*) coordinate system where the stratification plane is parallel to the s-t-plane. The orientation of this plane is defined by the *dip angle* and *dip direction* (see 4.3). As a consequence, the local material stiffness matrix has to be transformed from the local to the global coordinate system. Therefore we consider first a transformation of stresses and strains:

$$\underline{\sigma}_{nst} = \underline{\underline{R}}_{\sigma}\ \underline{\sigma}_{xyz} \qquad \underline{\sigma}_{xyz} = \underline{\underline{R}}_{\sigma}^{-1}\ \underline{\sigma}_{nst} \tag{4.2a}$$

$$\underline{\varepsilon}_{nst} = \underline{\underline{R}}_{\varepsilon}\ \underline{\varepsilon}_{xyz} \qquad \underline{\varepsilon}_{xyz} = \underline{\underline{R}}_{\varepsilon}^{-1}\ \underline{\varepsilon}_{nst} \tag{4.2b}$$

where

$$\underline{\underline{R}}_{\sigma} = \begin{bmatrix} n_x^2 & n_y^2 & n_z^2 & 2\ n_x\ n_y & 2\ n_y\ n_z & 2\ n_x\ n_z \\ s_x^2 & s_y^2 & s_z^2 & 2\ s_x\ s_y & 2\ s_y\ s_z & 2\ s_x\ s_z \\ t_x^2 & t_y^2 & t_z^2 & 2\ t_x\ t_y & 2\ t_y\ t_z & 2\ t_x\ t_z \\ n_x\ s_x & n_y\ s_y & n_z\ s_z & n_x\ s_y + n_y\ s_x & n_y\ s_z + n_z\ s_y & n_z\ s_x + n_x\ s_z \\ s_x\ t_x & s_y\ t_y & s_z\ t_z & s_x\ t_y + s_y\ t_x & s_y\ t_z + s_z\ t_y & s_x\ t_z + s_z\ t_x \\ n_x\ t_x & n_y\ t_y & n_z\ t_z & n_x\ t_y + n_y\ t_x & n_y\ t_z + n_z\ t_y & n_z\ t_x + n_x\ t_z \end{bmatrix} \tag{4.3}$$

and

$$\underline{\underline{R}}_{\varepsilon} = \begin{bmatrix} n_x^2 & n_y^2 & n_z^2 & n_x\ n_y & n_y\ n_z & n_x\ n_z \\ s_x^2 & s_y^2 & s_z^2 & s_x\ s_y & s_y\ s_z & s_x\ s_z \\ t_x^2 & t_y^2 & t_z^2 & t_x\ t_y & t_y\ t_z & t_x\ t_z \\ 2\ n_x\ s_x & 2\ n_y\ s_y & 2\ n_z\ s_z & n_x\ s_y + n_y\ s_x & n_y\ s_z + n_z\ s_y & n_z\ s_x + n_x\ s_z \\ 2\ s_x\ t_x & 2\ s_y\ t_y & 2\ s_z\ t_z & s_x\ t_y + s_y\ t_x & s_y\ t_z + s_z\ t_y & s_x\ t_z + s_z\ t_x \\ 2\ n_x\ t_x & 2\ n_y\ t_y & 2\ n_z\ t_z & n_x\ t_y + n_y\ t_x & n_y\ t_z + n_z\ t_y & n_z\ t_x + n_x\ t_z \end{bmatrix} \tag{4.4}$$

n_x, n_y, n_z, s_x, s_y, s_z, t_x, t_y and t_z are the components of the normalized n, s and t-vectors in global (*x,y,z*)-coordinates (i.e. 'sines' and 'cosines'; see 4.3). For plane condition $n_z = s_z = t_x = t_y = 0$ and $t_z = 1$.

It further holds that :

$$\underline{\underline{R}}_{\varepsilon}^{T} = \underline{\underline{R}}_{\sigma}^{-1} \qquad \underline{\underline{R}}_{\sigma}^{T} = \underline{\underline{R}}_{\varepsilon}^{-1} \tag{4.5}$$

A local stress-strain relationship in (*n,s,t*)-coordinates can be transformed to a global relationship in (*x,y,z*)-coordinates in the following way:

$$\left.\begin{aligned} \underline{\sigma}_{nst} &= \underline{\underline{D}}_{nst}^{*}\ \underline{\varepsilon}_{nst} \\ \underline{\sigma}_{nst} &= \underline{\underline{R}}_{\sigma}\ \underline{\sigma}_{xyz} \\ \underline{\varepsilon}_{nst} &= \underline{\underline{R}}_{\varepsilon}\ \underline{\varepsilon}_{xyz} \end{aligned}\right\} \quad \Rightarrow \quad \underline{\underline{R}}_{\sigma}\ \underline{\sigma}_{xyz} = \underline{\underline{D}}_{nst}^{*}\ \underline{\underline{R}}_{\varepsilon}\ \underline{\varepsilon}_{xyz} \tag{4.6}$$

Hence,

$$\underline{\sigma}_{xyz} = \underline{\underline{R}}_{\sigma}^{-1} \, \underline{\underline{D}}_{nst}^{*} \, \underline{\underline{R}}_{\varepsilon} \, \underline{\varepsilon}_{xyz} \tag{4.7}$$

Using to above condition (4.5):

$$\underline{\sigma}_{xyz} = \underline{\underline{R}}_{\varepsilon}^{T} \, \underline{\underline{D}}_{nst}^{*} \, \underline{\underline{R}}_{\varepsilon} \, \underline{\varepsilon}_{xyz} = \underline{\underline{D}}_{xyz}^{*} \, \underline{\varepsilon}_{xyz} \quad \text{or} \quad \underline{\underline{D}}_{xyz}^{*} = \underline{\underline{R}}_{\varepsilon}^{T} \, \underline{\underline{D}}_{nst}^{*} \, \underline{\underline{R}}_{\varepsilon} \tag{4.8}$$

Actually, not the D*-matrix is given in local coordinates but the inverse matrix $(\underline{\underline{D}}^*)^{-1}$.

$$\left.\begin{aligned} \underline{\varepsilon}_{nst} &= \underline{\underline{D}}_{nst}^{*\,-1} \, \underline{\sigma}_{nst} \\ \underline{\sigma}_{nst} &= \underline{\underline{R}}_{\sigma} \, \underline{\sigma}_{xyz} \\ \underline{\varepsilon}_{nst} &= \underline{\underline{R}}_{\varepsilon} \, \underline{\varepsilon}_{xyz} \end{aligned}\right\} \Rightarrow \underline{\varepsilon}_{xyz} = \underline{\underline{R}}_{\varepsilon}^{-1} \, \underline{\underline{D}}_{nst}^{*\,-1} \, \underline{\underline{R}}_{\sigma} \, \underline{\sigma}_{xyz} = \underline{\underline{R}}_{\sigma}^{T} \, \underline{\underline{D}}_{nst}^{*\,-1} \, \underline{\underline{R}}_{\sigma} \, \underline{\sigma}_{xyz} \tag{4.9}$$

Hence,

$$\underline{\underline{D}}_{xyz}^{*\,-1} = \underline{\underline{R}}_{\sigma}^{T} \, \underline{\underline{D}}_{nst}^{*\,-1} \, \underline{\underline{R}}_{\sigma} \quad \text{or} \quad \underline{\underline{D}}_{xyz}^{*} = \left[\underline{\underline{R}}_{\sigma}^{T} \, \underline{\underline{D}}_{nst}^{*\,-1} \, \underline{\underline{R}}_{\sigma} \right]^{-1} \tag{4.10}$$

Instead of inverting the $(\underline{\underline{D}}^{*}_{nst})^{-1}$-matrix in the first place, the transformation is considered first, after which the total is numerically inverted to obtain the global material stiffness matrix $\underline{\underline{D}}^{*}_{xyz}$.

4.2 PLASTIC BEHAVIOUR IN THREE DIRECTIONS

A maximum of 3 sliding directions (sliding planes) can be defined in the Jointed Rock model. The first sliding plane corresponds to the direction of elastic anisotropy. In addition, a maximum of two other sliding directions may be defined. However, the formulation of plasticity on all planes is similar. On each plane a local Coulomb condition applies to limit the shear stress, $|\tau|$. Moreover, a tension cut-off criterion is used to limit the tensile stress on a plane. Each plane, i, has its own strength parameters c_i, ϕ_i, ψ_i and $\sigma_{t,i}$.

In order to check the plasticity conditions for a plane with local (n,s,t)-coordinates it is necessary to calculate the local stresses from the Cartesian stresses. The local stresses involve three components, i.e. a normal stress component, σ_n, and two independent shear stress components, τ_s and τ_t.

$$\underline{\sigma}_i = \underline{\underline{T}}_i^{T} \, \underline{\sigma} \tag{4.11}$$

where

$$\underline{\sigma}_i = \left(\sigma_n \quad \tau_s \quad \tau_t\right)^T \tag{4.12a}$$

$$\underline{\sigma} = \left(\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{xy} \quad \sigma_{yz} \quad \sigma_{zx}\right)^T \tag{4.12b}$$

$\underline{\underline{T}}_i^T$ = transformation matrix (3x6), for plane i

As usual in PLAXIS, tensile (normal) stresses are defined as positive whereas compression is defined as negative.

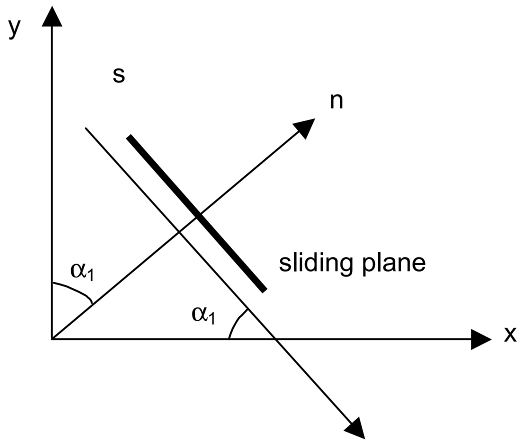

Figure 4.2 Plane strain situation with a single sliding plane and vectors n, s

Consider a plane strain situation as visualized in Figure 4.2. Here a sliding plane is considered under an angle α_1 (= *dip angle*) with respect to the *x*-axis. In this case the transformation matrix $\underline{\underline{T}}^T$ becomes:

$$T^T = \begin{bmatrix} s^2 & c^2 & 0 & -2sc & 0 & 0 \\ sc & -sc & 0 & -s^2+c^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c & -s \end{bmatrix} \tag{4.13}$$

where

$s = \sin \alpha_1$

$c = \cos \alpha_1$

In the general three-dimensional case the transformation matrix is more complex, since it involves both the *dip angle* and the *dip direction* (see 4.3):

$$\underline{\underline{T}}^T = \begin{bmatrix} n_x^2 & n_y^2 & n_z^2 & 2\,n_x\,n_y & 2\,n_y\,n_z & 2\,n_z\,n_x \\ n_x\,s_x & n_y\,s_y & n_z\,s_z & n_x\,s_y+n_y\,s_x & n_z\,s_y+n_y\,s_z & n_z\,s_x+n_x\,s_z \\ n_x\,t_x & n_y\,t_y & n_z\,t_z & n_y\,t_x+n_x\,t_y & n_y\,t_z+n_z\,t_y & n_z\,t_x+n_x\,t_z \end{bmatrix} \tag{4.14}$$

Note that the general transformation matrix, $\underline{\underline{T}}^T$, for the calculation of local stresses corresponds to rows 1, 4 and 6 of $\underline{\underline{R}}_\sigma$ (see Eq. 4.3).

After having determined the local stress components, the plasticity conditions can be checked on the basis of yield functions. The yield functions for plane *i* are defined as:

$$f_i = |\tau_s| + \sigma_n \tan\varphi_i - c_i \qquad \text{(Coulomb)} \tag{4.15a}$$

$$f_i^t = \sigma_n - \sigma_{t,i} \quad (\sigma_{t,i} \le c_i \cot\phi_i) \qquad \text{(Tension cut-off)} \tag{4.15b}$$

Figure 4.3 visualizes the full yield criterion on a single plane.

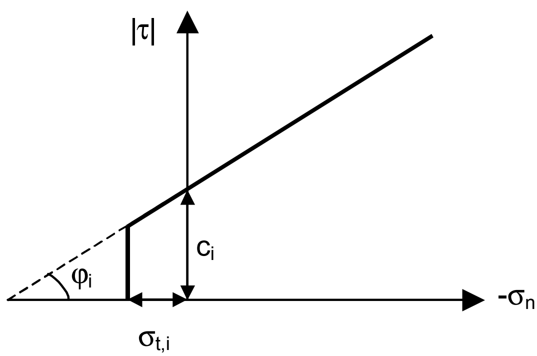

Figure 4.3 Yield criterion for individual plane

The local plastic strains are defined by:

$$\Delta\underline{\varepsilon}_j^p = \lambda_j \frac{\partial g_j}{\partial \underline{\sigma}_j} \tag{4.16}$$

where g_j is the local plastic potential function for plane *j*:

$$g_j = \left|\tau_j\right| + \sigma_n \tan\phi_j - c_j \qquad \text{(Coulomb)} \qquad (4.17a)$$

$$g_j^t = \sigma_n - \sigma_{t,j} \qquad \text{(Tension cut-off)} \qquad (4.17b)$$

The transformation matrix, $\underline{\underline{T}}$, is also used to transform the local plastic strain increments of plane *j*, $\Delta\underline{\varepsilon}^p{}_j$, into global plastic strain increments, $\Delta\underline{\varepsilon}^p$:

$$\Delta\underline{\varepsilon}^p = \underline{\underline{T}}_j \;\; \Delta\underline{\varepsilon}^p_j \qquad (4.18)$$

The consistency condition requires that at yielding the value of the yield function must remain zero for all active yield functions. For all planes together, a maximum of 6 yield functions exist, so up to 6 plastic multipliers must be found such that all yield functions are at most zero and the plastic multipliers are non-negative.

$$f_c^i = f_c^{ie} - \sum_{j=1}^{np} <\lambda_c^j> \frac{\partial f_c^{i\,T}}{\partial\sigma} T_i^T D\, T_j \frac{\partial g_c^j}{\partial\sigma} - \sum_{j=1}^{np} <\lambda_t^j> \frac{\partial f_c^{i\,T}}{\partial\sigma} T_i^T D\, T_j \frac{\partial g_t^j}{\partial\sigma} \qquad (4.19a)$$

$$f_t^i = f_t^{ie} - \sum_{j=1}^{np} <\lambda_c^j> \frac{\partial f_t^{i\,T}}{\partial\sigma} T_i^T D\, T_j \frac{\partial g_c^j}{\partial\sigma} - \sum_{j=1}^{np} <\lambda_t^j> \frac{\partial f_t^{i\,T}}{\partial\sigma} T_i^T D\, T_j \frac{\partial g_t^j}{\partial\sigma} \qquad (4.19b)$$

This means finding up to 6 values of $\lambda_i \geq 0$ such that all $f_i \leq 0$ and $\lambda_i f_i = 0$

When the maximum of 3 planes are used, there are $2^6 = 64$ possibilities of (combined) yielding. In the calculation process, all these possibilities are taken into account in order to provide an exact calculation of stresses.

4.3 PARAMETERS OF THE JOINTED ROCK MODEL

Most parameters of the jointed rock model coincide with those of the isotropic Mohr-Coulomb model. These are the basic elastic parameters and the basic strength parameters.

Elastic parameters as in Mohr-Coulomb model (see Section 3.3):

E_1	:	Young's modulus for rock as a continuum	[kN/m^2]
ν_1	:	Poisson's ratio for rock as a continuum	[−]

Anisotropic elastic parameters 'Plane 1' direction (e.g. stratification direction):

E_2	:	Young's modulus in 'Plane 1' direction	[kN/m^2]
G_2	:	Shear modulus in 'Plane 1' direction	[kN/m^2]
ν_2	:	Poisson's ratio in 'Plane 1' direction	[–]

Strength parameters in joint directions (Plane i=1, 2, 3):

c_i	:	Cohesion	[kN/m^2]
φ_i	:	Friction angle	[°]
ψ	:	Dilatancy angle	[°]
$\sigma_{t,i}$	:	Tensile strength	[kN/m^2]

*Definition of joint directions (Plane i=*1, 2, 3*):*

n	:	Numer of joint directions ($1 \leq n \leq 3$)	
$\alpha_{1,i}$	:	Dip angle	[°]
$\alpha_{2,i}$	:	Dip direction	[°]

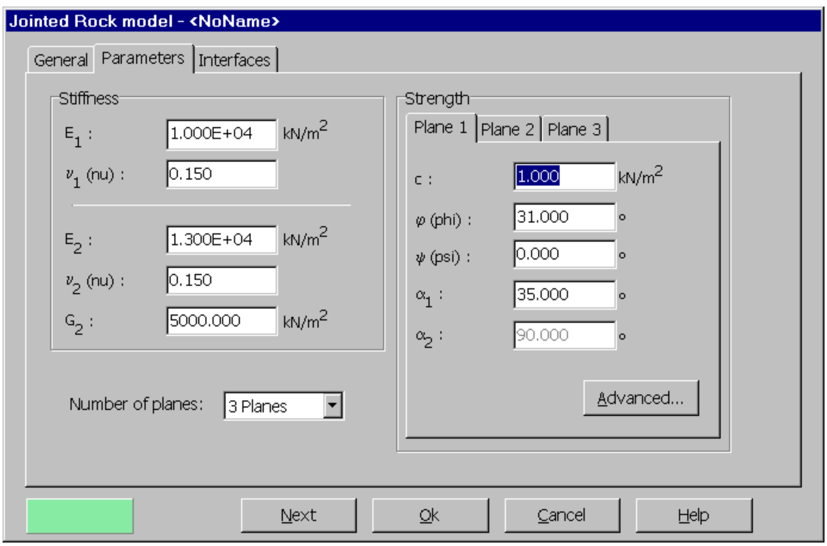

Figure 4.4 Parameters for the Jointed Rock model

Elastic parameters

The elastic parameters E_1 and ν_1 are the (constant) stiffness (Young's modulus) and Poisson's ratio of the rock as a continuum according to Hooke's law, i.e. as if it would not be anisotropic.

Elastic anisotropy in a rock formation may be introduced by stratification. The stiffness perpendicular to the stratification direction is usually reduced compared with the general stiffness. This reduced stiffness can be represented by the parameter E_2, together with a second Poisson's ratio, ν_2. In general, the elastic stiffness normal to the direction of elastic anisotropy is defined by the parameters E_2 and ν_2.

Elastic shearing in the stratification direction is also considered to be 'weaker' than elastic shearing in other directions. In general, the shear stiffness in the anisotropic direction can explicitly be defined by means of the elastic shear modulus G_2. In contrast to Hooke's law of isotropic elasticity, G_2 is a separate parameter and is not simply related to Young's modulus by means of Poisson's ratio (see Eq. 4.1d and e).

If the elastic behaviour of the rock is fully isotropic, then the parameters E_2 and ν_2 can be simply set equal to E_1 and ν_1 respectively, whereas G_2 should be set to $\frac{1}{2}E_1/(1+\nu_1)$.

Strength parameters

Each sliding direction (plane) has its own strength properties c_i, φ_i and $\sigma_{t,i}$ and dilatancy angle ψ_i. The strength properties c_i and φ_i determine the allowable shear strength according to Coulomb's criterion and σ_t determines the tensile strength according to the tension cut-off criterion. The latter is displayed after pressing <Advanced> button. By default, the tension cut-off is active and the tensile strength is set to zero. The dilatancy angle, ψ_i, is used in the plastic potential function g, and determines the plastic volume expansion due to shearing.

Definition of joint directions

It is assumed that the direction of elastic anisotropy corresponds with the first direction where plastic shearing may occur ('Plane 1'). This direction must always be specified. In the case the rock formation is stratified without major joints, the *number of sliding planes* (= sliding directions) is still 1, and strength parameters must be specified for this direction anyway. A maximum of three sliding directions can be defined. These directions may correspond to the most critical directions of joints in the rock formation.

The sliding directions are defined by means of two parameters: The *Dip angle* (α_1) (or shortly *Dip*) and the *Dip direction* (α_2). Instead of the latter parameter, it is also common in geology to use the *Strike*. However, care should be taken with the definition of *Strike*, and therefore the unambiguous *Dip direction* as mostly used by rock engineers is used in PLAXIS. The definition of both parameters is visualized in Figure 4.5.

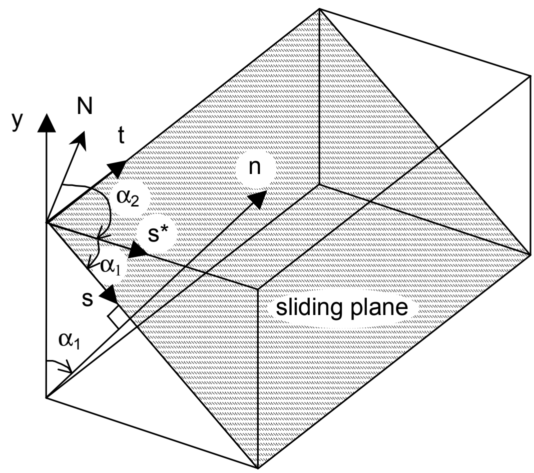

Figure 4.5 Definition of dip angle and dip direction

Consider a sliding plane, as indicated in Figure 4.5. The sliding plane can be defined by the vectors (*s,t*), which are both normal to the vector *n*. The vector *n* is the 'normal' to the sliding plane, whereas the vector *s* is the 'fall line' of the sliding plane and the vector *t* is the 'horizontal line' of the sliding plane. The sliding plane makes an angle α_1 with respect to the horizontal plane, where the horizontal plane can be defined by the vectors (*s*,t*), which are both normal to the vertical *y*-axis. The angle α_1 is the *dip angle*, which is defined as the positive 'downward' inclination angle between the horizontal plane and the sliding plane. Hence, α_1 is the angle between the vectors *s** and *s*, measured clockwise from *s** to s when looking in the positive *t*-direction. The dip angle must be entered in the range [0°, 90°].

The orientation of the sliding plane is further defined by the *dip direction*, α_2, which is the orientation of the vector *s** with respect to the North direction (*N*). The dip direction is defined as the positive angle from the North direction, measured clockwise to the horizontal projection of the fall line (=*s**-direction) when looking downwards. The dip direction is entered in the range [0°, 360°].

In addition to the orientation of the sliding planes it is also known how the global (*x,y,z*) model coordinates relate to the North direction. This information is contained in the *Declination* parameter, as defined in the *General settings* in the Input program. The *Declination* is the positive angle from the North direction to the positive *z*-direction of the model.

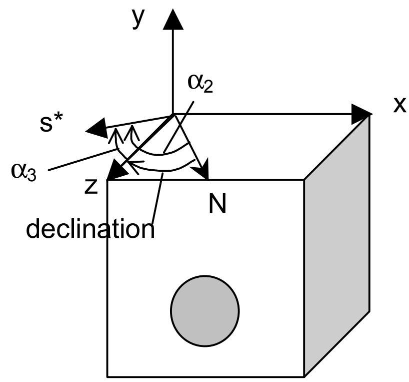

Figure 4.6 Definition of various directions and angles in the horiziontal plane

In order to transform the local (*n,s,t*) coordinate system into the global (*x,y,z*) coordinate system, an auxiliary angle α_3 is used internally, being the difference between the *Dip direction* and the *Declination*:

$$\alpha_3 = \alpha_2 - Declination \tag{4.19}$$

Hence, α_3 is defined as the positive angle from the positive *z*-direction clockwise to the *s**-direction when looking downwards.

From the definitions as given above, it follows that:

$$\underline{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} -\sin\alpha_1 \sin\alpha_3 \\ \cos\alpha_1 \\ \sin\alpha_1 \cos\alpha_3 \end{bmatrix} \tag{4.20a}$$

$$\underline{s} = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix} = \begin{bmatrix} -\cos\alpha_1 \sin\alpha_3 \\ -\sin\alpha_1 \\ \cos\alpha_1 \cos\alpha_3 \end{bmatrix} \tag{4.20b}$$

$$\underline{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} \cos\alpha_3 \\ 0 \\ \sin\alpha_3 \end{bmatrix} \tag{4.20c}$$

Below some examples are shown of how sliding planes occur in a 3D models for different values of α_1, α_2 and *Declination*:

$\alpha_1 = 45º$

$\alpha_2 = 0º$

Declination = 0º

$\alpha_1 = 45º$

$\alpha_2 = 90º$

Declination = 0º

$\alpha_1 = 45º$

$\alpha_2 = 0º$

Declination = 90º

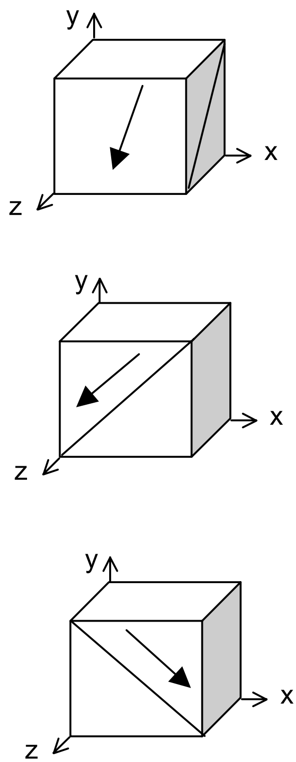

Figure 4.7 Examples of failure directions defined by α_1 , α_2 and *Declination*

As it can be seen, for plane strain conditions (the cases considered in Version 8) only α_1 is required. By default, α_2 is fixed at 90º and the declination is set to 0º.