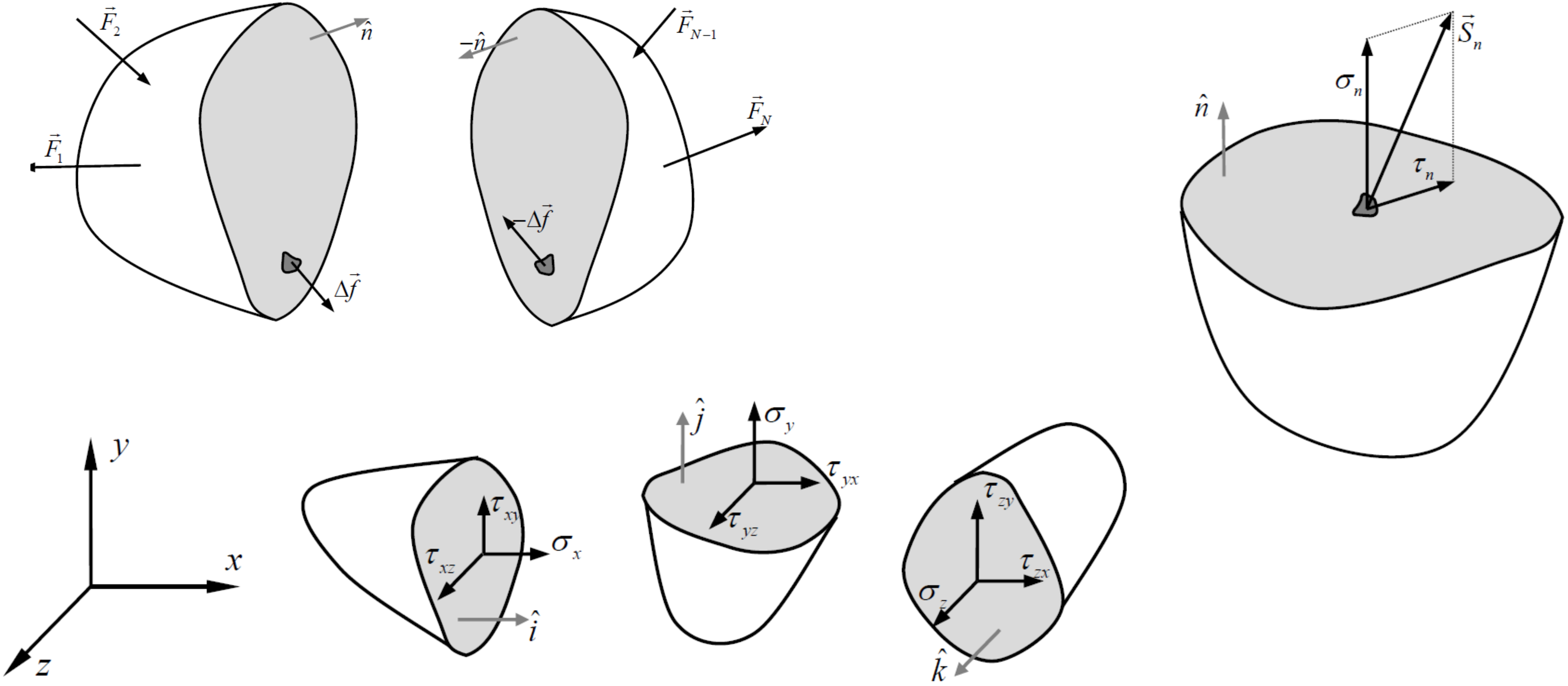
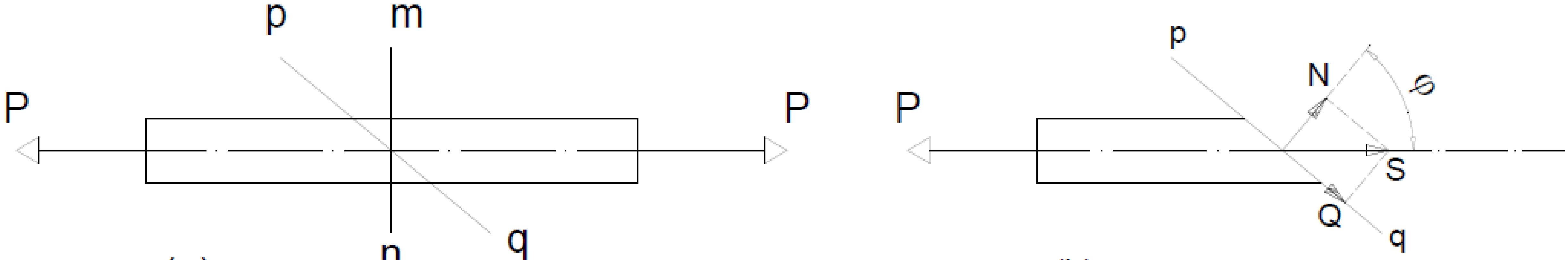
ANÁLISIS DE TENSIONES Y DEFORMACIONES

vector tensión

$$\vec{S}_n = \lim_{\Delta A \to 0} \frac{\Delta \vec{f}}{\Delta A}$$





(a) (b)

$$N = P \cdot \cos\varphi \qquad Q = P \cdot sen\varphi \qquad A` = \frac{A}{\cos\varphi}$$

$$\sigma_N = \frac{N}{A'} = \frac{P}{A} \cdot \cos^2\varphi \qquad \tau = \frac{Q}{A`} = \frac{P}{A} \cdot sen\varphi \cdot \cos\varphi$$

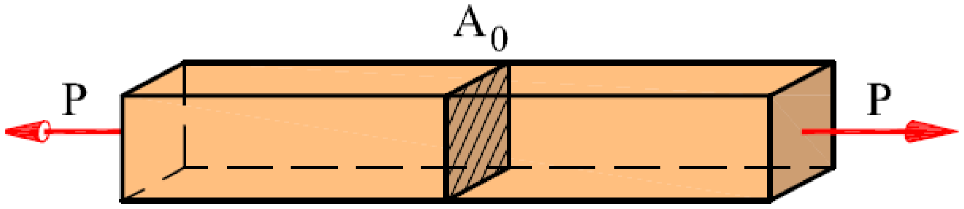
$$\sigma_x = \frac{P}{A} \qquad 2 \cdot sen\varphi \cdot \cos\varphi = sen(2\varphi)$$

$$\sigma_N = \sigma_x \cdot \cos^2\varphi \qquad \tau = \frac{\sigma_x \cdot sen(2\varphi)}{2}$$

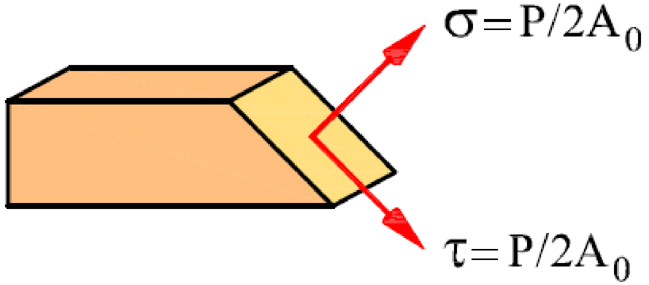
si $\varphi = 0^o$ $\sigma_N = \sigma_X$ y $\tau = 0$ (máxima tensión normal)

si $\varphi = 90^o$ $\sigma_N = 0$ y $\tau = 0$

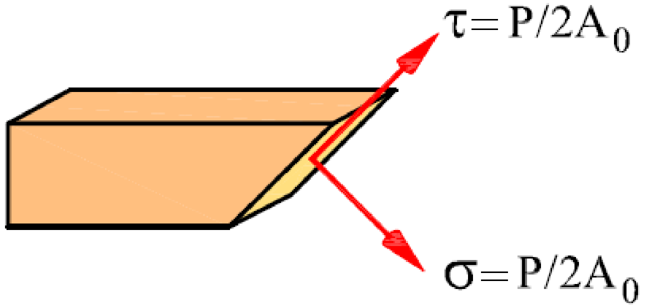
si $\varphi = 45^o$ $\tau = \sigma_X/2$ (máxima tensión tangencial)



CARGA AXIAL

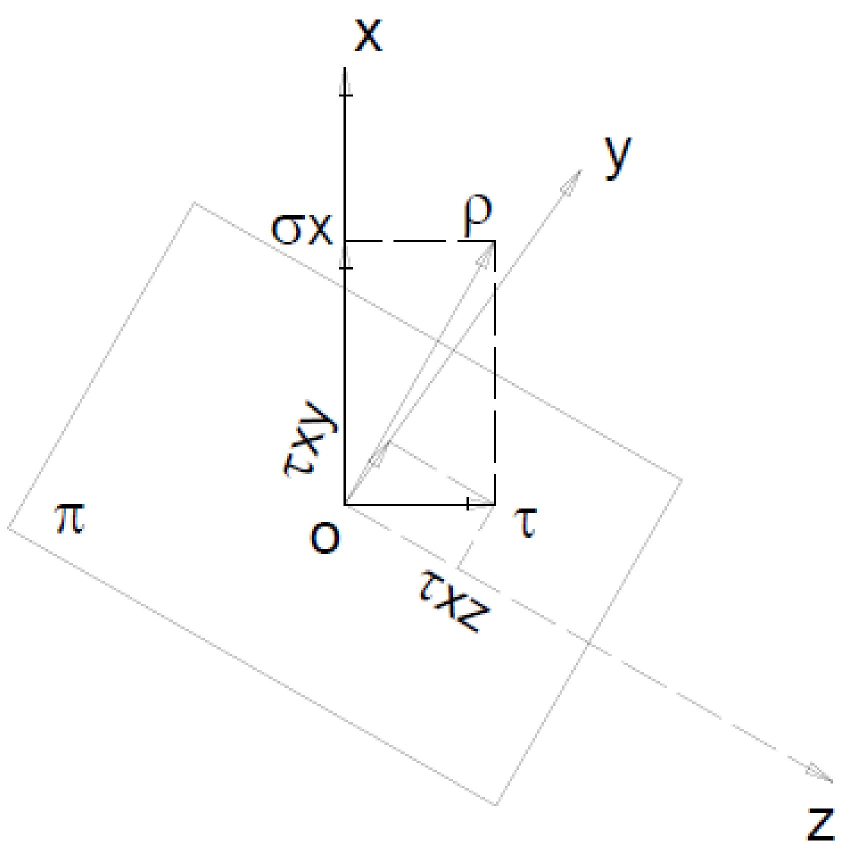


Tensiones para $\alpha = 45°$



Tensiones para $\alpha = -45°$

VARIACIÓN DE TENSIONES EN UN PUNTO

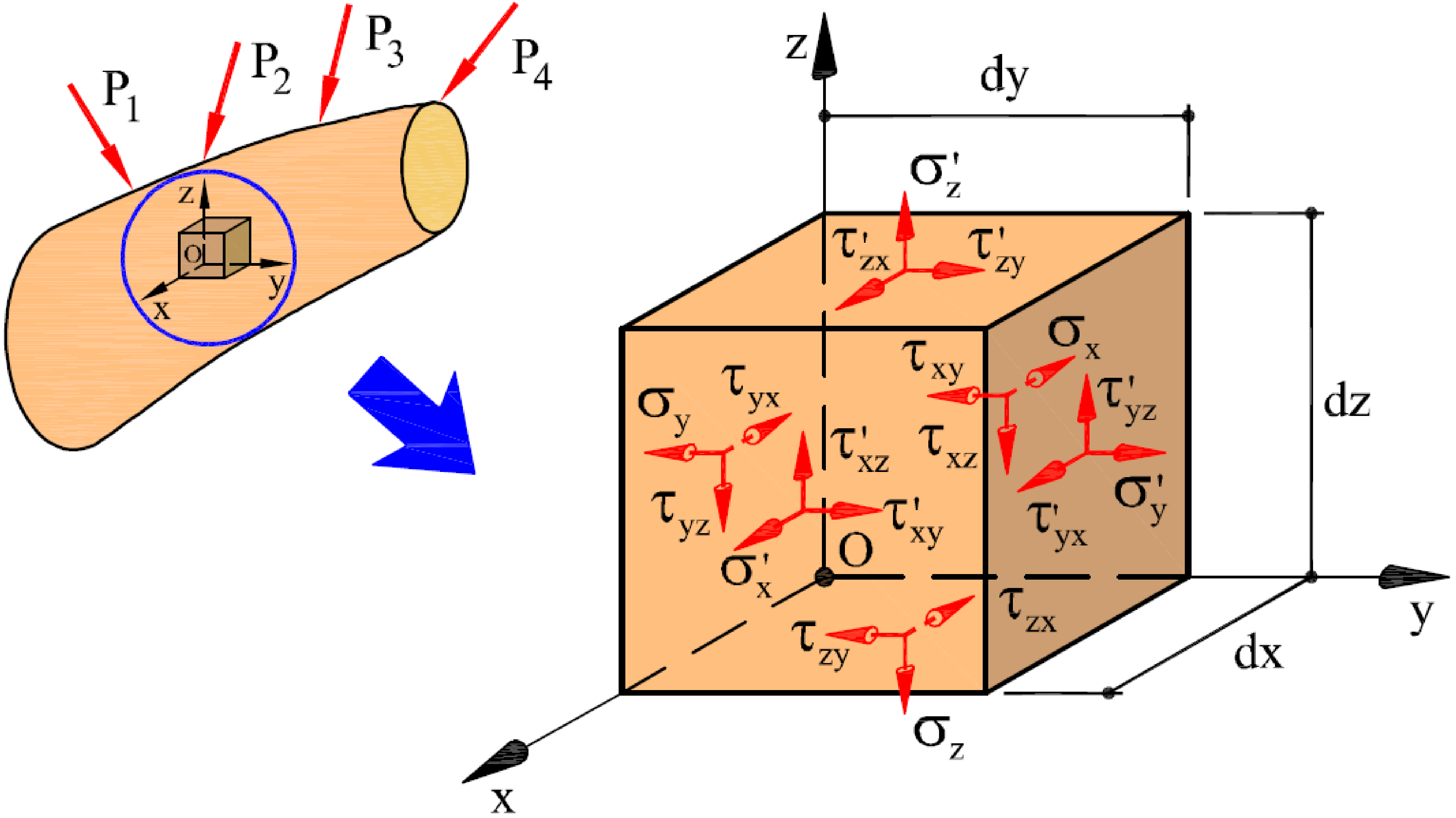


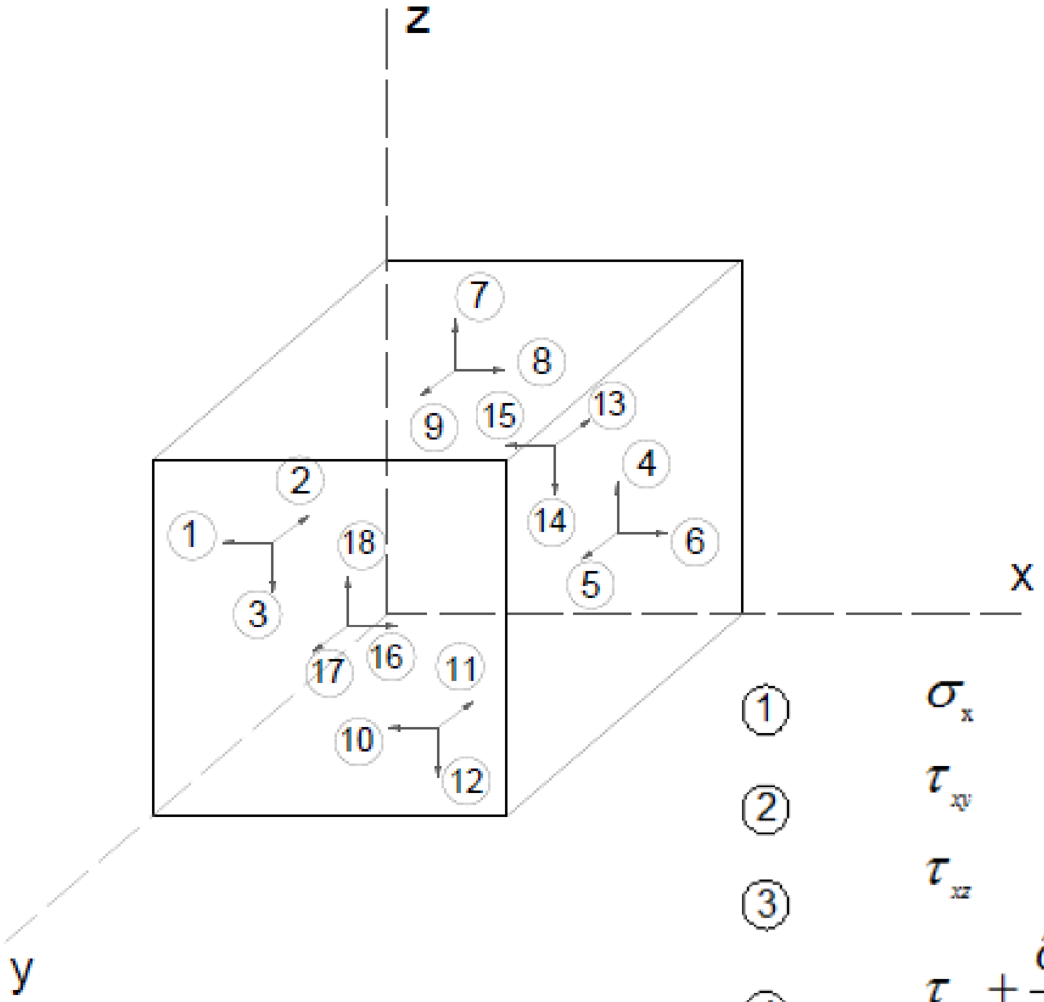
σ_x: tensión normal en la dirección x

τ: tensión tangencial ligada al plano π

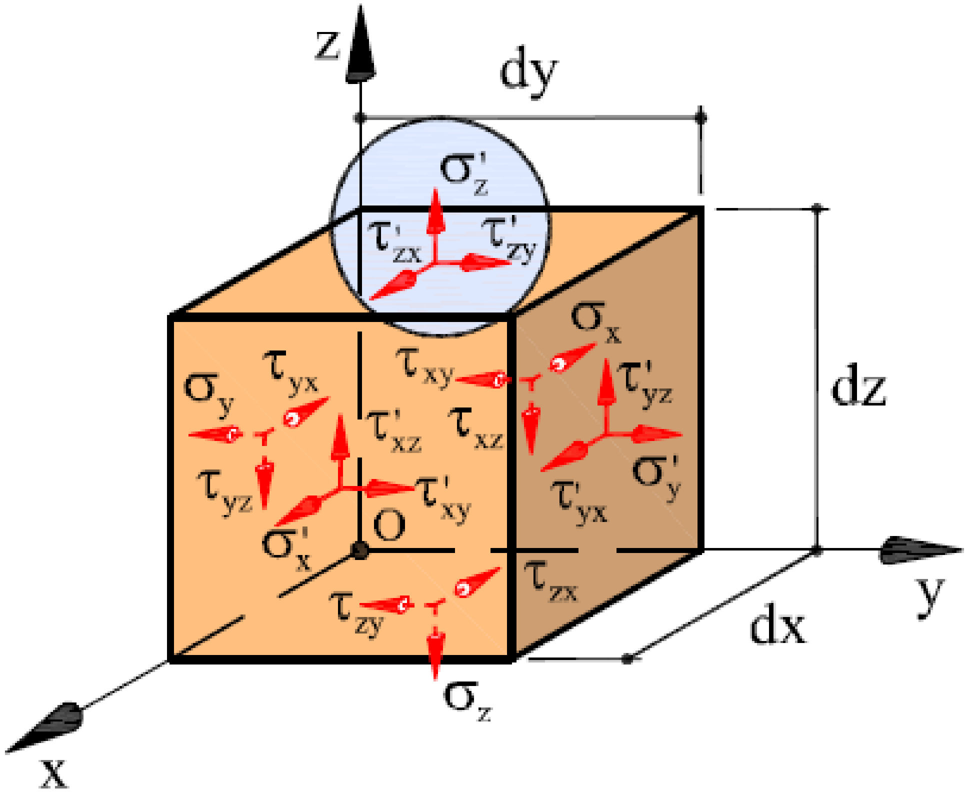
τ_{xy}: 1° subíndice: corresponde al de la tensión normal ligada al plano π

2° subíndice: corresponde a la dirección que tiene en el plano.

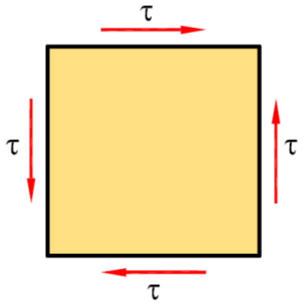

P1
P2
P3
P4
z
y
x
O
σ'z
τ'zx
τ'zy
σx
τxy
τ'yz
σy
τyx
τ'xz
τxz
σ'y
τyz
τ'xy
τ'yx
O
σ'x
τzx
τzy
σz
dy
dz
dx
z
y
x



1	σ_x	7	$\sigma_z + \dfrac{\partial \sigma_z}{\partial z} dz$	13	σ_y
2	τ_{xy}	8		14	τ_{yz}
3	τ_{xz}	9	$\tau_{zx} + \dfrac{\partial \tau_{zx}}{\partial z} dz$	15	τ_{yx}
4	$\tau_{xz} + \dfrac{\partial \tau_{xz}}{\partial x} dx$	10	$\tau_{zy} + \dfrac{\partial \tau_{zy}}{\partial z} dz$	16	$\tau_{yx} + \dfrac{\partial \tau_{yx}}{\partial y} dy$
5	$\tau_{xy} + \dfrac{\partial \tau_{xy}}{\partial x} dx$	11	τ_{zx}	17	$\sigma_y + \dfrac{\partial \sigma_y}{\partial y} dy$
6	$\sigma_x + \dfrac{\partial \sigma_x}{\partial x} dx$	12	τ_{zy} σ_z	18	$\tau_{yz} + \dfrac{\partial \tau_{yz}}{\partial y} dy$



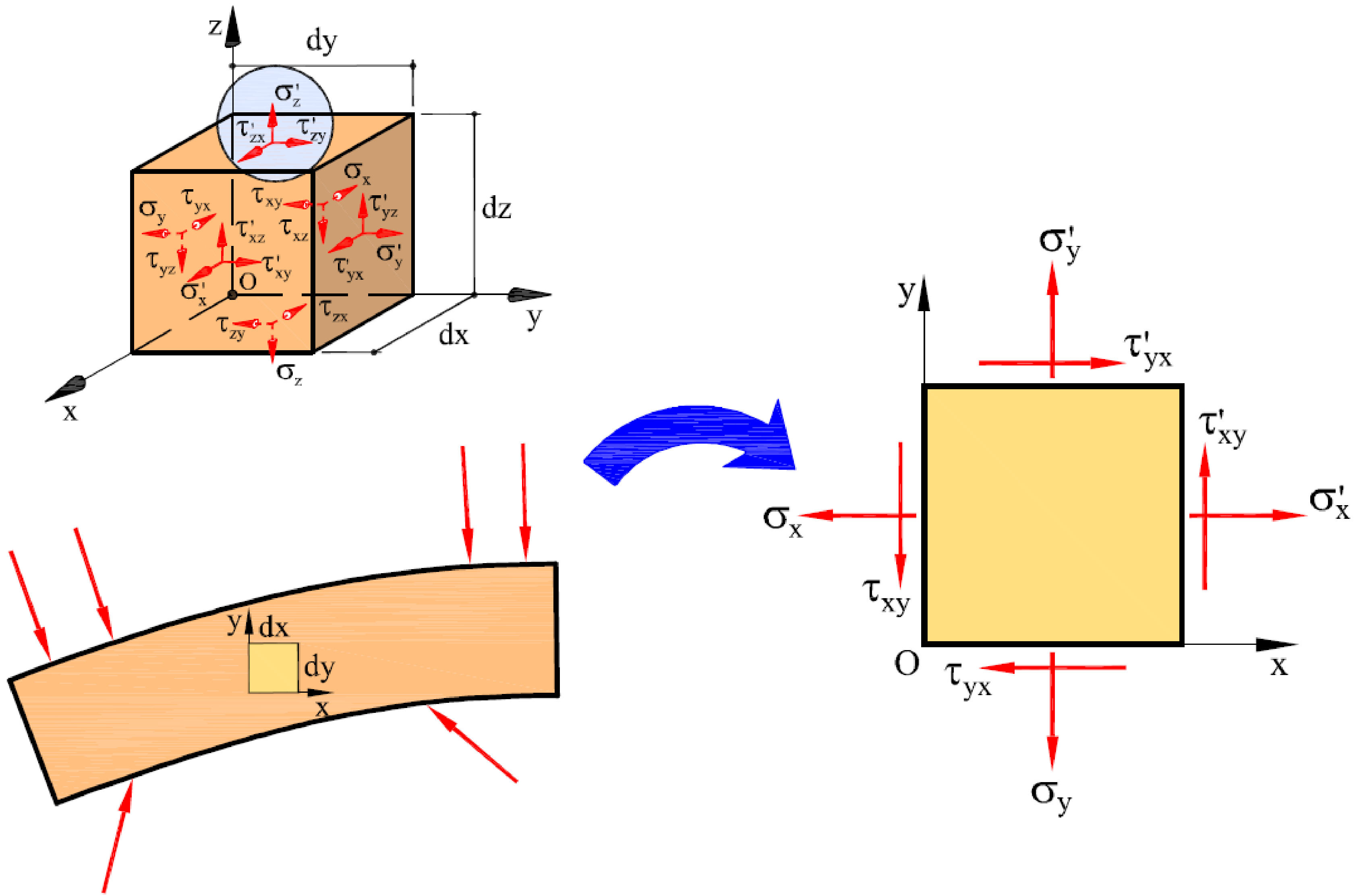
$$-2\cdot(\tau_{xz}\cdot dy\cdot dz)\cdot\frac{dx}{2}+2(\tau_{zx}\cdot dx\cdot dy)\cdot\frac{dz}{2}=0$$

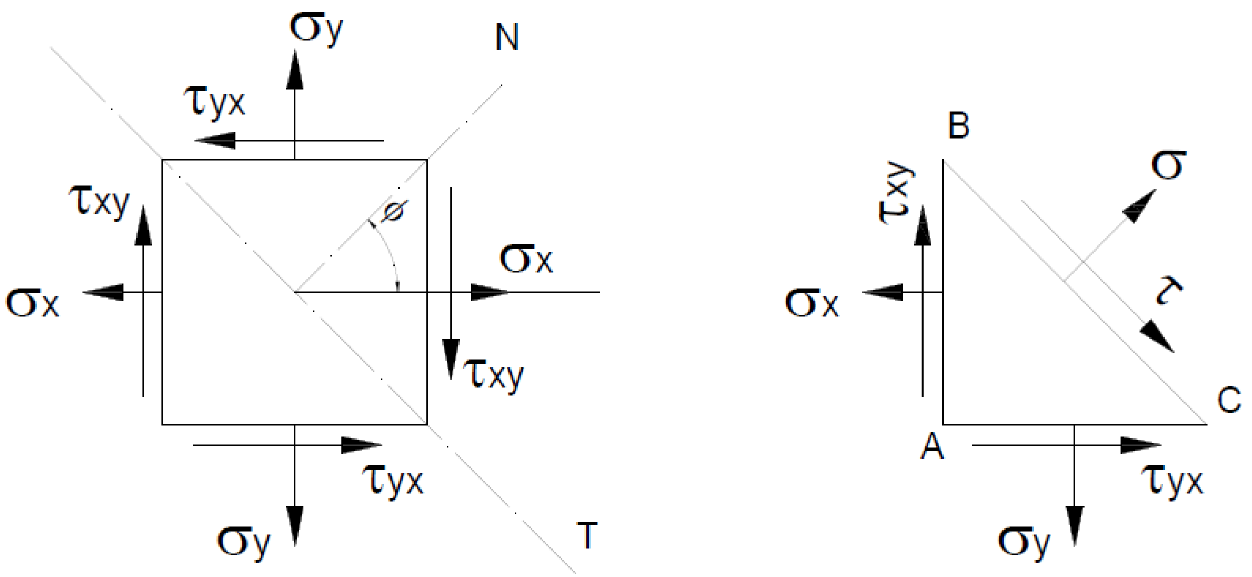


$\tau_{zy} = \tau_{yz}$; $\tau_{yx} = \tau_{xy}$

σ_x ; σ_y ; σ_z ; τ_{xy} ; τ_{yz} ; τ_{zx}

RÉGIMEN ELÁSTICO PLANO





BC = 1

AB = cosθ y AC = senθ

$$\sum N=0\ ;\quad \sigma=(\sigma_X\cdot\cos\theta)\cdot\cos\theta+(\sigma_y\cdot sen\theta)\cdot sen\theta-(\tau_{xy}\cdot\cos\theta)\cdot sen\theta-(\tau_{yx}\cdot sen\theta)\cdot\cos\theta$$

$$\sum T=0\ ;\quad \tau=(\sigma_X\cdot\cos\theta)\cdot sen\theta-(\sigma_y\cdot sen\theta)\cdot\cos\theta+(\tau_{xy}\cdot\cos\theta)\cdot\cos\theta-(\tau_{yx}\cdot sen\theta)\cdot sen\theta$$

Como $\tau_{xy} = \tau_{yx}$

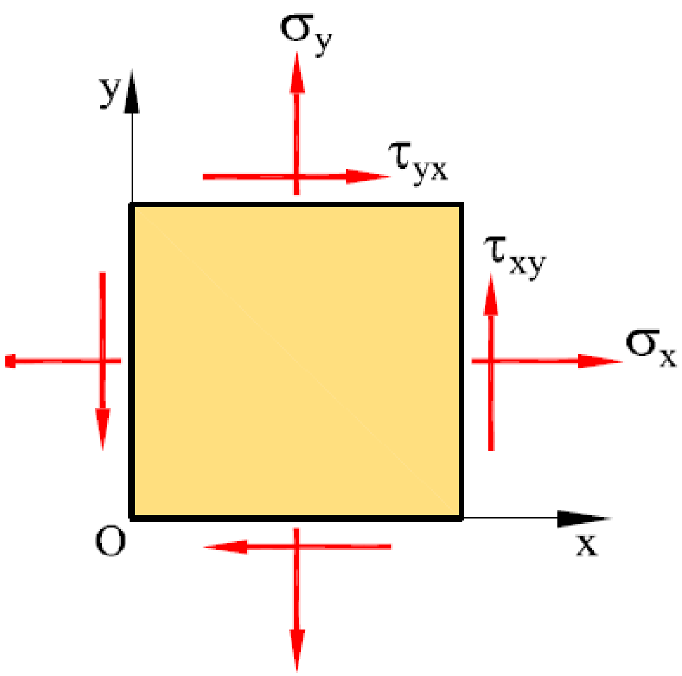
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

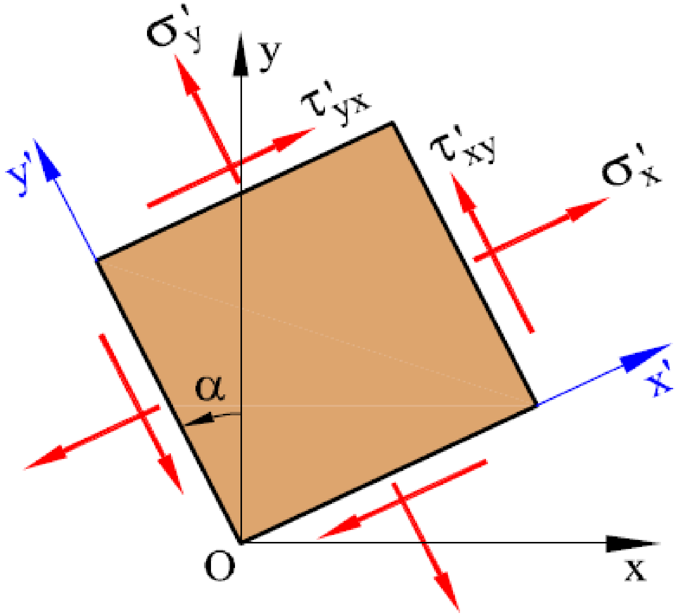
$$sen^2\theta = \frac{1-\cos 2\theta}{2}$$

$$2 \cdot sen\theta \cdot \cos\theta = sen2\theta$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta - \tau_{xy} \cdot sen2\theta$$

$$\tau = \left(\frac{\sigma_x - \tau_y}{2}\right) \cdot sen2\theta + \tau_{xy} \cdot \cos 2\theta$$





$$\boldsymbol{N} = \begin{bmatrix} \cos(x'x) & \cos(y'x) \\ \cos(x'y) & \cos(y'y) \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} l & -m \\ m & l \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{x'} & \tau_{y'x'} \\ \tau_{x'y'} & \sigma_{y'} \end{bmatrix} = \begin{bmatrix} l & m \\ -m & l \end{bmatrix} \begin{bmatrix} \sigma_x & \tau_{yx} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{bmatrix} l & -m \\ m & l \end{bmatrix}$$

$$\boldsymbol{T'} = \mathbf{N}^T \boldsymbol{T}\, \boldsymbol{N} \qquad \boldsymbol{T} = \mathbf{N}\, \boldsymbol{T'} \mathbf{N}^T$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta - \tau_{xy} \cdot sen 2\theta$$

$$\tau = \left(\frac{\sigma_x - \tau_y}{2} \right) \cdot sen 2\theta + \tau_{xy} \cdot \cos 2\theta$$

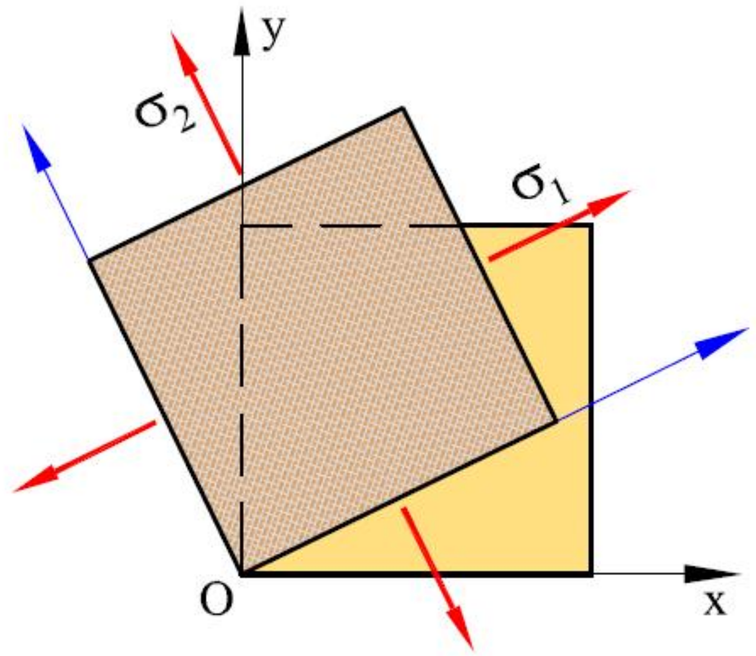
$\frac{d\sigma}{d\theta}=0$ para las tensiones normales máxima y mínima será:

$$tg 2\theta = \frac{-2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$\frac{d\tau}{d\theta}=0$ para la tensión tangencial

$$tg 2\theta = \frac{\sigma_x - \sigma_y}{2 \cdot \tau_{xy}}$$

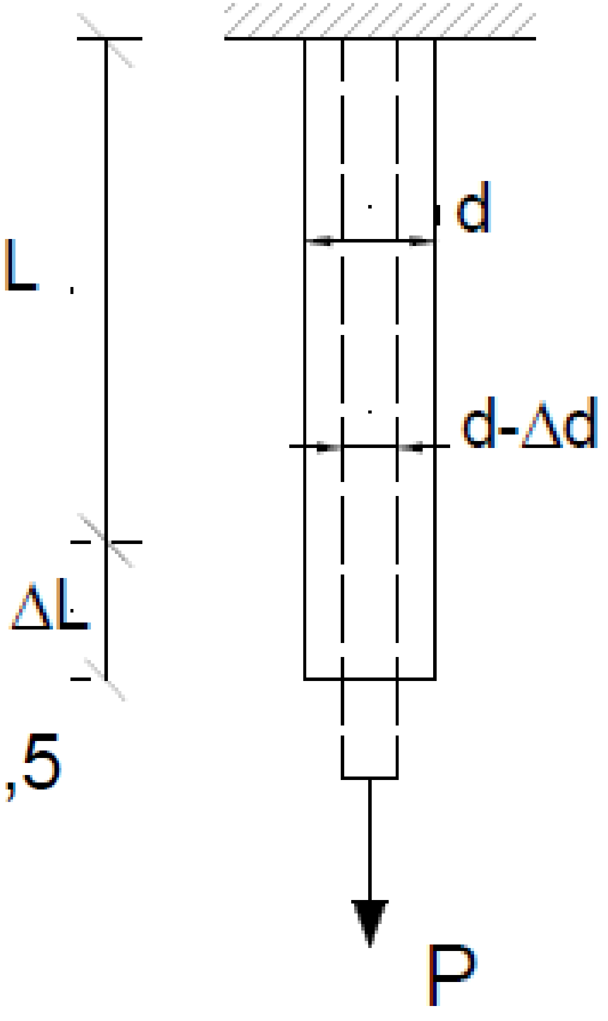
$$\sigma_{min}^{máx} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$\tau_{máx} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

DEFORMACIÓN TRASVERSAL- COEFICIENTE DE POISSON.

deformación longitudinal unitaria será: $\varepsilon = \frac{\Delta l}{l}$



deformación trasversal unitaria será: $\varepsilon_t = \frac{\Delta d}{d}$

$$\boxed{\mu = \frac{\varepsilon_t}{\varepsilon}}$$

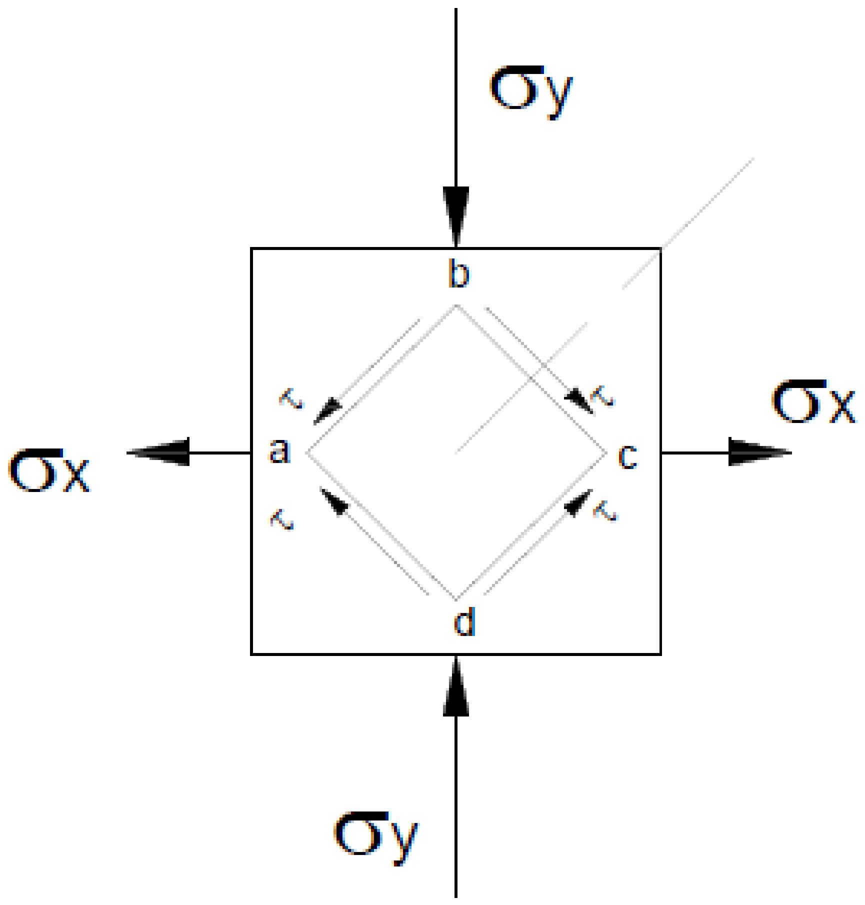
materiales isótropos μ varía entre 0 y 0,5

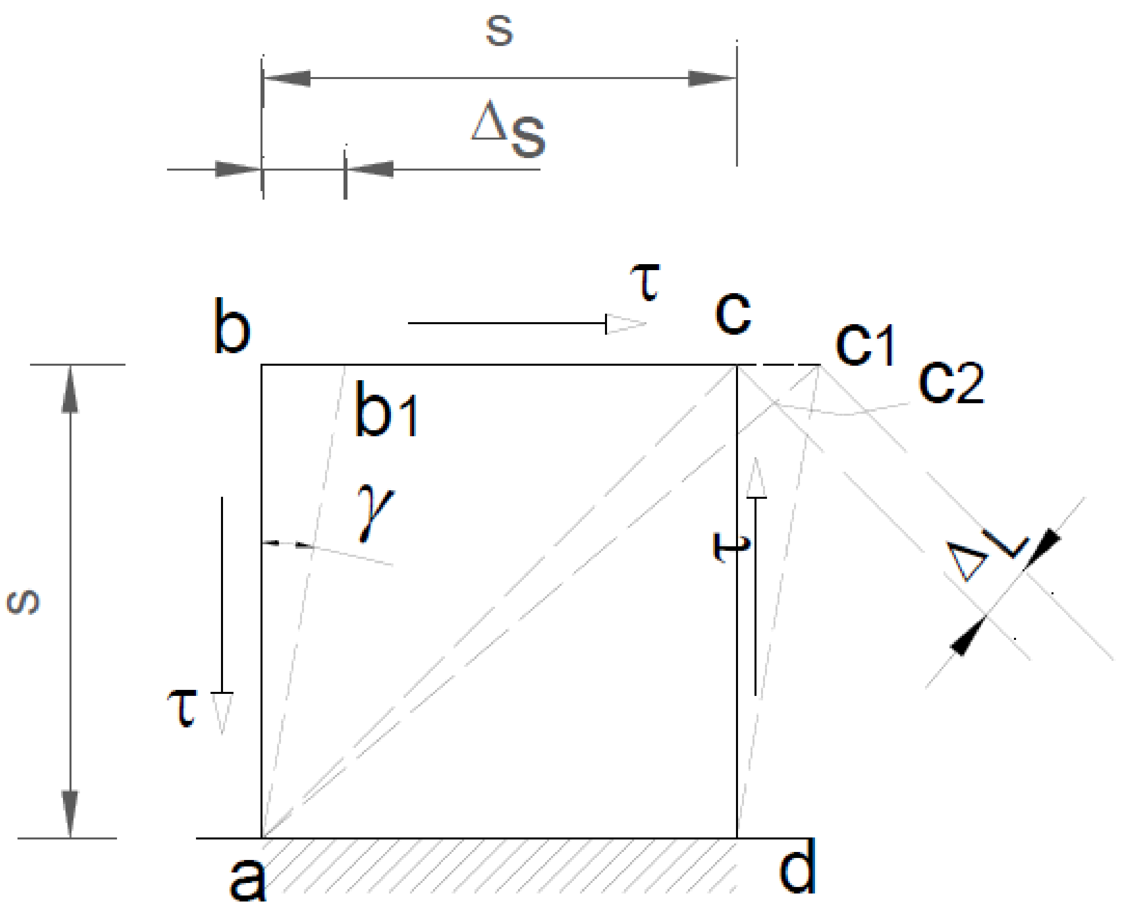
cerca de 0 para el corcho

0,5 para el caucho

acero $\mu \approx 0,3$

TENSIÓN CORTANTE PURA





$$tg\gamma = \frac{\Delta s}{s} \qquad tg\gamma \cong \gamma = \frac{\Delta s}{s}$$

$$\Delta l = cc_1 \cdot \cos\left(\frac{\pi}{4} - \frac{\gamma}{2}\right) \cong cc_1 \cdot \cos 45^{o} = \frac{\Delta s}{\sqrt{2}}$$

$$\varepsilon = \frac{\Delta l}{l} = \frac{\Delta s}{2 \cdot s} = \frac{\gamma}{2}$$

$$\varepsilon = \frac{\tau}{2 \cdot G}$$

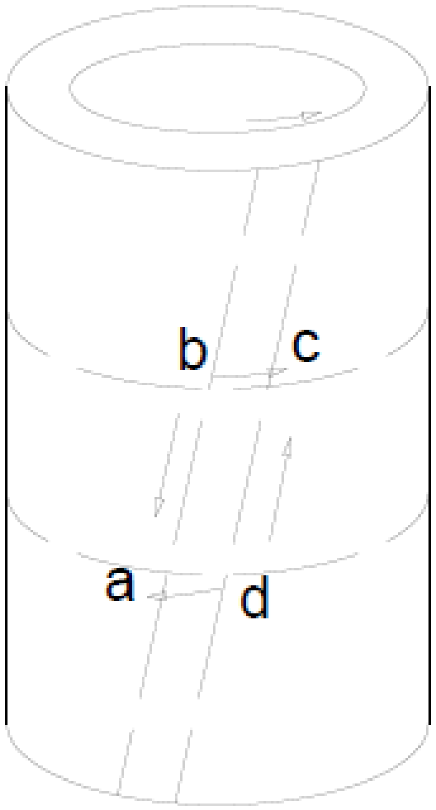
$$\sigma_x = -\sigma_y = \tau$$

en la dirección x: $\varepsilon_x = \frac{\sigma_x}{E} - \mu \cdot \frac{\sigma_y}{E}$

en la dirección y: $\varepsilon_y = \frac{\sigma_y}{E} + \mu \cdot \frac{\sigma_x}{E}$

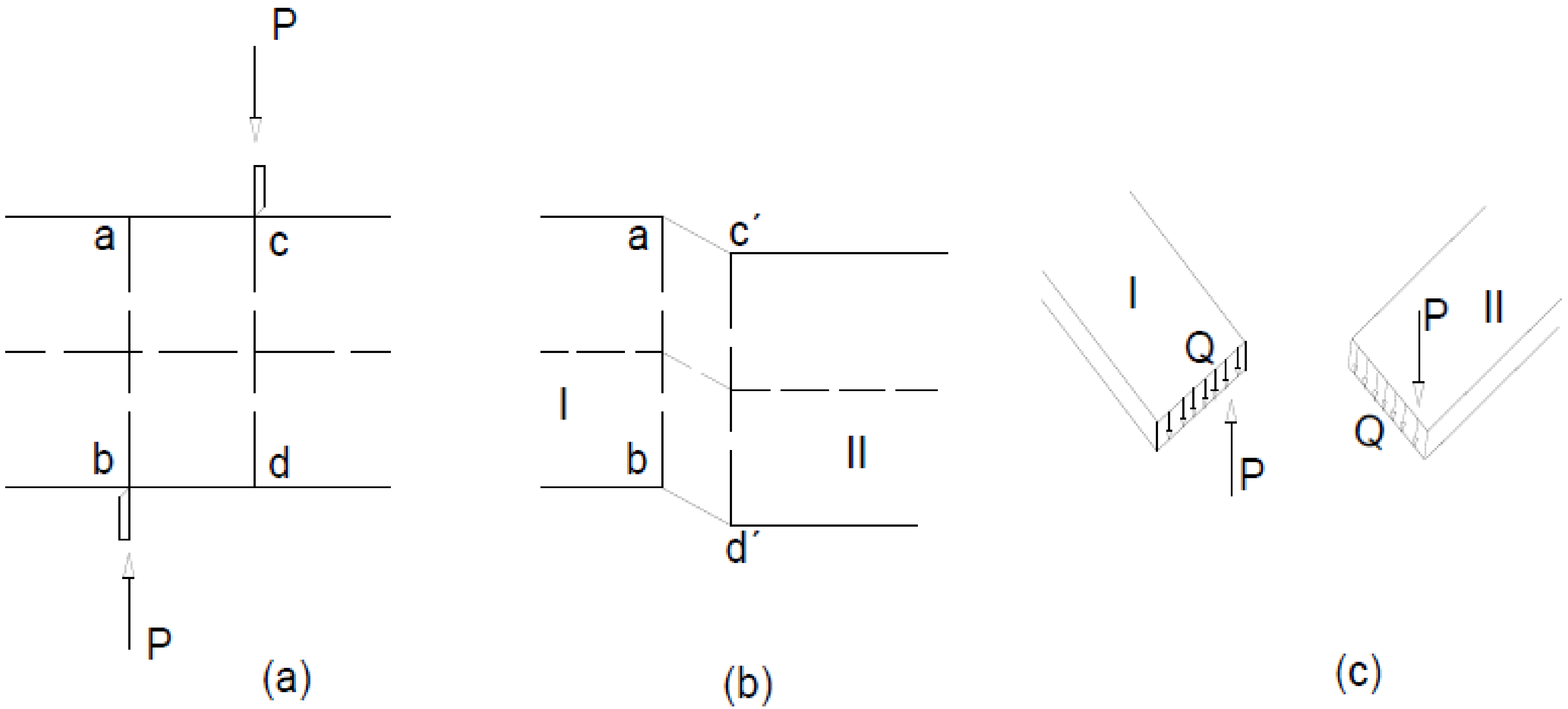
$$\varepsilon_x = \varepsilon_y = \frac{\tau}{E} \cdot (1 + \mu)$$

$$\boxed{G = \frac{E}{2 \cdot (1 + \mu)}}$$



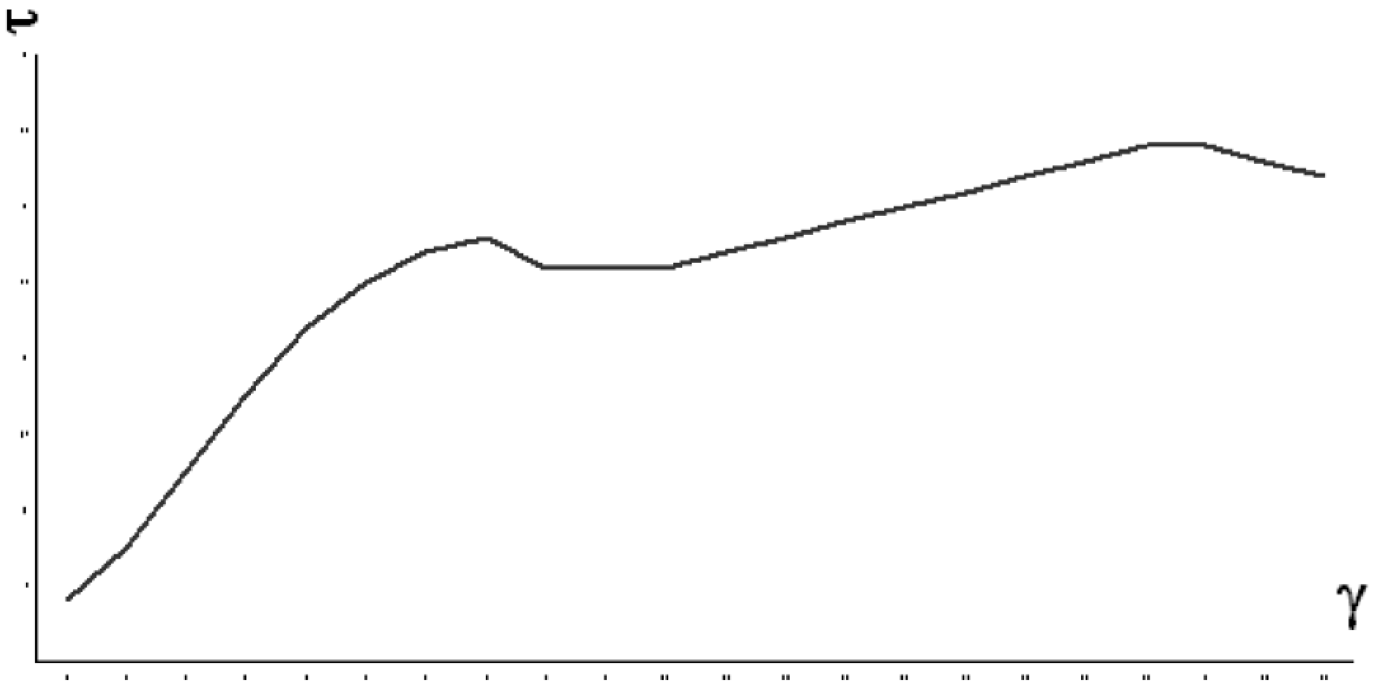
$$\boxed{\mu = \frac{E}{2 \cdot G} - 1}$$

ESFUERZO CORTANTE



Q = P siendo $Q = \int_A \tau \cdot dA$

$$\tau = \frac{Q}{A}$$



$$\tau_{ADM} = \frac{\tau_F}{s}$$

$$\tau_{ADM} = 0{,}55 \text{ á } 0{,}60\ \sigma_{ADM}$$